Les démonstrations des propositions énoncées sont presque identiques à celles des propositions 2-4 du travail [2] et c'est pourquoi nous les laissons.

Articles cités

[1] H. Pidek, *Sur les objets géométriques de la classe zéro qui admettent une algèbre*, Ann. Soc. Pol. Math. 24 (1952-3), p. 111-128.

[2] — *Sur un problème de l'algèbre des objets géométriques de classe zéro dans l'espace* X_1, Ann. Pol. Math., ce volume, p. 114-126.

[3] S. Gołąb, *O obiektach geometrycznych nieróżniczkowych*, Bull. Intern. de l'Acad. Pol. des Sc. (1949), p. 67-72.

A uniqueness theorem for the solution of a family of hyperbolic integro-differential equations

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In this note we shall give a proof of a uniqueness theorem for the two-parameter family of the integro-differential equations of string vibrations:

$$\frac{\partial^2}{\partial t^2}\int_a^b u(t,s)\,ds = u_s(t,b)-u_s(t,a)\ ^{1)} \tag{1}$$

i. e. we shall show, that *in the triangle* T $(0\leqslant t<k,\ |x|<k-t)$ $(k>0)$ *there is at most one function* $u(t,x)$ *of class* C^1 *such that the initial conditions*

$$u(0,x)=\varphi(x), \tag{2}$$

$$u_t(0,x)=\psi(x) \tag{3}$$

(*where* $\varphi(x),\psi(x)$ *are arbitrary functions defined for* $|x|<k$) *and equations* (1) (*for* $0\leqslant t<k$ *and for every pair of the real numbers* a,b *such that* $|a|<k-t$; $|b|<k-t$) *are satisfied.*

This theorem is the answer to the question put by T. Ważewski. Another proof of that theorem is included in the paper cited above.

The equations (1) are linear, hence for this purpose it will be enough to prove that the function of class C^1 satisfying the equations (1) in the triangle T (*i. e.* for $0\leqslant t<k$, $|a|<k-t$, $|b|<k-t$) and the initial conditions

$$u(0,x)=0, \tag{2'}$$

$$u_t(0,x)=0 \tag{3'}$$

vanishes identically in the triangle T.

[1]) Physical arguments, by which this equation of string vibrations is obtained are included in the paper: T. Ważewski, *Sur une relation entre la façon de la mise en équation du problème physique avec la notion des solutions généralisées des équations aux dérivées partielles du second ordre*, Bulletin de l'Académie Polonaise des Sciences, cl. trois., I (1953), p. 79-82.

Let us introduce the following notation:

(4) $$v^x(t,y)=\int_{x-y}^{x+y} u(t,s)\,ds, \qquad |x|<k, \quad 0\leqslant t<k-|x|, \quad |y|<k-|x|-t,$$

where the function $u(t,x)$ is an arbitrary solution of the equations (1) of class C^1 in the triangle T satisfying the initial conditions (2'), (3'). Let us substitute into the equation (1) $a=x-y$, $b=x+y$. We obtain the equality

$$\frac{\partial^2}{\partial t^2}v^x(t,y)=u_s(t,x+y)-u_s(t,x-y)$$

which by using the identity

$$u_s(t,x+y)-u_s(t,x-y)=\frac{\partial^2}{\partial y^2}\int_{x-y}^{x+y} u(t,s)\,ds=\frac{\partial^2}{\partial y^2}v^x(t,y)$$

implies that the function $v^x(t,y)$ of class C^2 [2]) satisfies, the partial differential equation of the hyperbolic type

$$\frac{\partial^2}{\partial t^2}v^x(t,y)=\frac{\partial^2}{\partial y^2}v^x(t,y)$$

in the triangle $0\leqslant t<k-|x|$, $|y|<k-|x|-t$.

The function $v^x(t,y)$ satisfies also (in consequence of (4), (2'), (3')) the initial conditions

$$\left.\begin{aligned} v^x(0,y)&=0\\ v^x_t(0,y)&=0\end{aligned}\right\} \quad \text{for} \quad |y|<k-|x|;$$

hence in virtue of the classical uniqueness theorem for the equation of string vibrations we have the equality

$$v^x(t,y)\equiv 0 \quad \text{for} \quad 0\leqslant t<k-|x|;\quad |y|<k-|x|-t,$$

and using the formula (4)

$$\int_{x-y}^{x+y} u(t,s)\,ds=0 \quad \text{for} \quad |x|<k, \quad 0\leqslant t<k-|x|, \quad |y|<k-|x|-t.$$

[2]) $\partial^2 v/\partial y^2$, $\partial^2 v/\partial y\partial t$ are continuous in virtue of the formula (4), $\partial^2 v/\partial t^2$ is continuous in virtue of the formula (4) and equations (1), since the function $u(t,x)$ is of class C^1.

The last formula divided by $2y$ gives for $y\to 0$

$$u(t,x)=0 \quad \text{for} \quad 0\leqslant t<k, \quad |x|<k-t$$

which was to be proved.

This method is of use also for hyperbolic equations of a more general type.