THEOREM 21. *If R is equicontinuous on $\overline{xR}$ and $xR_\omega \cap xR_a \neq 0$, then $\overline{xR} = xR_\omega = xR_a$ and R is recurrent at x.*

Proof. Assume $xR_\omega \neq 0$ and let $y \in xR_\omega$, $\varepsilon > 0$ be arbitrary. By hypothesis there is a $\delta > 0$ such that whenever $\varrho(y, u) < \delta$, then $\varrho(yr, ur) < \varepsilon$ for each $r \in R$. Since $y \in xR_\omega$, there is an unbounded increasing positive sequence $\{r_n\}$ such that $\varrho(y, xr_n) < \delta$ from which it follows that

$$\varrho[y(-r_n), xr_n(-r_n)] = \varrho[y(-r_n), x] < \varepsilon,$$

so that it is seen that $x \in yR_a \subset \overline{yR}$. Now since $y \in xR_\omega$, a closed and invariant set, then $\overline{yR} \subset xR_\omega$, and therefore $x \in xR_\omega$ which is also closed and invariant so that $\overline{xR} \subset xR_\omega$, and it follows that $\overline{xR} = xR_\omega$.

Since $x \in xR_\omega$, then $x \in xR_a$ from Theorem 19, and similarly $\overline{xR} = xR_a$. The proof is identical in case $xR_a \neq 0$.

All of the results in this section are established in the same manner when G is any simply ordered group, with appropriate modifications of the definitions.

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PROBLEMS ON SEMIGROUPS

BY

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P 326. Is it possible to construct a continuous associative multiplication on the closed n-cell ($n \geqslant 2$) such that the boundary consists of exactly those elements satisfying $x^2 = x$?

P 327. Is it possible to construct a continuous associative multiplication on an n-sphere in such a way that (i) every element is the product of two elements, (ii) there is a zero-element.

For $n = 1$ the answer is negative, see [3].

P 328. If G is a compact totally disconnected metrizable group does there exist a compact connected-acyclic one-dimensional metrizable space T and on T a continuous associative multiplication with a two-sided unit such that the maximal subgroup of T which contains the unit coincides with G and such that G is the set of endpoints of T?

If G is the Cantor group the answer is affirmative (unpublished). A related question has been considered and solved by Koch and McAuley (also unpublished).

P 329. Suppose that Euclidean n-space R^n is supplied with a continuous associative multiplication with unit and that there exists a compact connected subset G of R^n which contains the unit and which is a subgroup of R^n under the given multiplication. Is it possible that G can be "self-linked" in any reasonable way? (Cf. [1] for $n = 3$).

P 330. If S is a compact connected locally connected metrizable one-dimensional semigroup with unit, then it is known that S is either a dendrite or contains exactly one simple closed curve which coincides with the minimal ideal of S. (The details of the proof are unpublished but see [6]). Is there an analogous proposition for higher dimensions?

P 331. If S is a compact connected commutative semigroup with unit, all of whose elements satisfy $x^2 = x$, does S have the fixed point property?

P 332. If S is a compact semigroup then the minimal ideal K of S is a retract of S in the sense of Borsuk (see [9]). Examples will show that K need not be a deformation retract of S even if S has a unit, but in this case it is known that K and S have the same cohomology (see [5]). Does this last result hold if the assumption that S have a unit is replaced by the assumption that $S = ESE$ where E is the set of those elements satisfying $x^2 = x$?

P 333. It is a corollary to the result in P 332 that a compact connected semigroup with zero and unit is unicoherent. Is there a proof of this using only set-theoretic topology? A similar question arises concerning the result stated in P 330.

P 334. Suppose that S is a compact semigroup and let B denote the "boundary" of S is some suitable sense. For example, S might be homeomorphic with a subset of Euclidean n-space and B might be the ordinary boundary of S. The set B is known to play an important part in the determining the properties of S. (See [7] and [4].)

(a) If every element of S has a square-root in S does every element of B have a square-root in B (Problem of H. H. Corson)?

(b) Under some interpretations of "boundary" it is known that if S has a unit, then the unit lies in B (see [8]). Are there other useful interpretations of "boundary" for which this is true?

(c) If one assumes that the multiplication is commutative on B, are there agreeable conditions under which it may be shown to be commutative on S? (Cf. [2], where S is a dendrite and B is the set of endpoints of S.)

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НЕКОТОРЫЕ ЗАМЕЧАНИЯ О τ-КОЛЬЦАХ

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В этом сообщении даются некоторые замечания, относящиеся к τ-кольцам, рассматриваемым автором в работе [2], т. е. к кольцам, которые, вообще говоря, не предполагаются ассоциативными и в которых существует такой элемент τ, что равенства

(i) $$x(yz) = (x(\tau z))y,$$

(ii) $$\tau(\tau x) = x$$

выполняются для всех x, y и z, принадлежащих к данному кольцу.

В работе [2] было доказано, что *если R есть τ-кольцо, то τ не является левым делителем нуля в R*; в то же время *τ является правой единицей кольца, притом единственной*, откуда следует, что *в R может существовать лишь один элемент τ, удовлетворяющий условиям* (i) *и* (ii). Там-же было доказано, что *для любых $x, y, z \in R$*

(1) $$\tau(xy) = yx,$$

(2) $$(xy)z = x(z(\tau y)).$$

Основные результаты, полученные в [2], заключаются в следующем:

Если $R_\circ$ ассоциативное кольцо с инволюцией (см. [3] или [5]), *содержащее единицу, и если $\mathcal{K}(R_\circ)$ обозначает множество $R_\circ$ с обычным в кольце $R_\circ$ сложением и умножением*

(3) $$xy = y^* \circ x,$$

где $\circ$ обозначает умножение в $R_\circ$, а $*$ — инволюцию, то $\mathcal{K}(R_\circ)$ является τ-кольцом, в котором роль τ играет единица кольца $R_\circ$.

Далее доказывается теорема о точном представлении τ-колец: *для каждого τ-кольца R можно построить такое кольцо $R_\circ$, обладающее вышеуказанными свойствами, что $R = \mathcal{K}(R_\circ)$.*