References

[1] H. J. Cohen, *Sur un problème de M. Dieudonné*, C. R. Acad. Sci. Paris 23 (1952), pp. 290-292.

[2] H. H. Corson, *The determination of paracompactness by uniformities*, Amer. J. Math. 80 (1958), pp. 185-190.

[3] J. Dieudonné, *Un critère de normalité pour les espaces produits*, Colloq. Math. 6 (1958), pp. 29-32.

[4] C. H. Dowker, *On countably paracompact spaces*, Can. J. Math. 3 (1951) pp. 219-224.

[5] — *On a theorem of Hanner*, Ark. Mat. 2 (1952), pp. 307-313.

[6] M. Katětov, *Extension of locally finite coverings*, Colloq. Math. 6 (1958), pp. 141-151.

[7] M. J. Mansfield, *Some generalizations of full normality*, Trans. Amer. Math. Soc. 86 (1957), pp. 487-505.

[8] E. Michael, *A note on paracompact spaces*, Proc. Amer. Math. Soc. 4 (1953), pp. 831-838.

[9] — *Continuous selections I*, Ann. of Math. 63 (1956), pp. 361-382.

[10] — *Another note on paracompact spaces*, Proc. Amer. Math. Soc. 8 (1957), pp. 822-828.

[11] K. Morita, *Star-finite coverings and the star-finite property*, Math. Japonicae, 1 (1948), pp. 60-68.

[12] — *On the dimension of normal spaces II*, J. Math. Soc. Japan, 2 (1950), pp. 16-33.

[13] — *On spaces having the weak topology with respect to a closed covering I, II*, Proc. Japan Acad. 29 (1953), pp. 537-543; 30 (1954), pp. 711-717.

[14] Y. Smirnov, *On normally disposed sets of normal spaces*, Mat. Sbornik N. S. 29 (1951), pp. 173-176.

[15] A. H. Stone, *Paracompactness and product spaces*, Bull. Amer. Math. Soc. 54 (1948), pp. 977-982.

[16] H. Tamano, *On paracompact spaces*, Pacific J. Math. 10 (1960), pp. 1043-1047.

[17] J. W. Tukey, *Convergence and uniformity in topology*, Princeton 1940.

Reçu par la Rédaction le 23. 11. 1960

A fixed point theorem for the hyperspace of a snake-like continuum

by

J. Segal (Seattle, Wash.)

Introduction. If X is a metric continuum, $C(X)$ denotes the space of subcontinua of X with the finite topology. As a partial answer to question 186 (due to B. Knaster 4/29/52) of the New Scottish Book it is shown that $C(X)$ has fixed point property if X is a snake-like continuum. This is done by showing that $C(X)$ is a quasi-complex and since $C(X)$ is acyclic (see [9]) it has fixed point property by the Lefschetz Fixed Point Theorem.

Definition 1. If G is a finite collection of open sets of X let $\Omega(G)$ denote $\{K \in C(X) \mid K \cap g \neq \emptyset$ for each $g \in G$ and $K \subset \bigcup_{g \in G} (g)\}$. The finite topology on $C(X)$ is the one generated by open sets of the form $\Omega(G.)$ (See [8], pp. 153.) If U is a finite open covering of X define U^* to be $\{\Omega(G) \mid G$ is a finite subset of $U\}$.

Lemma 1. *If U is a finite open covering of X, then U^* is a finite open covering of $C(X)$.*

Proof. The elements of U^* are open by the definition of the finite topology, and since U is finite, so is U^*. If $A \in C(X)$, there is a subcollection G of U which irreducibly covers A, so $A \in \Omega(G)$. Hence U^* covers $C(X)$.

Lemma 2. *If U is a finite collection of open sets, then* $\operatorname{mesh} U^* \leqslant \operatorname{mesh} U$.

Proof. Suppose that G is a subcollection of U and K and L are elements of $\Omega(G)$. If $x \in K$, there is an element g_x of G containing x. Given $L \cap g_x \neq \emptyset$ and $\operatorname{diam} g_x \leqslant \operatorname{mesh} U$, there is a point y of L such that $d(x, L) \leqslant \operatorname{mesh} U$. Hence for each x in K, $d(x, L) \leqslant \operatorname{mesh} U$. Therefore since $d'(K, L) = \max\big(\max_{x \in K} d(x, L), \max_{y \in L} d(y, K)\big)$, $d'(K, L) \leqslant \operatorname{mesh} U$, and hence $\operatorname{diam} G \leqslant \operatorname{mesh} U$.

Lemma 3. *If $\{U_a\}$ is a cofinal sequence of open coverings of X, then $\{U_a^*\}$ is a cofinal sequence of open coverings of $C(X)$.*

Proof. A sequence $\{U_a\}$ of open coverings of a compact space X is cofinal (in the set of all open coverings of X) if and only if $\operatorname{mesh} U_a \to 0$. By Lemma 2 if $\operatorname{mesh} U_a \to 0$ then $\operatorname{mesh} U_a^* \to 0$.

DEFINITION 2. A finite collection of open subsets of X, which covers X such that U^i intersects U^j if and only if $i = j$, $j-1$, or $j+1$ is called a *chain covering* X. If each element of the chain is of diameter less than ε, the chain is called an *ε-chain*. A continuum is called *snake-like* if for each positive number ε it can be covered by an ε-chain.

In the remainder of this paper, X will be assumed to be snake-like and $\{U_a\}$ will denote a sequence of chains covering X such that mesh $U_a \to 0$ and such that for each a, (1) the closure of each element of U_{a+1} lies in an element of U_a and each element of U_a contains an element of U_{a+1} and (2) every two non-adjacent elements of U_a have disjoint closures. (It is shown in [1], p. 654, that every snake-like continuum has such a sequence of coverings.)

Suppose that $U = (U_1, \ldots, U_n)$ is a chain covering X. For each ordered pair (i, j) of integers such that $1 \leqslant i \leqslant j \leqslant n$, let $U(i, j) = (U_i, \ldots, U_j)$ and let $V(i, j) = \Omega\big(U(i, j)\big) \in U^*$. Two elements $V(i_1, j_1)$ and $V(i_2, j_2)$ of U^* are said to be *Δ-related* provided $|i_1 - i_2| \leqslant 1$ and $|j_1 - j_2| \leqslant 1$.

LEMMA 4. (1) $U^* = \{V(i, j) \mid 1 \leqslant i \leqslant j \leqslant n\}$.

(2) *If $V \subset U^*$, then $\bigcap V \neq \emptyset$ if and only if every two elements of V are Δ-related.*

(3) *No five elements of U^* have a common point.*

Proof. (1) If $G \subset U$, then $\Omega(G) \neq \emptyset$ implies that G is a subchain of U; i.e. $G = U(i, j)$ for some (i, j).

(2) (a) Suppose that every two elements of V are Δ-related. Let $i_{\min} = \min\{i \mid \text{for some } j, V(i, j) \in V\}$, $i_{\max} = \max\{i \mid \text{for some } j, V(i, j) \in V\}$, $j_{\min} = \min\{j \mid \text{for some } i, V(i, j) \in V\}$, and $j_{\max} = \max\{j \mid \text{for some } j, V(i, j) \in V\}$. Since every two elements of V are Δ-related, $i_{\max} - i_{\min} \leqslant 1$ and $j_{\max} - j_{\min} \leqslant 1$. We need to show that there is a subcontinuum K of X such that $K \subset U_{i_{\max}} \cup \ldots \cup U_{j_{\min}}$ and K intersects each $U_{i_{\min}}, \ldots, U_{j_{\max}}$.

Let $A = U_1 \cup U_2 \cup \ldots \cup \big(U_i - (U_i \cap U_{i+1})\big)$ and $B = \big(U_j - (U_j \cap U_{j-1}) \cup U_{j+1} \cup \ldots \cup U_n\big)$ where $j+1 > i$. Since A is a closed subset of the open set $U_1 \cup \ldots \cup U_i$, there is an open set A' containing A such that $\bar{A}' \subset U_1 \cup \ldots \cup U_i$. Similarly, there is an open set B' containing B such that $\bar{B}' \subset U_j \cup \ldots \cup U_n$. There is a subcontinuum K of X irreducible from $\bar{A}'$ to $\bar{B}'$ and so K intersects $U_i \cap U_{i+1}$ and $U_j \cap U_{j-1}$ for $i \neq j$ and consequently each U_r, $i \leqslant r \leqslant j$. Then $K' = K - (K \cap \bar{A}')$ is connected. Since $X - (A' \cap U_{i+1}) = A \cup \big(U_{i+1} - (A' \cap U_{i+1})\big) \cup (U_{i+2} \cup \ldots \cup U_n) = A \cup C$, where A and C are closed and disjoint and $K' \cap C \neq \emptyset$, it follows that $K' \subset C$. Hence since $K = \bar{K}'$, $K \subset C$ and consequently $K \cap A = \emptyset$. Similarly $K \cap B = \emptyset$. Hence $K \subset U_{i+1} \cup \ldots \cup U_{j-1}$. Since the above holds for any i, j, it holds for $i_{\max}$ and $j_{\min}$.

If $V(i, j) \in V$, then $i \leqslant i_{\max}$ and $j \geqslant j_{\min}$ so $U_{i_{\max}} \cup \ldots \cup U_{j_{\min}} \subset U_i \cup \ldots \cup U_j$ and hence $K \subset U_i \cup \ldots \cup U_j$. Also, $i \geqslant i_{\min}$ and $j \leqslant j_{\max}$ so the sequence $U_i, \ldots, U_j$ is a subsequence of $U_{i_{\min}}, \ldots, U_{j_{\max}}$ and hence K intersects each of $U_i, \ldots, U_j$. Hence $K \in \Omega\big(U(i, j)\big) = V(i, j)$.

(b) Suppose that V contains two elements $V(i_1, j_1)$ and $V(i_2, j_2)$ which are not Δ-related. Then either $|i_1 - i_2| \geqslant 2$ or $|j_1 - j_2| \geqslant 2$. If $i_1 \leqslant i_2 - 2$ then $U_{i_2} \cup \ldots \cup U_{j_2}$ does not intersect U_{i_1} and hence no continuum lying in $U_{i_2} \cup \ldots \cup U_{j_2}$ can intersect U_{i_1}; consequently no element of $V(i_2, j_2)$ belongs to $V(i_1, j_1)$. The other cases are similar.

(3) Suppose $V \subset U^*$ and $\bigcap V \neq \emptyset$. Let $i_0 = \min\{i \mid \text{for some } j, V(i, j) \in V\}$ and let $j_0 = \min\{j \mid V(i_0, j) \in V\}$. Then if $V(i, j) \in V$, $i = i_0$ or $i_0 + 1$ and $j = j_0$ or $j_0 + 1$, so the only possible elements of V are $V(i_0, j_0)$, $V(i_0, j_0+1)$, $V(i_0+1, j_0)$ and $V(i_0+1, j_0+1)$.

DEFINITION 3. The *nerve* of a finite collection U of sets (denoted by $N(U)$) is an abstract complex C whose vertices are in 1-1 correspondence with the elements of U and which is such that a subset of the vertices of C is the set of vertices of a simplex of C if and only if the intersection of the corresponding elements of U is non-empty.

Remark. Let R denote the set of all lattice points of the plane lying in the region bounded by the lines $x = 1$, $y = x$, $y = n$; two points of R will be said to be Δ-related if neither their ordinates nor their abscissas differ by more than 1.

If U is a chain covering X with n elements, a 1-1 correspondence between the elements of U^* and the points of R is obtained by letting the element $V(i, j)$ of U^* correspond to the point (i, j) of R. Hence R may be considered as the set of vertices $\{a_j^i\}$ of $N(U^*)$.

A subset R' of R has the property that every two of its elements are Δ-related if and only if R' is a subset of the vertices of a unit square in the plane; hence a "topological realization" of $N(U^*)$ can be obtained by adjoining to R the solid triangle bounded by the lines $x = 1$, $y = x$ and $y = n$ together with a collection G of "topological tetrahedrons" (i.e. closed 3-cells) given that each element of G contains a solid unit square with vertices in R and every such square is contained in an element of G, and such that no two elements of G have a point in common not in the xy-plane.

The following is a special case of the Mayer-Vietoris Theorem.

LEMMA 5. ([3], p. 39) *If K, A, and B are simplicial complexes such that $K = A \cup B$ and $A, B, A \cap B$ are acyclic, then K is acyclic.*

THEOREM 1. *$N(U^*)$ is acyclic.*

Proof. If U has one element $N(U)$ is acyclic being just one vertex. Assume that the theorem is true for V which has k elements. Suppose that U has $k+1$ elements. Then $N(U^*) = N(V^*) \cup M$ where M is the simplicial complex composed of those simplexes added to $N(V^*)$ to obtain $N(U^*)$. So M is composed of $k-1$ tetrahedrons and one triangle each joined to the next along one edge. So $|M|$ is contractible and hence M is acyclic. $|N(U^*) \cap M|$ is the union of $k-1$ segments and is homeomorphic to an arc and so $N(U^*) \cap M$ is acyclic Applying Lemma 5 we have $N(U^*)$ is acyclic.

DEFINITION OF A QUASI-COMPLEX. The following definition is in [6], p. 322.

Let X be a compact space, $\{U_i \mid i \in M\}$ a cofinal set of open coverings of X. For each pair i, j of elements of M such that $i > j$ let π_{ij}: $N(U_i) \to N(U_j)$ be one of the projections induced by the inclusion relations associated with the refinement of U_j by U_i.

Further for each $j \in M$ there exists an $i \in M$ and one or more chain mappings ω_{ji}: $N(U_j) \to N(U_i)$, called antiprojections, such that

(a) $\omega_{ji}\pi_{ij} \sim 1$,

(b) if ω_{kj}, ω_{ji} are antiprojections, then so is $\omega_{ji}\omega_{kj}$,

(c) if ω_{ji} and $\bar{\omega}_{ji}$ are antiprojections, then $\omega_{ji} \sim \bar{\omega}_{ji}$.

If σ_j is a simplex of $N(U_j)$, let $[\sigma_j]$ denote the kernel of σ_j, that is, the intersection of the sets of U_j corresponding to vertices of σ_j. Further, if c^p is a chain of $N(U_j)$, let $[c^p]$ denote the union of the kernels of simplexes in the carrier of c^p.

(d) all indices being understood in M, for every i there is a j, $j > i$, and for every k an m, $m > k, j$ (m depending on i and j) such that ω_{jm} exists, satisfies (a), (b), (c), and if the simplex $\sigma_j \in N(U_j)$, then $[\sigma_j] \cup [\omega_{jm}\sigma_j]$ is contained in a set of U_i.

The collection $(X; \{U_i\}; \{\pi_{ij}\}; \{\omega_{ji}\})$ defines a quasi-complex X.

DEFINITION OF ω. The following definition of ω is in [2], p. 666. If α and β are arc-like finite simplicial complexes and π is a simplicial mapping of β onto α, there exists a chain mapping of α onto β which is defined as follows. Let $a_1, a_2, \ldots, a_n$ denote the vertices of α ordered as on α. There is a subarc β' of β such that $\pi(\beta') = \alpha$ and there is no proper subarc γ of β' such that $\pi(\gamma) = \alpha$. Let b_1 denote the vertex of β' such that $\pi(b_1) = a_1$ and let $b_1, b_2, \ldots, b_u$ denote the vertices of β' ordered as on β'. There is a subsequence $b_{k_1}, b_{k_2}, \ldots, b_{k_v}$ of $b_1, b_2, \ldots, b_u$ such that

(1) $\pi(b_{k_1}) = a_1$ and $\pi(b_{k_v}) = a_n$;

(2) if $\pi(b_{k_i}) = a_p$ and $\pi(b_{k_{i+1}}) = a_r$, then $|p-r| \leqslant 1$; and

(3) for each i, k_{i+1} is the greatest integer j such that
 (a) $k_i \leqslant j \leqslant k_v$ and
 (b) if $k_i < q \leqslant j$, $\pi(b_q) \in \{\pi(b_{k_i})\} \cup \{\pi(b_{k_{i+1}})\}$.

Define $\omega(1 \cdot a_p)$ to be $\sum_{s=1}^{v} X_p^s \cdot b_{k_s}$, where $X_p^s = 0$ if $\pi(b_{k_s}) \neq a_p$, and

$$\text{if} \quad \pi(b_{k_s}) = a_p \text{ and } \begin{cases} \pi(b_{k_{s+1}}) = a_{p+1}, & \text{then} \quad X_p^s = +1, \\ \pi(b_{k_{s+1}}) = a_{p-1}, & \text{then} \quad X_p^s = -1, \\ s = v, & \text{then} \quad X_p^s = +1. \end{cases}$$

Define $\omega\big(1 \cdot (a_p a_{p+1})\big) = \sum_{s=1}^{u-1} Y_p^s \cdot (b_s b_{s+1})$, where if $k_\sigma \leqslant s < s+1 \leqslant k_{\sigma+1}$ and for all θ, $k_\sigma \leqslant \theta \leqslant k_{\sigma+1}$, $\pi(b_\theta) \in \{a_p\} \cup \{a_{p+1}\}$, then $Y_p^s = +1$; otherwise $Y_p^s = 0$.

DEFINITION OF ω_{ji}. Let ω_{ji} denote the chain mapping of $N(U_j)$ onto $N(U_i)$ defined for π_{ij} in the preceding definition. ω_{ji} and finite products $\omega_{i_{n-1}i_n} \ldots \omega_{i_2i_3}\omega_{i_1i_2}$, where $i_1 < i_2 < \ldots < i_n$, are antiprojections and satisfy (a), (b), (c) and (d) (see Definition of quasi-complex). Moreover, ω_{ji} is an algebraic map.

DEFINITION OF π_{ji}^*. Since X is a snake-like continuum, we have by [2], pp. 666-667 that $(X; \{U_i\}; \{\pi_{ij}\}; \{\omega_{ji}\})$ defines a quasi-complex X. For simplicity we now write a_p^p as a_p and use i, j as indexes on the coverings. We define an extension of π_{ij}, π_{ij}^*: $C^p\big(N(U_i^*)\big) \to C^p\big(N(U_j^*)\big)$. For each member of U_i^* we select a member of U_j^* containing it. This gives a simplicial mapping of $N(U_i^*)$ in $N(U_j^*)$ which is a projection. First we specify a particular projection π_{ij}' as follows. Let $\psi(s)$ be the subscript of the vertex in $N(U_j)$ which is the image of b_s under π_{ij}, i.e. $\pi_{ij}(b_s) = a_{\psi(s)}$. Let $\eta(q, s) = \min[\psi(\xi) \mid q \leqslant \xi \leqslant s]$ and $\mu(q, s) = \max[\psi(\xi) \mid q \leqslant \xi \leqslant s]$. Now we define a simplicial set transformation π_{ij}': $N(U_i^*) \to N(U_j^*)$ by

$$\pi_{ij}'(b_s^q) = a_{\mu(q,s)}^{\eta(q,s)} \quad \text{and} \quad \pi_{ij}'(b_{s_1}^{q_1} \ldots b_{s_t}^{q_t}) = a_{\mu(q_1,s_1)}^{\eta(q_1,s_1)} \ldots a_{\mu(q_t,s_t)}^{\eta(q_t,s_t)}.$$

If $K \in \Omega(U_j^q, \ldots, U_j^s)$ then $K \subset \bigcup_{\xi=q}^{s} U_j^\xi$ and $K \cap U_q^\xi \neq \emptyset$ for $q \leqslant \xi \leqslant s$. Since $\pi_{ij}(b_\xi) = a_{\psi(\xi)}$, $U_j^\xi \subset U_i^{\psi(\xi)}$ for each ξ. Hence $K \in \Omega(U_i^{\eta(q,s)}, \ldots, U_i^{\mu(q,s)})$ and $\Omega(U_j^q, \ldots, U_j^s) \subset \Omega(U_i^{\eta(q,s)}, \ldots, U_i^{\mu(q,s)})$. Therefore π_{ij}' is induced by one of the inclusion relations associated with the refinement of U_j^* by U_i^*. π_{ij}' induces a chain mapping π_{ij}^* on $N(U_i^*)$, i.e. for a simplex $b_{s_1}^{q_1} \ldots b_{s_t}^{q_t}$ we have

$$\pi_{ij}^*(b_{s_1}^{q_1} \ldots b_{s_t}^{q_t}) = \begin{Bmatrix} a_{\mu(q_1,s_1)}^{\eta(q_1,s_1)} \ldots a_{\mu(q_t,s_t)}^{\eta(q_t,s_t)} & \text{when the } a_\mu^\eta\text{'s are distinct} \\ 0 & \text{otherwise} \end{Bmatrix}.$$

So π_{ij}^* is actually an algebraic map of $N(U_i^*)$ onto $N(U_j^*)$ which is an extension of π_{ij} and the carrier of π_{ij}^* is π_{ij}' (see [6], p. 146, (9.13)).

DEFINITION 4. If v is a vertex of a simplicial complex K, then the $\overline{\mathrm{St}}(v)$ is the subcomplex of K consisting of all simplexes having v as a vertex and all faces of such simplexes.

DEFINITION 5. If $U = \{U^i\}$ is a finite covering of a continuum X and v_i is the vertex associated with U^i in $N(U)$, then the $\overline{\mathrm{St}}(U^i) = \bigcup\{U^j \mid U^j \in U \text{ and } v_j \in \overline{\mathrm{St}}(v_i)\}$.

Notation. In the following $K = N(U_j^*)$, $L = N(U_j)$, $K' = N(U_i^*)$, and $L' = N(U_i)$, where U_i is finer than U_j so that π_{ij}: $C^q(N(U_i)) \to C^p(N(U_j))$ and ω_{ji}: $C^p(N(U_j)) \to C^p(N(U_i))$.

DEFINITION OF c. Define c mapping simplexes of L onto subcomplexes of L' by $c(a_p) = \bigcup\{\overline{\mathrm{St}}(b_q) \mid I(p) \leqslant q \leqslant R(p)\}$ where $I(p) = \min\{k \mid \pi(b_k) = a_p\}$, $R(p) = \max\{k \mid \pi(b_k) = a_p\}$, the $\overline{\mathrm{St}}(b_q)$ is taken in L' and the a's are vertices of L and the b's are vertices of L'. Also $c(a_pa_{p+1}) = c(a_p) \cup c(a_{p+1})$.

LEMMA 6. *c is a carrier of ω.*

Proof.

$$c(a_p) = \bigcup\{\overline{\mathrm{St}}(b_q) \mid I(p) \leqslant q \leqslant R(p)\} \supset \sum_{s=1}^{v} X_p^s b_{k_s} = \omega(1 \cdot a_p)\,,$$

since $X_p^s = 0$ if $\pi(b_{k_s}) \neq a_p$ (see the definition of ω).

$$\omega(1 \cdot a_p + a_{p+1}) = \omega(1 \cdot a_p) + \omega(1 \cdot a_{p+1}) \subset c(a_p) \cup c(a_{p+1}) = c(a_pa_{p+1})\,,$$

$$\omega(1 \cdot a_pa_{p+1}) = \sum_{s=1}^{u-1} Y_p^s(b_sb_{s+1})$$

where if $Y_p^s \neq 0$ then $\{\pi(b_s)\} \cup \{\pi(b_{s+1})\} \subset \{a_p\} \cup \{a_{p+1}\}$, so $c(a_p)$ or $c(a_{p+1})$ contains $\overline{\mathrm{St}}(b_s)$ or $\overline{\mathrm{St}}(b_{s+1})$ and hence $c(a_p)$ or $c(a_{p+1})$ contains b_sb_{s+1}. Therefore $c(a_p) \cup c(a_{p+1}) \supset \omega(1 \cdot a_pa_{p+1})$. Hence c is a carrier of ω, since for any chain e contained in a simplex t we have $\omega(e) \subset c(t)$.

DEFINITION OF c^*. c^* maps the simplexes of K into the subcomplexes of K' and $c^*|L = c$ and is defined as follows on the rest of K: $c^*(a_r^p) = \bigcup\{\overline{\mathrm{St}}(b_s^q) \mid I(p) \leqslant q \leqslant s \leqslant R(r)\}$ where I and R are defined above and $\overline{\mathrm{St}}(b_s^q)$ is taken in K', and c^* of a simplex is the union of the images of its vertices under c^*.

LEMMA 7. *c^* is an acyclic carrier function.*

Proof. We need to show that if t is a simplex of K, then $c^*(t)$ is a subcomplex of K' and if $t' \subset t$ then $c^*(t') \subset c^*(t)$ and $c^*(t)$ is acyclic. By definition, $c^*(t)$ is a subcomplex of K' and if $t' \subset t$ then $c^*(t') \subset c^*(t)$, so that c^* is a carrier function. The images of simplexes of L are clearly acyclic, so we consider simplexes in K and not in L.

Now $c^*(a_r^p)$ is $N(V^*)$, where $V = \{U^{I(p)-\delta}, \ldots, U^{R(r)+\varepsilon}\}$, $\delta = 0$ if $I = 1$ and $\delta = 1$ otherwise, $\varepsilon = 0$ if $R = u$ and $\varepsilon = 1$ otherwise. These U's are elements of the covering of which L' is the nerve and there are u elements in this covering. In each case $c^*(a_r^p)$ is acyclic, so c^* of any 0-simplex is acyclic. $c^*(a_r^p a_{r+1}^{p+1}) = c^*(a_r^p) \cup c^*(a_{r+1}^{p+1})$ which is $N(V^*) \cup N(W^*)$ where V is as above and $W = \{U^{I(p+1)-\delta}, \ldots, U^{R(r+1)+\varepsilon}\}$. $N(V^*) \cap N(W^*) = N((V \cap W)^*)$, where $V \cap W = \{U^{I(p+1)-\delta}, \ldots, U^{R(r)+\varepsilon}\}$. So that $N(V^*) \cap N(W^*)$ is acyclic and therefore since $N(V^*)$ and $N(W^*)$ are acyclic, by the Mayer-Vietoris Theorem we have that $N(V^*) \cap N(W^*)$ is acyclic. Therefore $c^*(a_r^p a_{r+1}^{p+1})$ is acyclic. Likewise if $a_r^p \Delta a_t^s$ (i.e. the open sets associated with these vertices are Δ-related), then $c^*(a_r^p a_t^s) = c^*(a_r^p) \cup c^*(a_t^s)$ which are each acyclic and $c^*(a_r^p) \cap c^*(a_t^s) = c^*(a_{\min(r,t)}^{\max(p,s)})$, which is acyclic. Therefore, by the Mayer-Vietoris Theorem, $c^*(a_r^p a_t^s)$ is acyclic so that c^* of any 1-simplex is acyclic.

If each pair of vertices of $a_r^p a_t^s a_w^v$ are Δ-related, then $c^*(a_r^p a_t^s a_w^v) = c^*(a_r^p) \cup c^*(a_t^s) \cup c^*(a_w^v)$ which are each acyclic and the intersection of any two is acyclic. Moreover $c^*(a_r^p) \cap c^*(a_t^s) \cap c^*(a_w^v) = c^*(a_{\min(r,t,w)}^{\max(p,s,v)})$, which is acyclic. Therefore, by the Mayer-Vietoris Theorem, $c^*(a_r^p a_t^s a_w^v)$ is acyclic and so c^* of any 2-simplex is acyclic.

If each pair of vertices of $a_r^p a_t^s a_w^v a_z^y$ are Δ-related then $c^*(a_r^p a_t^s a_w^v a_z^y) = c^*(a_r^p) \cup c^*(a_t^s) \cup c^*(a_w^v) \cup c^*(a_z^y)$ which are each acyclic and the intersection of any two is acyclic. Moreover $c^*(a_r^p) \cap c^*(a_t^s) \cap c^*(a_w^v) \cap c^*(a_z^y) = c^*(a_{\min(r,t,w,z)}^{\max(p,s,v,y)})$ which is acyclic. Therefore $c^*(a_r^p a_t^s a_w^v a_z^y)$ is acyclic and so c^* of any 3-simplex is acyclic. Hence c^* is an acyclic carrier function.

LEMMA 8. ([3], p. 171, Theorem 5.7) *Let K and K' be simplicial complexes, let c^* be an acyclic carrier function defined on K with values in K', and let L be a subcomplex of K. Any algebraic map $L \to K'$ with carrier c can be extended to an algebraic map $K \to K'$ with carrier c^*. If f, g: $K \to K'$ are algebraic maps with carrier c^*, then any algebraic homotopy between $f|L$ and $g|L$ with carrier c can be extended to an algebraic homotopy between f and g with carrier c^*.*

DEFINITION OF ω^*. Let a_r^p and b_t^q denote vertices of $N(U_j^*)$ and $N(U_i^*)$ respectively and $\gamma(p) = \max\{k_s \mid X_p^s \neq 0\}$ and $\tau(p, r) = \min\{k_s \mid X_r^s \neq 0 \text{ and } k_s \geqslant \gamma(p)\}$ (see definition of ω). Following the construction of the extension in Lemma 8 we extend ω_{ji} to ω_{ji}^* where $\omega_{ji}^* \mid N(U_j) = \omega_{ji}$ and in $N(U_j^*) - N(U_j)$ on 0-chains we have $\omega_{ji}^*(1 \cdot a_r^p) = b_{\tau(p,r)}^{\gamma(p)}$. In $N(U_j^*) - N(U_j)$ on 1-chains we have

$$\omega^*(1 \cdot a_{r+1}^p a_r^p) = \sum_{g=\tau(p,r)}^{\tau(p,r+1)-1} (b_{g+1}^{\gamma(p)} b_g^{\gamma(p)}) + \delta_{pr} \sum_{q=1}^{t-2} \Big(\sum_{g=k_{s_q}}^{k_{s_{q+1}}-1} (b_{g+1} b_g) \Big)$$

where $\delta_{pr} = \begin{Bmatrix} 1 & p = r \\ 0 & p \neq r \end{Bmatrix}$ and $b_{k_{s_1}}, \ldots, b_{k_{s_t}}$ are b_{k_s}'s such that $X_p^s \neq 0$ and $k_{s_t} < k_{s_{t+1}}$;

$$\omega^*(1\cdot a_r^{p+1}a_r^p) = \sum_{g=\gamma(p)}^{\gamma(p+1)-1}(b_{\tau(p,r)}^{g+1}b_{\tau(p,r)}^{g}) + \sum_{g=\tau(p,r)}^{\tau(p+1,r)-1}(b_{g+1}^{\gamma(p+1)}b_g^{\gamma(p+1)});$$

$$\omega^*(1\cdot a_{r-1}^{p+1}a_r^p) = \omega^*(1\cdot a_{r-1}^{p}a_r^p) + \omega^*(1\cdot a_{r-1}^{p+1}a_{r-1}^{p});$$

$$\omega^*(1\cdot a_r^{p}a_{r+1}^{p+1}) = \omega^*(1\cdot a_r^{p}a_{r+1}^{p}) + \omega^*(1\cdot a_{r+1}^{p}a_{r+1}^{p+1}).$$

In $N(U_j^*)-N(U_j)$ on 2-chains we have

$$\omega^*(1\cdot a_r^p a_{r+1}^p a_{r+1}^{p+1}) = 0; \qquad \omega^*(1\cdot a_r^p a_{r+1}^p a_r^{p+1}) = 0;$$

$$\omega^*(1\cdot a_r^{p+1}a_r^p a_{r-1}^{p+1}) = \sum_{h=\gamma(p)}^{\gamma(p+1)-1}\Big(\sum_{g=\tau(p,r)}^{\tau(p+1,r+1)-1}(b_g^h b_{g+1}^h b_{g+1}^{h+1} + b_{g+1}^{h+1}b_g^{h+1}b_g^h)\Big);$$

$$\omega^*(1\cdot a_r^p a_r^{p-1}a_{r-1}^p) = -\omega^*(1\cdot a_{r-1}^p a_{r-1}^{p-1}a_r^p).$$

In $N(U_j^*)-N(U_j)$ on 3-chains we have

$$\omega^*(1\cdot a_r^p a_{r+1}^{p+1}a_{r+1}^p a_r^{p+1}) = 0.$$

LEMMA 9. *Condition* (a) *for a quasi-complex is satisfied.*

Proof. By Lemma 8, ω^* is an algebraic map of K to K' which is an extension of ω and has carrier c^*. We now denote a carrier of f by c_f. If t is a simplex in K' then $c_{\omega^*}c_{\pi^*}(t) = c_{\omega^*}\pi'(t) \supset c_{1^*}(t)$, where 1^* is the identity map on K'. Since $c_{\omega^*\pi^*} = c_{\omega^*}c_{\pi^*}$, we have that $\omega^*\pi^*$ and 1^* have the same carrier $c_{\omega^*\pi^*}$. Also in the same way $c_{\omega\pi}(t)\supset c_1(t)$ so that $\omega\pi$ and 1 have the same carrier $c_{\omega\pi}$.

Let D be the algebraic homotopy between $\omega\pi$ and 1 so $D\colon C^r(L') \to C^{r+1}(L')$. We wish to show that $c_{\omega\pi}$ is a carrier of D also. We need only consider the case when $r=0$ since L' has no 2-chains. Now

$$\partial Db_k = b_k - \omega\pi b_k = b_k - \omega a_p = b_k - \sum_{s=1}^{v} X_p^s b_{k_s}$$

so

$$Db_k \subset (b_{k-1}b_k) + (b_k b_{k+1}) - \sum_{s=1}^{v}(b_{k_s-1}b_{k_s}) + (b_{k_v}b_{k_v+1}) \subset$$

$$\bigcup\{\overline{\mathrm{St}}(b_q)\mid \min_{\pi(b_k)=a_p} k \leqslant q \leqslant \max_{\pi(b_k)=a_p} k\} = c_\omega(a_p) = c_\omega\pi'(b_k) = c_\omega c_\pi(b_k) = c_{\omega\pi}(b_k).$$

Now if $\pi'(b_{k+1}) = a_{p+1}$ then

$$\begin{aligned} D(b_k+b_{k+1}) &= D(b_k)+D(b_{k+1}) \subset c_\omega(a_p)\cup c_\omega(a_{p+1}) \\ &= c_\omega(a_p a_{p+1}) = c_\omega\pi'(b_k b_{k+1}) = c_\omega c_\pi(b_k b_{k+1}) = c_{\omega\pi}(b_k b_{k+1}). \end{aligned}$$

Likewise if $\pi'(b_{k+1}) = a_{p-1}$ then $D(b_k+b_{k+1})\subset c_{\omega\pi}(b_k b_{k+1})$. So we have shown for any $e\in C^0(L)$ that if $e\subset t$ then $De\subset c_{\omega\pi}(t)$, therefore $c_{\omega\pi}$ is a carrier of D. Since $(X; \{U_i\}; \{\pi_{ij}\}; \{\omega_{ji}\})$ is a quasi-complex, we have $\omega\pi\sim 1$.

By part 2 of Lemma 8 the algebraic homotopy between $\omega\pi$ and 1 can be extended to an algebraic homotopy between $\omega^*\pi^*$ and 1^* with carrier c^*. Hence $\omega^*\pi^*\sim 1^*$, i.e., condition (a) is satisfied.

LEMMA 10. *Condition* (b) *for a quasi-complex is satisfied.*

Proof. If ω_{kj}^*, ω_{ji}^* are antiprojections we wish to show $\omega_{ji}^*\omega_{kj}^*$ is also. $\omega_{ji}^*\omega_{kj}^*$ is a chain map from $N(U_k^*)$ to $N(U_i^*)$. So we need to show that $\omega_{ji}^*\omega_{kj}^*\pi_{ik}^*\sim 1_i^*$. First

$$c_{(\omega_{ji}\omega_{kj}\pi_{ik})}\supset c_{1_i}$$

and

$$\begin{aligned} c_{(\omega_{ji}^*\omega_{kj}^*\pi_{ik}^*)}(t) &= c_{(\omega_{ji}^*\omega_{kj}^*)}c_{\pi_{ik}^*}(t) = c_{(\omega_{ji}^*\omega_{kj}^*)}c_{\pi_{jk}^*\pi_{ij}^*)}(t) \\ &= c_{\omega_{ji}^*}c_{\omega_{kj}^*}c_{\pi_{jk}^*}c_{\pi_{ij}^*}(t)\supset c_{\omega_{ji}^*}c_{1_j^*}c_{\pi_{ij}^*}\supset c_{\omega_{ji}^*}(t')\supset c_{1_j^*}(t), \end{aligned}$$

where $t'\supset c_{\pi_{ji}^*}(t)$ since $c_{1_j^*}$ only makes the domain of $c_{\omega_{ji}^*}$ larger. Hence by part 2 of Lemma 8 the algebraic homotopy between $\omega_{ji}\omega_{kj}\pi_{ik}$ and 1_i can be extended to an algebraic homotopy between $\omega_{ji}^*\omega_{kj}^*\pi_{ik}^*$ and 1_i^*. Hence $\omega_{ji}^*\omega_{kj}^*$ is also an antiprojection, i.e. condition (b) is satisfied.

LEMMA 11. *Condition* (c) *for a quasi-complex is satisfied.*

Proof. If ω_{ji}^* and $\overline{\omega}_{ji}^*$ are antiprojections, then since by Theorem 1 $N(U_i^*)$ is acyclic, we have $\omega_{ji}\sim\overline{\omega}_{ji}^*$, i.e. condition (c) is satisfied.

Notation. The star of a simplex σ, $\mathrm{St}(\sigma)$, is the union of open sets corresponding to the vertices of σ. U_j is a star refinement of U_i if the star of every vertex corresponding to elements of U_j is contained in some element of U_i.

LEMMA 12. *Condition* (d) *for a quasi-complex is satisfied.*

Proof. For any i we choose j sufficiently large so that U_j^* is a star-refinement of U_i^* (see [6], p. 324). Then for any U_k^* let U_m^* be the one of the two U_j^* and U_k^* which is a refinement of both. Now we show that condition (d) is satisfied, i.e. for any i there exists $j>i$, and for any k an $m>k$, j (depending on i and k) such that ω_{jm}^* exists, satisfies (a), (b), and (c) and if $\sigma_j\in N(U_j^*)$, then $[\sigma_j]\cup[\omega_{jm}^*\sigma_j]$ is contained in a set of U_i^*. We have that if $\sigma_j = a_p$ or $a_p a_{p+1}$ then condition (d) holds since $(X; \{U_i\}; \{\pi_{ij}\}; \{\omega_{ji}\})$ defines a quasi-complex X (see [2], p. 667).

In the following a with subscripts and superscripts will denote a vertex of $N(U_j^*)$ and b with subscripts and superscripts will denote a vertex of $N(U_m^*)$. We show for any simplex σ that $[\sigma]\cup[\omega_{jm}^*\sigma]$ is contained in the star of some vertex of $N(U_j^*)$. Since U_j^* is a star refinement of U_i^*, we have that the star of a vertex of $N(U_j^*)$ is contained on some element of U_i^* and hence condition (d) will be satisfied. We need only consider chains whose images are non-zero.

0-chains: $[a_r^p]\cup[\omega_{jm}^*a_r^p] = [a_r^p]\cup[b_{\tau(p,r)}^{\gamma(p)}] = \Omega(U_j^p, \ldots, U_j^r)\cup\Omega(U_m^{\gamma(p)}, \ldots, U_m^{\tau(p,r)}) = \Omega(U_s^p, \ldots, U_j^r)\subset\mathrm{St}(a_r^p)$ since if $\gamma(p)\leqslant\xi\leqslant\tau(p,r)$ then $U_m^\xi\subset U_j^q$, $p\leqslant q\leqslant r$.

1-chains: If $p \neq r$ then

$$[a_{r+1}^p a_r^p] \cup [\omega_{jm}^* a_{r+1}^p a_r^p] = [a_{r+1}^p a_r^p] \cup \Big[\sum_{g=\tau(p,r)}^{\tau(p,r+1)-1} (b_{g+1}^{\gamma(p)} b_g^{\gamma(p)})\Big]$$

$$= \big(\Omega(U_j^p, \ldots, U_j^r) \cap \Omega(U_j^p, \ldots, U_j^{r+1})\big) \cup \Big(\bigcup_{g=\tau(p,r)}^{\tau(p,r+1)-1} \{\Omega(U_m^{\gamma(p)}, \ldots, U_m^g) \cap (U_m^{\gamma(p)}, \ldots, U_m^{g+1})\}\Big) \subset \mathrm{St}(a_r^p) \cap \mathrm{St}(a_{r+1}^p)$$

since if $\tau(p, r) \leqslant \xi \leqslant \tau(p, r+1)$ then $\Omega(U_m^{\gamma(p)}, \ldots, U_m^{\xi}) \subset \Omega(U_j^p, \ldots, U_j^r) \cup \Omega(U_j^p, \ldots, U_j^{r+1})$.

If $p = r$ then

$$[a_{r+1}^p a_r^p] \cup [\omega_{jm}^* a_{r+1}^p a_r^p]$$

$$= [a_{r+1}^p a_r^p] \cup \Big[\sum_{g=\tau(p,r)}^{\tau(p,r+1)-1} (b_{g+1}^{\gamma(p)} b_g^{\gamma(p)}) + \sum_{q=1}^{t-2} \sum_{g=k_{s_q}}^{k_{s_{q+1}}-1} (b_{g+1} b_g)\Big]$$

$$= \big(\Omega(U_j^p, U_j^{p+1}) \cap \Omega(U_j^p)\big) \cup \Big(\bigcup_{g=\tau(p,r)}^{\tau(p,r+1)-1} \{\Omega(U_m^{\gamma(p)}, \ldots, U_m^g) \cap \Omega(U_m^{\gamma(p)}, \ldots, U_m^{g+1})\}\Big)$$
$$\cup \Big(\bigcup_{q=1}^{t-2} \Big\{ \bigcup_{g=k_{s_q}}^{k_{s_{q+1}}-1} \big(\Omega(U_m^g) \cap \Omega(U_m^{g+1})\big)\Big\}\Big)$$

$$\subset \Omega(U_j^p) \cup \Big(\bigcup_{g=\tau(p,r)}^{\tau(p,r+1)-1} \{\Omega(U_m^{\gamma(p)}, \ldots, U_m^g) \cap \Omega(U_m^{\gamma(p)}, \ldots, U_m^{g+1})\}\Big)$$

$$\subset \Omega(U_j^p) \cup \Omega(U_j^p, U_j^{p+1}) \subset \mathrm{St}(a_r^p) \cap \mathrm{St}(a_{r+1}^p).$$

$$[a_r^{p+1} a_r^p] \cup [\omega_{jm}^* a_r^{p+1} a_r^p] = [a_r^{p+1} a_r^p] \cup \Big[\sum_{g=\gamma(p)}^{\gamma(p+1)-1} (b_{\tau(p,r)}^{g+1} b_{\tau(p,r)}^{g}) + \sum_{g=\tau(p,r)}^{\tau(p+1,r)-1} (b_{g+1}^{\gamma(p+1)} b_g^{\gamma(p+1)})\Big] \subset \mathrm{St}(a_r^p)$$

since if $\gamma(p) \leqslant \lambda \leqslant \gamma(p+1)$ then

$$\Omega(U_m^{\lambda}, \ldots, U_m^{\tau(p,r)}) \subset \Omega(U_j^{p-1}, \ldots, U_j^r) \cup \Omega(U_j^p, \ldots, U_j^r) \cup \Omega(U_j^{p+1}, \ldots, U_j^r)$$

and if $\tau(p, r) \leqslant \xi \leqslant \tau(p+1, r)$ then

$$\Omega(U_m^{\gamma(p+1)}, \ldots, U_m^{\xi}) \subset \Omega(U_j^{p+1}, \ldots, U_j^{r-1}) \cup \Omega(U_j^{p+1}, \ldots, U_j^r) \cup \Omega(U_j^{p+1}, \ldots, U_j^{r+1}).$$

$$[a_{r-1}^{p+1} a_r^p] \cup [\omega_{jm}^* a_{r-1}^{p+1} a_r^p] = [a_{r-1}^{p+1} a_r^p] \cup [\omega_{jm}^* a_{r-1}^p a_r^p] \cup [\omega_{jm}^* a_{r-1}^{p+1} a_{r-1}^p] \subset \mathrm{St}(a_{r-1}^p),$$

$$[a_r^p a_{r+1}^{p+1}] \cup [\omega_{jm}^* a_r^p a_{r+1}^{p+1}] = [a_r^p a_{r+1}^{p+1}] \cup [\omega_{jm}^* a_r^p a_{r+1}^p] \cup [\omega_{jm}^* a_{r+1}^p a_{r+1}^{p+1}] \subset \mathrm{St}(a_{r+1}^p).$$

2-chains:

$$[a_r^{p+1} a_r^p a_{r+1}^{p+1}] \cup [\omega_{jm}^* a_r^{p+1} a_r^p a_{r+1}^{p+1}]$$

$$= [a_r^{p+1} a_r^p a_{r+1}^{p+1}] \cup \Big[\sum_{h=\gamma(p)}^{\gamma(p+1)-1} \Big(\sum_{g=\tau(p,r)}^{\tau(p+1,r+1)-1} (b_g^h b_{g+1}^h b_{g+1}^{h+1} + b_{g+1}^{h+1} b_g^{h+1} b_g^h)\Big)\Big]$$

$$= \big(\Omega(U_j^{p+1}, \ldots, U_j^r) \cap \Omega(U_j^p, \ldots, U_j^r) \cap \Omega(U_j^{p+1}, \ldots, U_j^{r+1})\big)$$
$$\cup \Big(\bigcup_{h=\gamma(p)}^{\gamma(p+1)-1} \Big\{ \bigcup_{g=\tau(p,r)}^{\tau(p+1,r+1)-1} \Big\{\big(\Omega(U_m^h, \ldots, U_m^g) \cap \Omega(U_m^h, \ldots, U_m^{g+1}) \cap \Omega(U_m^{h+1}, \ldots, U_m^{g+1})\big) \cup \big(\Omega(U_m^{h+1}, \ldots, U_m^{g+1}) \cap \Omega(U_m^{h+1}, \ldots, U_m^g) \cap \Omega(U_m^h, \ldots, U_m^g)\big)\Big\}\Big\}\Big) \subset \mathrm{St}(a_r^p)$$

since if $\gamma(p) \leqslant \lambda \leqslant \gamma(p+1)$ then

$$\Omega(U_m^{\lambda}, \ldots, U_m^{\tau(p,r)}) \subset \Omega(U_j^{p-1}, \ldots, U_j^r) \cup \Omega(U_j^p, \ldots, U_j^r) \cup \Omega(U_j^{p+1}, \ldots, U_j^r)$$

and if $\tau(p, r) \leqslant \xi \leqslant \tau(p+1, r)$ then

$$\Omega(U_m^{\gamma(p+1)}, \ldots, U_m^{\xi}) \subset \Omega(U_j^{p+1}, \ldots, U_j^{r-1}) \cup \Omega(U_j^{p+1}, \ldots, U_j^r) \cup \Omega(U_j^{p+1}, \ldots, U_j^{r+1}).$$

$$[a_r^p a_r^{p-1} a_{r-1}^p] \cup [\omega_{jm}^* a_r^p a_r^{p-1} a_{r-1}^p] = [a_r^p a_r^{p-1} a_{r-1}^p] \cup [-\omega_{jm}^* a_{r-1}^p a_{r-1}^{p-1} a_r^p] \subset \mathrm{St}(a_{r-1}^{p-1}).$$

Hence condition (d) is satisfied.

From lemmas 9, 10, 11, and 12 follows

THEOREM 2. $(C(X); \{U_i^*\}; \{\pi_{ij}^*\}; \{\omega_{ji}^*\})$ *defines a quasi-complex* $C(X)$.

THEOREM 3. $C(X)$ *has fixed point property.*

Proof. By [9] $C(X)$ is acyclic and since it is connected, it is a zero-cyclic quasi-complex. By [6], p. 326, (36.4), a zero-cyclic quasi-complex has fixed point property.

Remark. For any continuum Y, $C(Y)$ is an absolute retract if and only if Y is locally connected (see [5], Theorem 4.4). Hence if Y is locally connected, $C(Y)$ has the fixed point property. It follows from a theorem of Lefschetz ([7], p. 46] that if $C(Y)$ is an absolute neighborhood retract, then it has the fixed point property. By the following theorem if $C(Y)$ is finite dimensional (in particular, if Y is snake-like), then $C(Y)$ is an absolute neighborhood retract only in case Y is locally connected; hence neither of the above results applies when Y is a non-locally connected snake-like continuum.

THEOREM 4. *If $C(Y)$ is a finite dimensional absolute neighborhood retract then it is an absolute retract.*

Proof. Fox [4] has shown that any m-dimensional absolute neighborhood retract which is simply connected and acyclic in all dimensions

$\leqslant k$ ($\leqslant m$) can be covered by $m-k+1$ contractible open sets. Hence since $C(Y)$ is simply connected ([5], Theorem 4.5) and acyclic in all dimensions, it follows that $C(Y)$ is contractible and hence is an absolute retract.

QUESTION. For what class of continua is $C(X)$ a quasi-complex? We know that if X is locally connected (in which case $C(X)$ is an absolute retract) or a snake-like continuum that $C(X)$ is a quasi-complex.

References

[1] R. H. Bing, *Snake-like continua*, Duke Math. J. 18 (1951), pp. 653-663.
[2] E. Dyer, *A fixed point theorem*, Proc. Amer. Math. Soc., (1956), pp. 662-672.
[3] S. Eilenberg and N. Steenrod, *Foundations of algebraic topology*, Princeton 1952.
[4] R. H. Fox, *On the Lusternik Schnirelmann category*, Annals of Math. (1941), pp. 333-370.
[5] J. L. Kelley, *Hyperspaces of a continuum*, Trans. Amer. Math. Soc. 52 (1942), pp. 22-36.
[6] S. Lefschetz, *Algebraic topology*, Amer. Math. Soc. Colloquium Publications, vol. 27, New York 1942.
[7] — *Topics in topology*, Annals of Math. Studies, no. 10, 1942.
[8] E. Michael, *Topologies on spaces of subsets*, Trans. Amer. Math. Soc. 71 (1951), pp. 152-182.
[9] J. Segal, *Hyperspaces of the inverse limit space*, Proc. Amer. Math. Soc. 10 (1959), pp. 706-709.

THE UNIVERSITY OF WASHINGTON
SEATTLE, WASHINGTON, U. S. A.

Reçu par la Rédaction le 26. 11. 1960

Sur la représentation topologique des graphes

par

Á. Császár (Budapest)

1. Nous entendons par *graphe* (abstrait) le système (S, A, e) composé d'un ensemble S (dont les éléments sont appelés *sommets* du graphe), d'un ensemble A (dont les éléments s'appellent *arêtes* du graphe) et d'une application e qui fait correspondre à chaque arête $a \in A$ un ensemble $e(a) = \{s_1, s_2\} \subset S$ composé de deux sommets distincts, appelés *extrémités* de l'arête a (1).

Nous dirons qu'un graphe (S, A, e) est *fini* si les ensembles S et A sont finis, et qu'il est *dénombrable* si S et A sont *dénombrables* (2). Le graphe (S, A, e) est dit *connexe* si, deux sommets distincts $s, s' \in S$, $s \neq s'$ étant donnés, on peut toujours trouver une suite finie d'arêtes $a_1, \ldots, a_n$ telles que $s \in e(a_1)$, $s' \in e(a_n)$ et que $e(a_i) \cap e(a_{i+1}) \neq 0$ pour $1 \leqslant i \leqslant n-1$.

2. On a l'habitude de représenter un graphe *fini* (S, A, e) par un sous-ensemble G de l'espace euclidien E^3, composé de certains points $s'_1, \ldots, s'_m$ et de certains arcs $a'_1, \ldots, a'_n$ (où m est égal à la puissance de l'ensemble S et n à celle de l'ensemble A), de manière que les points s'_i correspondent biunivoquement aux sommets $s_i \in S$ et les arcs a'_k aux arêtes $a_k \in A$, l'arc a'_k ayant pour extrémités les points s'_i et s'_j si et seulement si l'arête correspondante a_k a pour extrémités les sommets s_i et s_j qui correspondent à s'_i et s'_j respectivement, et deux arcs a'_k et a'_l n'ayant d'autres points communs que leurs extrémités au plus. Beaucoup de propriétés du graphe (S, A, e) peuvent être formulées au moyen des propriétés topologiques de l'ensemble G; p.ex. le graphe (S, A, e) est connexe si et seulement si l'ensemble G est connexe (au sens topologique).

Pour un graphe (S, A, e) quelconque (fini ou non), on peut définir une représentation topologique analogue de la façon suivante. Considérons un ensemble G dont les éléments sont d'une part les sommets $s \in S$ du graphe, de l'autre les couples (a, x) formés par une arête $a \in A$

(1) D'après la terminologie adoptée par D. König ([3], pp. 1 et 2), il faudrait encore postuler que, pour $s \in S$, il existe au moins un $a \in A$ tel que $s \in e(a)$; d'après C. Berge ([1], p. 27), on devrait dire *multi-graphe* au lieu de graphe. Cependant, la terminologie que nous venons d'introduire conviendra mieux à nos buts.

(2) C'est-à-dire finis ou dénombrablement infinis.