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STUDIA MATHEMATICA, T. XX. (1961)

Extinguishing a class of functions

by

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Let E be a set of real positive numbers. By $L(E)$ we shall denote the family of all intervals of the form

$$I = \{(x, y): ax+y = t,\ x \geqslant 0,\ y \geqslant 0\},$$

where $a \in E$ and $0 < t < \infty$. A complex-valued continuous function φ of two variables defined on the first quadrant is said to be *extinguished* by the set E if $\int_I \varphi(x, y)\, ds = 0$ for any interval $I \in L(E)$. It is well known ([2], p. 63) that

(*) *The unique function extinguished by the right half-line is the function identically equal to* 0.

Let $\mathcal{A}_n$ denote the class of all complex-valued functions φ of two variables defined on the first quadrant and having the representation

$$\varphi(x, y) = \sum_{j=1}^{n} f_j(x) g_j(y),$$

where all the functions $f_1, f_2, \ldots, f_n, g_1, g_2, \ldots, g_n$ are continuous on the right half-line. By $\mathfrak{E}_n$ we shall denote the class of all sets E of positive numbers such that the unique function belonging to $\mathcal{A}_n$ and extinguished by E is the function identically equal to 0. From Titchmarsh's Theorem on convolution ([3], p. 327) it follows that all one-point sets belong to $\mathfrak{E}_1$. Indeed, if a function φ is extinguished by a set $\{a\}$ and $\varphi(x, y) = f(x) g(y)$, then we have the equality

$$\int_{ax+y=t} f(x) g(y)\, ds = 0 \qquad (t > 0).$$

Hence for any positive t we get the equality

$$\int_0^t f(x) g\big(a(t-x)\big)\, dx = 0,$$

which, according to Titchmarsh's Theorem, implies either $f(x) = 0$ for $x \geqslant 0$ or $g(y) = 0$ for $y \geqslant 0$. Thus $\varphi(x, y)$ vanishes in the whole first quadrant.

Let P_n denote the least power of sets belonging to $\mathfrak{E}_n$, i. e. $P_n = \min_{E \in \mathfrak{E}_n} \overline{\overline{E}}$, where $\overline{\overline{E}}$ is the power of the set E. We have proved above that $P_1 = 1$. The aim of our note is to prove the inequality

$$(**) \qquad n < P_n \leqslant \tfrac{1}{2}(n^2 - n + 4) \qquad (n \geqslant 2),$$

which for $n = 2$ implies the equality $P_2 = 3$.

In the proof of inequality (**) Mikusiński's Operational Calculus will be used [1].

Let us consider the set of all complex-valued continuous functions defined on the right half-line. This set is a commutative ring with respect to usual addition and convolution as multiplication:

$$(fg)(t) = \int_0^t f(x) g(t-x) dx.$$

By Titchmarsh's Theorem on convolution the ring in question has no divisors of zero. Therefore it can be extended to a quotient field. The elements of that quotient field are called operators.

For any positive number α we put

$$(1) \qquad f^\alpha(t) = f(\alpha t).$$

Let us introduce a family of transformations T^α $(0 < \alpha < \infty)$ defining them for every operator $a = \frac{f}{g}$, where f and g are continuous functions, by the equality

$$T^\alpha a = \frac{f^\alpha}{g^\alpha}.$$

It is easy to verify that this definition does not depend on the choice of the representation of the operator by a quotient of continuous functions. Moreover, we have the equalities

$$(2) \qquad T^{\alpha\beta} a = T^\alpha(T^\beta a) = T^\beta(T^\alpha a),$$

$$(3) \qquad T^1 a = a,$$

$$T^\alpha(ab) = T^\alpha a \cdot T^\alpha b$$

for all operators a and b and all positive numbers α and β.

A system $\alpha_1, \alpha_2, \ldots, \alpha_n$ of positive numbers is said to be *independent* if from the equality $\alpha_1^{m_1} \alpha_2^{m_2} \cdot \ldots \cdot \alpha_n^{m_n} = 1$, where $m_1, m_2, \ldots, m_n$ are integers, follows the equality $m_1 = m_2 = \ldots = m_n = 0$.

LEMMA 1. *The only invariant operators under two transformations T^α and T^β, where α and β are independent, are constant operators.*

Proof. Let us assume that an operator a satisfies the equalities $T^\alpha a = a$ and $T^\beta a = a$, where α and β are independent numbers. By properties (2) and (3), the operator a satisfies also the equalities

$$(4) \qquad T^{\alpha^k \beta^s} a = a \qquad (k, s = 0, \pm 1, \pm 2, \ldots).$$

Writing the operator a in the form $\frac{f}{g}$, where f and g are continuous functions and g is not identically equal to 0, and using notation (1) we have, according to (4), the following equalities:

$$\frac{f^{\alpha^k \beta^s}}{g^{\alpha^k \beta^s}} = \frac{f}{g} \qquad (k, s = 0, \pm 1, \pm 2, \ldots),$$

or

$$(5) \qquad g f^{\alpha^k \beta^s} - f g^{\alpha^k \beta^s} = 0 \qquad (k, s = 0, \pm 1, \pm 2, \ldots).$$

It is easy to see that for any continuous function h the convergence to γ of a sequence $\gamma_1, \gamma_2, \ldots$ of positive numbers implies the convergence to h^γ, uniform in every finite interval, of the sequence $h^{\gamma_1}, h^{\gamma_2}, \ldots$ Since for independent α and β the set $\{\alpha^k \beta^s : k, s = 0, \pm 1, \pm 2, \ldots\}$ is dense on the right half-line, we have according to (5) $g f^\lambda - f g^\lambda = 0$ for each positive number λ. This means that

$$(6) \qquad \int_0^t \Big(g(x) f\big(\lambda(t-x)\big) - f(x) g\big(\lambda(t-x)\big)\Big) dx = 0$$

for all positive t and λ. Introducing the auxiliary function

$$(7) \qquad \varphi(x, y) = g(x) f(y) - f(x) g(y),$$

we have, according to (6),

$$\int_{\lambda x + y = t} \varphi(x, y) ds = 0$$

for every positive t and λ, In other words, the function φ is extinguished by the right half-line. Thus, by theorem (*),

$$(8) \qquad \varphi(x, y) = 0 \text{ in the first quadrant.}$$

We have assumed that the function g is not identically equal to 0. Let y_0 be a positive number for which $g(y_0) \neq 0$. From (7) and (8) we get the equality $f(x) = \frac{f(y_0)}{g(y_0)} g(x)$ for any non-negative x. Thus, $a = \frac{f(y_0)}{g(y_0)}$, which proves that a is a constant operator.

For every system $a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n$ of operators we shall denote by $\Lambda(a_1, a_2, \ldots, a_n; b_1, b_2, \ldots, b_n)$ the set of all positive numbers λ for which the equality $\sum_{j=1}^{n} a_j T^\lambda b_j = 0$ holds. Further, for any pair α and β of positive numbers we put $E_1(\alpha, \beta) = \{1\}$ and $E_n(\alpha, \beta) = \{\alpha^k \beta^s\colon k \geqslant 0, s \geqslant 0, k+s \leqslant n-2$ or $k = 0, s = n-1$ and $s = 0, k = n-1\}$ if $n \geqslant 2$. For example, $E_2(\alpha, \beta) = \{1, \alpha, \beta\}$, $E_3(\alpha, \beta) = \{1, \alpha, \beta, \alpha^2, \beta^2\}$.

LEMMA 2. *If $E_n(\alpha, \beta) \subset \Lambda(a_1, a_2, \ldots, a_n; b_1, b_2, \ldots, b_n)$ and $n \geqslant 2$, then both $\alpha\beta^{n-2}$ and $\alpha^{n-2}\beta$ belong to $\Lambda(a_1, a_2, \ldots, a_n; b_1, b_2, \ldots, b_n)$.*

Proof. For $n = 2$ our assertion is obvious because $\alpha\beta^{n-2} = \alpha \in E_2(\alpha, \beta)$ and $\alpha^{n-2}\beta = \beta \in E_2(\alpha, \beta)$. Therefore we may suppose that $n \geqslant 3$. Moreover, if $a_1 = a_2 = \ldots = a_n = 0$, then every positive number belongs to $\Lambda(a_1, a_2, \ldots, a_n; b_1, b_2, \ldots, b_n)$. Consequently, we may assume that at least one operator $a_1, a_2, \ldots, a_n$ is different from 0. Hence it follows that the rank of the matrix $[T^\lambda b_j]$ $(j = 1, 2, \ldots, n;\ \lambda \in \Lambda(a_1, a_2, \ldots, a_n; b_1, b_2, \ldots, b_n))$ is not greater than $n-1$.

First let us assume that the rank of the matrix $[T^{\alpha^k} b^j]$ $(j = 1, 2, \ldots, n;\ k = 0, 1, \ldots, n-2)$ is equal to $n-1$. Since for every $\mu \in \Lambda(a_1, a_2, \ldots, a_n;\ b_1, b_2, \ldots, b_n)$ the rank of the matrix $[T^\lambda b_j]$ $(j = 1, 2, \ldots, n;\ \lambda = 1, \alpha, \alpha^2, \ldots, \alpha^{n-2}, \mu)$ is also $n-1$, there is a system of operators $c_0, c_1, \ldots, c_{n-2}$ such that

$$T^\mu b_j = \sum_{s=0}^{n-2} c_s T^{\alpha^s} b_j \quad (j = 1, 2, \ldots, n).$$

Hence we get the equality

$$T^{\alpha\mu} b_j = \sum_{s=0}^{n-2} T^\alpha c_s T^{\alpha^{s+1}} b_j \quad (j = 1, 2, \ldots, n),$$

which implies

$$\sum_{j=1}^{n} a_j T^{\alpha\mu} b_j = \sum_{s=0}^{n-2} T^\alpha c_s \sum_{j=1}^{n} a_j T^{\alpha^{s+1}} b_j = 0$$

because $\alpha, \alpha^2, \ldots, \alpha^{n-1} \in E_n(\alpha, \beta) \subset \Lambda(a_1, a_2, \ldots, a_n; b_1, b_2, \ldots, b_n)$. In other words we have got the relation $\alpha\mu \in \Lambda(a_1, a_2, \ldots, a_n;\ b_1, b_2, \ldots, b_n)$, provided $\mu \in \Lambda(a_1, a_2, \ldots, a_n; b_1, b_2, \ldots, b_n)$. In particular, $\alpha^{n-2}\beta$ and $\alpha\beta^{n-2}$ belong to $\Lambda(a_1, a_2, \ldots, a_n; b_1, b_2, \ldots, b_n)$, because for $n \geqslant 3$ $\alpha^{n-3}\beta$ and β^{n-2} belong to $E_n(\alpha, \beta)$. By symmetry it follows that $\alpha^{n-2}\beta$ and $\alpha\beta^{n-2}$ also belong to $\Lambda(a_1, a_2, \ldots, a_n;\ b_1, b_2, \ldots, b_n)$ if the rank of the matrix $[T^{\beta^s} b_j]$ $(j = 1, 2, \ldots, n;\ s = 0, 1, \ldots, n-2)$ is equal to $n-1$.

Now let us suppose that the rank of the matrices $[T^{\alpha^s} b_j]$ and $[T^{\beta^s} b_j]$ $(j = 1, 2, \ldots, n;\ s = 0, 1, \ldots, n-2)$ is smaller than $n-1$. By symmetry it suffices to show that

$$\beta\alpha^{n-2} \in \Lambda(a_1, a_2, \ldots, a_n; b_1, b_2, \ldots, b_n).$$

Since the rank of $[T^{\alpha^s} b_j]$ $(j = 1, 2, \ldots, n;\ s = 0, 1, \ldots, n-2)$ is smaller than $n-1$, there are an index $k \leqslant n-2$ and a system of operators $d_0, d_1, \ldots, d_k$, where $d_k \neq 0$, such that

$$\sum_{s=0}^{k} d_s T^{\alpha^s} b_j = 0 \quad (j = 1, 2, \ldots, n).$$

Hence we get the equality

$$(9)\quad T^{\beta\alpha^{n-2}} b_j = T^{\beta\alpha^{n-2-k}} (T^{\alpha^k} b_j) = -T^{\beta\alpha^{n-2-k}} \left(\sum_{s=0}^{k-1} \frac{d_s}{d_k} T^{\alpha^k} b_j\right)$$
$$= -\sum_{s=0}^{k-1} T^{\beta\alpha^{n-2-k}} \frac{d_s}{d_k} T^{\beta\alpha^{n-2-k+s}} b_j \quad (j = 1, 2, \ldots, n).$$

Further, taking into account the inequalities $0 \leqslant n-2-k+s \leqslant n-3$ for $0 \leqslant k \leqslant n-2$ and $0 \leqslant s \leqslant k-1$, and the definition of $E_n(\alpha, \beta)$, we have the relation $\beta\alpha^{n-2-k+s} \in E_n(\alpha, \beta)$ for $0 \leqslant k \leqslant n-2$ and $0 \leqslant s \leqslant k-1$. Hence and from (9) we get the equality

$$\sum_{j=1}^{n} a_j T^{\beta\alpha^{n-2}} b_j = -\sum_{s=0}^{k-1} T^{\beta\alpha^{n-2-k}} \frac{d_s}{d_k} \sum_{j=1}^{n} a_j T^{\beta\alpha^{n-2-k+s}} b_j = 0.$$

Thus $\beta\alpha^{n-2} \in \Lambda(a_1, a_2, \ldots, a_n;\ b_1, b_2, \ldots, b_n)$, which completes the proof of the Lemma.

Since for $n \geqslant 2$ $\alpha\lambda \in E_n(\alpha, \beta) \cup \{\alpha\beta^{n-2}\}$ and $\beta\lambda \in E_n(\alpha, \beta) \cup \{\alpha^{n-2}\beta\}$ for any $\lambda \in E_{n-1}(\alpha, \beta)$, we get, as a direct consequence of Lemma 2, the following

COROLLARY. *If $E_n(\alpha, \beta) \subset \Lambda(a_1, a_2, \ldots, a_n;\ b_1, b_2, \ldots, b_n)$ and $n \geqslant 2$, then $\alpha\lambda$ and $\beta\lambda$ belong to $\Lambda(a_1, a_2, \ldots, a_n; b_1, b_2, \ldots, b_n)$ for any $\lambda \in E_{n-1}(\alpha, \beta)$.*

LEMMA 3. *Let α and β be a pair of independent positive numbers. If the operators $b_1, b_2, \ldots, b_n$ are linearly independent with respect to the field of complex numbers and $E_n(\alpha, \beta) \subset \Lambda(a_1, a_2, \ldots, a_n;\ b_1, b_2, \ldots, b_n)$, then $a_1 = a_2 = \ldots = a_n = 0$.*

Proof. We shall prove our Lemma by induction with respect to the index n. For $n = 1$ our statement is a direct consequence of Titchmarsh's Theorem. Now let us suppose that $n \geqslant 2$ and for all indices smaller than n the statement of our Lemma is true. Further, let us suppose that not all operators $a_1, a_2, \ldots, a_n$ vanish. Without loss of generality of our considerations we may assume that $a_n \neq 0$. From the linear independence of $b_1, b_2, \ldots, b_n$ we infer that $b_n \neq 0$. Putting

$$\tilde{a}_j = \frac{a_j}{a_n}, \quad \tilde{b}_j = \frac{b_j}{b_n} \quad (j = 1, 2, \ldots, n-1),$$

we have the equalities

$$(10) \qquad \sum_{j=1}^{n-1} \tilde{a}_j T^{\lambda} b_j + T^{\lambda} b_n = 0,$$

$$(11) \qquad \sum_{j=1}^{n-1} \tilde{a}_j T^{\lambda} \tilde{b}_j + 1 = 0$$

for any $\lambda \in \Lambda(a_1, a_2, \ldots, a_n; b_1, b_2, \ldots, b_n)$. Hence, by the Corollary to Lemma 2, we get the equalities

$$(12) \qquad \sum_{j=1}^{n-1} \tilde{a}_j T^{\alpha\lambda} \tilde{b}_j + 1 = 0,$$

$$(13) \qquad \sum_{j=1}^{n-1} \tilde{a}_j T^{\beta\lambda} \tilde{b}_j + 1 = 0$$

for any $\lambda \in E_{n-1}(\alpha, \beta)$. Applying the transformations $T^{1/\alpha}$ and $T^{1/\beta}$ to equations (12) and (13) respectively, we get the following system of equations:

$$\sum_{j=1}^{n-1} T^{1/\alpha} \tilde{a}_j T^{\lambda} \tilde{b}_j + 1 = 0, \quad \sum_{j=1}^{n-1} T^{1/\beta} \tilde{a}_j T^{\lambda} \tilde{b}_j + 1 = 0,$$

for any $\lambda \in E_{n-1}(\alpha, \beta)$. Hence and from (11) we obtain the equations

$$\sum_{j=1}^{n-1} (T^{1/\alpha} \tilde{a}_j - \tilde{a}_j) T^{\lambda} \tilde{b}_j = 0, \quad \sum_{j=1}^{n-1} (T^{1/\beta} \tilde{a}_j - \tilde{a}_j) T^{\lambda} \tilde{b}_j = 0$$

for every $\lambda \in E_{n-1}(\alpha, \beta)$. Thus, by the linear independence of $\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_{n-1}$ and the induction assumption, we have the equalities $T^{1/\alpha}\tilde{a}_j = \tilde{a}_j = T^{1/\beta}\tilde{a}_j$ $(j = 1, 2, \ldots, n-1)$. The numbers $1/\alpha$ and $1/\beta$ are independent. Consequently, in view of Lemma 1, all the operators $\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_{n-1}$ are constant, i. e. are complex numbers. Hence and from (10) follows the linear dependence of the operators $b_1, b_2, \ldots, b_n$, which is impossible. The Lemma is thus proved.

Proof of inequality (**). It is very easy to verify that $\overline{\overline{E}}_n(\alpha, \beta) = \frac{1}{2}(n^2 - n + 4)$ for independent α and β and $n \geqslant 2$. Consequently, to prove the inequality

$$P_n \leqslant \tfrac{1}{2}(n^2 - n + 4) \quad (n \geqslant 2)$$

it is sufficient ot show that for independent α and β the relation

$$(14) \qquad E_n(\alpha, \beta) \in \mathfrak{E}_n \quad (n \geqslant 1)$$

holds.

For $n = 1$ the last relation is evident. Now let us suppose that $n \geqslant 2$ and $E_k(\alpha, \beta) \in \mathfrak{E}_k$ for $k < n$. Let φ be a function belonging to $\mathcal{A}_n$, extinguished by the set $E_n(\alpha, \beta)$ and having the representation $\varphi(x, y) = \sum_{j=1}^{n} f_j(x) g_j(y)$. If the functions $g_1, g_2, \ldots, g_n$ are linearly dependent, then $\varphi \in \mathcal{A}_{n-1}$ and, by the inclusion $E_{n-1}(\alpha, \beta) \subset E_n(\alpha, \beta)$, the function φ is extinguished by the set $E_{n-1}(\alpha, \beta)$. Consequently, $\varphi(x, y) = 0$ in the whole first quadrant. Finally let us suppose that the functions $g_1, g_2, \ldots, g_n$ are linearly independent. Then we have the operational equality

$$\sum_{j=1}^{n} f_j T^{\lambda} g_j = 0 \quad \text{for any} \quad \lambda \in E_n(\alpha, \beta).$$

Applying Lemma 3 we get $f_1 = f_2 = \ldots = f_n = 0$ and, consequently, $\varphi(x, y) = 0$ in the whole first quadrant. Thus we have proved relation (14).

Now we shall prove the inequality $P_n > n$ $(n \geqslant 2)$. Let E be an arbitrary n-point set: $E = \{\gamma_1, \gamma_2, \ldots, \gamma_n\}$. Put

$$g_j(x) = \sin^j \left\{ 2\pi \left(\log \frac{\gamma_1}{\gamma_2} \right)^{-1} \log x \right\} \quad (j = 1, 2, \ldots, n).$$

It is easy to see that all the functions $g_1, g_2, \ldots, g_n$ are linearly independent and

$$T^{\gamma_1/\gamma_2} g_j = \frac{\gamma_1}{\gamma_2} g_j \quad (j = 1, 2, \ldots, n).$$

Hence

$$T^{\gamma_1} g_j = \frac{\gamma_1}{\gamma_2} T^{\gamma_2} g_j \quad (j = 1, 2, \ldots, n)$$

and, consequently, the rank of the matrix $[T^{\gamma_s} g_j]$ $(j = 1, 2, \ldots, n;$ $s = 1, 2, \ldots, n)$ is smaller than n. There exists then a system of operators $a_1, a_2, \ldots, a_n$ satisfying the equalities

$$(15) \qquad \sum_{j=1}^{n} a_j T^{\gamma_s} g_j = 0 \quad (s = 1, 2, \ldots, n),$$

where at least one operator a_j $(1 \leqslant j \leqslant n)$ is different from 0. Writing the operators a_j in the form $a_j = \dfrac{f_j}{f}$ $(j = 1, 2, \ldots, n)$, where $f, f_1, f_2, \ldots, f_n$

are continuous functions, we have, according to (15), the following equalities

$$\sum_{j=1}^{n} f_j T^{\prime s} g_j = 0 \qquad (s = 1, 2, \ldots, n).$$

In other words the function

$$\psi(x, y) = \sum_{j=1}^{n} f_j(x) g_j(y)$$

is extinguished by the set E. Since not all function $f_1, f_2, \ldots, f_n$ vanish and $g_1, g_2, \ldots, g_n$ are linearly independent, $\psi(x, y)$ is not identically equal to 0 in the first quadrant. Thus $E \notin \mathfrak{E}_n$ and, consequently, $P_n > n$.

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A proof of Schwartz's theorem on kernels

by

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L. Schwartz has shown that every bilinear continuous functional $B(\varphi_1, \varphi_2)$ on the space $D(\Omega_1)\times D(\Omega_2)$ (see the definition below) may be represented by a linear continuous functional T on the space $D(\Omega_1\times\Omega_2)$, i. e.

(1) $$B(\varphi_1, \varphi_2) = T(\varphi_1\times\varphi_2) \quad \text{for} \quad \varphi_i \in D(\Omega_i), \quad i = 1, 2,$$

where $(\varphi_1\times\varphi_2)(x_1, x_2) = \varphi_1(x_1)\cdot\varphi_2(x_2)$ for $x_i \in \Omega_i$, $i = 1, 2$.

Since every such functional corresponds to a linear continuous map L of $D(\Omega_1)$ into $D'(\Omega_2)$ defined by

$$(L\varphi_1)(\varphi_2) = B(\varphi_1, \varphi_2),$$

equality (1) may be written symbolically in the form

(2) $$L(\varphi_1)(x_2) = \int T(x_1, x_2)\varphi_1(x_1)\,dx_1 \quad \text{for any} \quad \varphi_1 \in D(\Omega_1)$$

and therefore Schwartz's theorem may be interpreted as a theorem concerning representation of linear continuous operations by kernels. The theorem is a special case of a general theorem of A. Grothendieck on topological tensor products.

The purpose of this paper is to give a simple proof of Grothendieck's theorem for a special case which often occurs in applications. The proof is based only on elementary properties of (F)-spaces ((B_0)-spaces in the Polish terminology) and (LF)-spaces.

For the convenience of the reader we shall make a short review of the properties to be used in the paper.

1. Let X be a linear space over the complex field. Given a family of seminorms $\|x\|_a$ $(a \in A)$ on X, we can define a topology on X taking the family of sets $\{x: \|x-x_0\|_{a_i} < \varepsilon, i = 1, 2, \ldots, n\}$ as a fundamental system of neighbourhoods of the point x_0.

This topology is a Hausdorff topology if and only if the family of semi-norms is separating, i. e. if, for every $x \neq 0$, there is an $a \in A$ such