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Perfect transfinite numbers

by

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1. Introduction. The well-known concept of a perfect (finite) number can easily be extended to include transifinite ordinal numbers provided some care is taken to take account of the non-commutativity of addition and multiplication of ordinals. This is done as follows.

1.1. DEFINITION. An ordinal number is *left perfect* if and only if it is equal to the sum of its proper left hand divisors, arranged in increasing order.

Right perfect is similarly defined. As an example, ω has the proper left hand divisors $1, 2, 3, \ldots$, and $1+2+3+\ldots=\omega$, so ω is left perfect. For finite ordinals, it is clear that the concepts of left perfect, right perfect, and the usual concept of perfect coincide.

In this paper it is proved that there are no transfinite right perfect ordinals, and all transfinite left perfect ordinals are determined to within certain questions of finite arithmetic (see Section 4).

2. Triangular numbers. In this section we develop certain properties of triangular numbers which will be needed later. All variables $\alpha, \delta, \sigma, \ldots$ will denote ordinals.

2.1. DEFINITION. $T(\alpha)=\sum\limits_{\zeta<\alpha}\zeta$, all summations being understood to be taken in increasing order.

$T(\alpha)$ is simply the triangular number associated with α. The function T is obviously increasing and continuous. We wish to determine $T(\varrho)$ explicitly whenever ϱ is a prime component of addition.

2.2. LEMMA. *Let ϱ be a prime component of addition, ζ_φ be a non-decreasing set of ordinals defined for $\varphi<\varrho$. Then for any $\beta<\varrho$ we have* $\sum\limits_{\beta\leqslant\varphi<\varrho}\zeta_\varphi=\sum\limits_{\varphi<\varrho}\zeta_\varphi$.

Proof. $\beta<\varrho$ implies $\varrho=\beta+\varrho$. The ζ_φ are non-decreasing, hence $\sum\limits_{\varphi<\beta}\zeta_\varphi\leqslant\zeta_\beta\beta$.

Similarly, $\sum\limits_{\beta\leqslant\varphi<\varrho}\zeta_\varphi\geqslant\zeta_\beta(\varrho-\beta)=\zeta_\beta\varrho$, say

$$\sum_{\beta\leqslant\varphi<\varrho}\zeta_\varphi=\zeta_\alpha\varrho+\delta\,.$$

Now

$$\sum_{\beta\leqslant\varphi<\varrho}\zeta_\varphi\leqslant\sum_{\varphi<\varrho}\zeta_\varphi=\sum_{\varphi<\beta}\zeta_\varphi+\sum_{\beta\leqslant\varphi<\varrho}\zeta_\varphi=\sum_{\varphi<\beta}\zeta_\varphi+\zeta_\beta\varrho+\delta\leqslant\zeta_\beta\beta+\zeta_\beta\varrho+\delta$$
$$=\zeta_\beta(\beta+\varrho)+\delta=\zeta_\beta\varrho+\delta=\sum_{\beta\leqslant\varphi<\varrho}\zeta_\varphi\,,$$

therefore

$$\sum_{\beta\leqslant\varphi<\varrho}\zeta_\varphi=\sum_{\varphi<\varrho}\zeta_\varphi\,.$$

2.3. LEMMA. *If ϱ is a prime component then so is $T(\varrho)$.*

Proof. Let $\alpha<T(\varrho)=\sum_{\zeta<\varrho}\zeta$. Since ϱ is of the second kind, there exists a $\beta<\varrho$ such that $\alpha\leqslant\sum_{\zeta<\beta}\zeta$. Using Lemma 2.2 we have

$$T(\varrho)\leqslant\alpha+T(\varrho)\leqslant\sum_{\zeta<\beta}\zeta+\sum_{\zeta<\varrho}\zeta=\sum_{\zeta<\beta}\zeta+\sum_{\beta\leqslant\zeta<\varrho}\zeta=\sum_{\zeta<\varrho}\zeta=T(\varrho)\,,$$

so $T(\varrho)=\alpha+T(\varrho)$. This proves that $T(\varrho)$ is a prime component.

COROLLARY. *There exists a unique function f such that $T(\omega^\alpha)=\omega^{f(\alpha)}$ for every ordinal α.*

This follows because prime components are identical with powers of ω. It thus suffices to find the function f in order to determine $T(\varrho)$ for all prime components ϱ. It is clear that f is an increasing function. It is also continuous; indeed, let α be of the second kind. Then using the continuity of T and the continuity of exponentiation with a fixed base,

$$\omega^{\lim\limits_{\varphi<\alpha}f(\varphi)}=\lim_{\varphi<\alpha}\omega^{f(\varphi)}=\lim_{\varphi<\alpha}T(\omega^\varphi)=T(\lim_{\varphi<\alpha}\omega^\varphi)=T(\omega^\alpha)\,,$$

so $\lim_{\varphi<\alpha}f(\varphi)=f(\alpha)$ by the definition of f.

2.4. THEOREM. *For $\alpha=\delta+1$ of the first kind, $f(\alpha)=\delta\cdot 2+1$. For α of the second kind, let $\alpha=\omega^{\alpha_1}a_1+\ldots+\omega^{\alpha_n}a_n$ be its normal expansion (i.e. $\alpha_1>\alpha_2>\ldots>\alpha_n>0$, $a_1,\ldots,a_n$, are positive integers). Then $f(\alpha)=\omega^{\alpha_1}(2a_1-1)$ if $n=1$, and $f(\alpha)=\omega^{\alpha_1}(2a_1)+\omega^{\alpha_2}a_2+\ldots+\omega^{\alpha_n}a_n$ if $n>1$.*

Proof. Let $\alpha=\delta+1$ be of the first kind. By Lemma 2.2,

$$T(\omega^\alpha)=\sum_{\zeta<\omega^\delta}\zeta+\sum_{\omega^\delta\leqslant\zeta<\omega^\alpha}\zeta=\sum_{\omega^\delta\leqslant\zeta<\omega^\alpha}\zeta\,.$$

Hence

$$T(\omega^\alpha)=\sum_{\beta<\omega^\delta}(\omega^\delta+\beta)+\sum_{\beta<\omega^\delta}(\omega^\delta\cdot 2+\beta)+\sum_{\beta<\omega^\delta}(\omega^\delta\cdot 3+\beta)+\ldots$$
$$=\sum_{\beta<\omega^\delta}\omega^\delta+\sum_{\beta<\omega^\delta}\omega^\delta\cdot 2+\sum_{\beta+\omega^\delta}\omega^\delta\cdot 3+\ldots$$
$$=\omega^\delta\cdot\omega^\delta+\omega^\delta\cdot 2\cdot\omega^\delta+\omega^\delta\cdot 3\cdot\omega^\delta+\ldots=\omega^{\delta 2}\cdot\omega=\omega^{\delta 2+1}\,.$$

This proves that $f(\alpha)=\delta\cdot 2+1$. Having $f(\alpha)$ for all α of the first kind determines $f(\alpha)$ for all α by continuity. Indeed, for α of the second kind,

$$f(\alpha)=\lim_{\gamma<\alpha}f(\gamma)=\lim_{\gamma<\alpha}f(\gamma+1)=\lim_{\gamma<\alpha}(\gamma\cdot 2+1)=\lim_{\gamma<\alpha}(\gamma\cdot 2)\,.$$

Let $\alpha=\omega^{\alpha_1}a_1+\ldots+\omega^{\alpha_n}a_n$, $a_n>0$. If $n>1$ we now have

$$f(\alpha)=\lim_{\gamma<\alpha}(\gamma\cdot 2)=\lim_{\beta<\omega^{\alpha_2}a_2+\ldots+\omega^{\alpha_n}a_n}[(\omega^{\alpha_1}a_1+\beta)\cdot 2]=\lim_{\beta<\omega^{\alpha_2}a_2+\ldots+\omega^{\alpha_n}a_n}(\omega^{\alpha_1}\cdot 2a_1+\beta)$$
$$=\omega^{\alpha_1}(2a_1)+\omega^{\alpha_2}a_1+\ldots+\omega^{\alpha_n}a_n\,.$$

Finally, suppose $n=1$, i.e. $\alpha=\omega^{\alpha_1}a_1$. Then

$$f(\alpha)=\lim_{\gamma<\alpha}(\gamma\cdot 2)=\lim_{\beta<\omega^{\alpha_1}}\left[\left(\omega^{\alpha_1}(a_1-1)+\beta\right)\cdot 2\right]=\lim_{\beta<\omega^{\alpha_1}}\left(\omega^{\alpha_1}(a_1-1)\cdot 2+\beta\right)$$
$$=\omega^{\alpha_1}(2a_1-2)+\omega^{\alpha_1}=\omega^{\alpha_1}(2a_1-1)\,.$$

This completes the proof of Theorem 2.4.

2.5. COROLLARY. *$f(\alpha)=\alpha$ if and only if $\alpha=\omega^\beta$ for some β.*

3. Transfinite perfect nunbers.

3.1. DEFINITION.

$$\tau_l(\alpha)=\sum\{\zeta\colon\ \zeta<\alpha\text{ and there exists }\gamma,\ \zeta\gamma=\alpha)\}\,,$$
$$\tau_r(\alpha)=\sum\{\zeta\colon\ \zeta<\alpha\text{ and there exists }\gamma,\ \gamma\zeta=\alpha)\}\ (^1)\,.$$

Hence α is left (right) perfect if and only if $\alpha=\tau_l(\alpha)$ $(\tau_r(\alpha))$. For α finite we denote $\tau_l(\alpha)=\tau_r(\alpha)$ by $\tau(\alpha)$, and also use $\sigma(\alpha)=\tau(\alpha)+\alpha$.

3.2. THEOREM. *If $\alpha\geqslant\omega$, then $\alpha\neq\tau_r(\alpha)$.*

Proof. Let $\alpha\geqslant\omega$, $\alpha=\omega^{\alpha_1}a_1+\ldots+\omega^{\alpha_n}a_n$ be its normal expansion. Then $\alpha_1\geqslant 1$. It is known ([1], p. 298) that α has only a finite number of proper right-hand divisors, say $\beta_0,\beta_1,\ldots,\beta_k$, where $\beta_0=1<\beta_1<\ldots<\beta_k<\alpha$. If $\beta_k<\omega^{\alpha_1}$ then

$$\tau_r(\alpha)=\sum_{i\leqslant k}\beta_i\leqslant\beta_k\cdot(k+1)<\omega^{\alpha_1}\leqslant\alpha\,,$$

so it remains to consider the case $\omega^{\alpha_1}\leqslant\beta_k<\alpha$.

We may write $\beta_i=\omega^{\alpha_1}m_i+\beta_i'$, $0\leqslant i\leqslant k$, where each m_i is a finite non-negative integer and each $\beta_i'<\omega^{\alpha_1}$. By assumption $m_k\geqslant 1$.

Now suppose $\tau_r(\alpha)=\alpha$. Then

$$\alpha=\sum_{i\leqslant k}\beta_i=\omega^{\alpha_1}\sum_{i\leqslant k}m_i+\beta_k'\,,\qquad\omega^{\alpha_1}a_1+\omega^{\alpha_2}a_2+\ldots+\omega^{\alpha_n}a_n=\omega^{\alpha_1}\sum_{i\leqslant k}m_i+\beta_k'\,.$$

(1) The sums are understood to be taken in increasing order, as mentioned in Definition 2.1.

By the uniqueness of the normal form,

$$\beta'_k = \omega^{a_2}a_2 + \ldots + \omega^{a_n}a_n .$$

Then

$$\beta_k = \omega^{a_1}m_k + \omega^{a_2}a_2 + \ldots + \omega^{a_n}a_n .$$

Now $a = \gamma\beta_k$ for some γ, say $\gamma = \omega^{\gamma_1}c_1 + \ldots + \omega^{\gamma_p}c_p$ in normal form. Thus

$$\omega^{a_1}a_1 + \ldots + \omega^{a_n}a_n = \gamma\omega^{a_1}m_k + \gamma\omega^{a_2}a_2 + \ldots + \gamma\omega^{a_n}a_n .$$

This implies $\gamma\omega^{a_1} \leqslant a < \omega^{a_1+1}$. But $\gamma\omega^{a_1} = \omega^{\gamma_1}\omega^{a_1} = \omega^{\gamma_1+a_1}$ (since $a_1 \geqslant 1$), which gives $\omega^{\gamma_1+a_1} < \omega^{a_1+1}$, $a_1 \leqslant \gamma_1 + a_1 < a_1+1$, $\gamma_1 + a_1 = a_1$, i.e. $\gamma\omega^{a_1} = \omega^{a_1}$. We now have $\omega^{a_1} = \gamma\omega^{a_1} > \gamma\omega^{a_2} > \ldots > \gamma\omega^{a_n}$, and a comparision of normal forms now tells us that $m_k = a_1$. But then $\beta = a$, contradicting the fact that β is a proper divisor.

3.3. Theorem. *Let $a = \omega^{a_1}a_1 + \ldots + \omega^{a_{n-1}}a_{n-1} + a_n$ (normal form) be any ordinal of the first kind. Then $\tau_1(a) = a$ if and only if $\tau(a_1) = a_1$.*

Proof. Let a be as stated. It is known ([1], p. 298), that a has only a finite number of proper left hand divisors, say $\beta_0, \beta_1, \ldots, \beta_m$, where $1 = \beta_0 < \beta_1 < \ldots < \beta_k < \omega^{a_1} \leqslant \beta_{k+1} < \ldots < \beta_m < a$. It may happen that there are no $\beta_{k+1}, \ldots, \beta_m \geqslant \omega^{a_1}$, but in this case $\tau_1(a) = \sum_{i\leqslant k}\beta_i < \omega^{a_1} \leqslant a$, so we exclude this possibility from now on. Then

$$\tau_1(a) = \sum_{i\leqslant m}\beta_i = \sum_{k+1\leqslant i\leqslant m}\beta_i .$$

It remains to find $\beta_{k+1}, \ldots, \beta_m$, that is, we wish to find all β such that $\omega^{a_1} \leqslant \beta < a$ and there exists γ with $a = \beta\gamma$. If $\gamma \geqslant \omega$ then $\beta\gamma \geqslant \omega^{a_1}\cdot\omega = \omega^{a_1+1} > a$, therefore γ must be finite, and of course $\gamma > 1$. Put $\beta = \omega^{a_1}b_1 + \beta'$, where b_1 is a positive integer and $\beta' < \omega^{a_1}$, then $a = \beta\gamma = \omega^{a_1}b_1\gamma + \beta'$ if and only if $\beta' = \omega^{a_2}a_2 + \ldots + \omega^{a_{n-1}}a_{n-1} + a_n$ and $b_1\gamma = a_1$. Thus $\beta_{k+1}, \ldots, \beta_m$ consist of all numbers $\omega^{a_1}b_1 + \omega^{a_2}a_2 + \ldots + \omega^{a_{n-1}}a_{n-1} + a_n$, where b_1 is a proper divisor of a_1; their sum is obviously $\omega^{a_1}\tau(a_1) + \omega^{a_2}a_2 + \ldots + a_n$, which equals a if and only if $\tau(a_1) = a_1$.

3.4. Theorem. *Let $a = \omega^{a_1}a_1 + \ldots + \omega^{a_n}a_n$, $a_n \geqslant 1$, be any ordinal of the second kind. Then $\tau_1(a) = a$ if and only if one of the following hold:*

(a) *$a_1 > 1$, $f(a_n) < a_1$, and $\tau(a_1) = a_1$,*

(b) *$a_1 > 1$, $f(a_n) = a_1$, and $\tau(a_1) = a_1 - 1$,*

(c) *$a_1 = 1 = a_2$ and $f(a_n) = a_1$,*

(d) *$a = 1$, $n = 1$, and $a_1 = \omega^\beta$ for some β (i.e. $a = \omega^{\omega^\beta}$).*

Proof. It is known ([1], p. 328), that the proper left hand divisors of a consist of all $a < \omega^{a_n}$ and a finite number of $\beta \geqslant \omega^{a_n}$, say $\omega^{a_n} \leqslant \beta_0 < \ldots < \beta_k < a$. Hence

$$\tau_1(a) = \sum_{\beta<\omega^{a_n}}\beta + \sum_{i\leqslant k}\beta_i = T(\omega^{a_n}) + \sum_{i\leqslant k}\beta_i = \omega^{f(a_n)} + \sum_{i\leqslant k}\beta_i .$$

It follows that $\tau_1(a) > a$ whenever $f(a_n) > a_1$, so from now on we may assume $f(a_n) \leqslant a_1$.

If $a_1 > 1$ then a has proper left divisors $\beta > \omega^{a_1}$, namely all $\omega^{a_1}b_1 + \omega^{a_2}a_2 + \ldots + \omega^{a_n}a_n$ where b_1 is a proper divisor of a_1 (this is proved exactly as in the proof of Theorem 3.3, since the corresponding right hand divisor must be finite). It is then easy to see that

$$\tau_1(a) = \omega^{f(a_n)} + \sum_{b_1|a_1,\, b_1<a_1}(\omega^{a_1}b_1 + \omega^{a_2}a_2 + \ldots + \omega^{a_n}a_n) = \omega^{f(a_n)} + \omega^{a_1}\tau(a_1) + \omega^{a_2}a_2 + \ldots + \omega^{a_n}a_n .$$

Parts (a) and (b) are now obvious.

If $a_1 = 1$ then a has no proper left divisors $\beta > \omega^{a_1}$ ([1], p. 329), whence $\sum_{i\leqslant k}\beta_i < \omega^{a_1}$. It follows that $f(a_n) = a_1$ is a necessary condition that $\tau_1(a) = a$, so we assume this from now on. The proper left divisors of a, $\beta_0, \ldots, \beta_k$, are ([1], p. 329) precisely all numbers of the form

$$\omega^{a_{n-r+1}}b_1 + \omega^{a_{n-r+2}}a_{n-r+2} + \ldots + \omega^{a_n}a_n , \quad \text{where} \quad r < n \text{ and } b_1 | a_{n-r+1} .$$

As usual, only those $\beta_i \geqslant \omega^{a_2}$ will affect $\sum_{i\leqslant k}\beta_i$, since ω^{a_2} is the largest prime component which can occur among this finite set of divisors. Thus

$$\sum_{i\leqslant k}\beta_i = \sum_{b_1|a_2}(\omega^{a_2}b_1 + \omega^{a_3}a_3 + \ldots + \omega^{a_n}a_n) = \omega^{a_2}\sigma(a_2) + \omega^{a_3}a_3 + \ldots + \omega^{a_n}a_n ,$$

$$\tau_1(a) = \omega^{a_1} + \omega^{a_2}\sigma(a_2) + \omega^{a_3}a_3 + \ldots + \omega^{a_n}a_n .$$

If $n > 1$ we see that $\tau_1(a) = a$ if and only if $\sigma(a_2) = a_1$, which holds if and only if $a_2 = 1$. This proves (c). Finally, if $n = 1$ we already have $\tau_1(a) = a$. The only condition imposed in this case was that $f(a_1) = a_1$ (since $a_n = a_1$), which by Corollary 2.5 is equivalent to $a_1 = \omega^\beta$ for some β. This proves (d) and completes the proof of Theorem 3.4.

4. Examples and further questions. Using Theorem 3.3 it is easy to furnish examples of perfect ordinals of the first kind (in virtue of Theorem 3.2 we can omit the "left" without ambiguity). The smallest transfinite one is $\omega\cdot 6+1$. Since this number is odd, it follows that there do exist odd perfect ordinals, the smallest transfinite one being $\omega\cdot 6+1$.

Using Theorem 3.4 it is easy to furnish examples of perfect ordinals of the second kind. First let us note that $f(1) = 1$, $f(2) = 3$, $f(3) = 5$; $\tau(1) = 0$, $\tau(2) = 1$, $\tau(4) = 3$; $\tau(6) = 6$, $\tau(28) = 28$. Then examples satisfying Theorem 3.4 (a)-(d) respectively are

(a) $\omega^2\cdot 6 + \omega$, $\omega^4\cdot 28 + \omega^2$,

(b) $\omega^3\cdot 4 + \omega^2$, $\omega^3\cdot 2 + \omega^2$, $\omega\cdot 2$, $\omega\cdot 4$,

(c) $\omega^3 + \omega^2$, $\omega^5 + \omega^4 + \omega^3$, $\omega^5 + \omega^3$,

(d) ω, ω^ω ω^{ω^2}.

In particular, it follows from (d) that any initial ordinal ω_γ is perfect, since for $\gamma = 0$, $\omega_0 = \omega$ is perfect and for $\gamma > 0$ $\omega_\gamma = \omega^{\omega_\gamma} = \omega^{\omega^{\omega_\gamma}}$.

From theorems 3.3 and 3.4 we see that in order to find all transfinite perfect numbers it suffices to solve the following two problems of finite arithmetic:

(1) Find all finite numbers N such that $\tau(N) = N$.

(2) Find all finite numbers N such that $\tau(N) = N-1$.

Problem (1) is simply the well-known problem of determining all finite perfect numbers. Problem (2) seems to be new; I call such numbers *almost perfect*. Clearly, $N = 2^n$ is almost perfect for any non-negative integer n. The question arises whether there are any almost perfect integers which are not of the form 2^n. It is easy to obtain necessary conditions for such an N, but the general problem seems to be at least as difficult as (1). In particular, I have been able to prove that if N is almost perfect and not a power of 2 then N has at least three distinct prime divisors, either N or $2N$ is a perfect square, and $N > 10^7$, with stronger results if N is odd.

One may also consider the question of perfect cardinals, the definition being the same as in the finite case. The axiom of choice must be assumed in making this definition since the proper divisors of a cardinal will usually form an infinite family, so we now assume the axiom of choice. Every cardinal is then finite or an aleph, and it is easy to prove that a cardinal n is perfect if and only if n is finite and perfect or $n = \aleph_\alpha$ where α is an ordinal of the second kind. It follows from the continuum hypothesis that $2^{\aleph_0}$ is not perfect. It is interesting that this also follows from the weaker hypothesis $2^{\aleph_0} < \aleph_{\omega_1}$, for $2^{\aleph_0}$ is not the sum of a denumerable family of smaller cardinals ([1], p. 401) whereas any $\aleph_\alpha$, $\alpha < \omega_1$ and α of the second kind, is such a sum.

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Eine Anwendung der unendlichwertigen Logik auf topologische Räume

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1. Einleitung. In [1] wurde gezeigt, daß die Menge der „wahren“ Formeln des unendlichwertigen Prädikatenkalküls von Łukasiewicz-Tarski nicht axiomatisierbar ist. Dabei wurde eine Formel als wahr bezeichnet, wenn sie bei jeder Bewertung immer den Wert Null annimmt. Zu analogen Fragestellungen gelangt man, wenn man statt beliebiger Mengen E und beliebiger n-stelliger Abbildungen $\varphi(x, \ldots, x_n)$ von E in $[0,1]$ nur topologische Räume T und n-stellige stetige Funktionen von T in $[0,1]$ zuläßt.

Dabei kann man etwa folgende Probleme betrachten:

a) *Welches ist der Kompliziertheitsgrad (in bezug auf die Kleene-Hierarchie) der Menge der Formeln, die auf T immer den Wert Null annehmen.*

b) *Lassen sich zwei Räume T_1, T_2 durch eine Formel F in dem Sinne unterscheiden, daß etwa F identisch Null auf T_1, hingegen erfüllbar größer Null auf T_2 ist.*

c) *Welches ist der Kompliziertheitsgrad, wenn statt der Menge der Formeln, die identisch Null sind, die Menge der Formeln, die erfüllbar Null sind, betrachtet wird.*

d) *Sind zwei Räume T_1, T_2 durch eine Formel F in dem Sinne unterscheidbar, daß etwa F erfüllbar Null auf T_1, hingegen immer größer Null auf T_2 ist.*

Von diesen vier Problemen erweist sich das letzte (d) als das interessanteste. In dieser Arbeit soll gezeigt werden, daß zu einem Raum T jedenfalls dann eine Formel F mit den in d) gewünschten Eigenschaften konstruiert werden kann, wenn T einigen spezifischen logischen Forderungen genügt. Die Fragen a), b), c) werden in 6. kurz gestreift.

Als Aussagenkalkül wurde hier eine etwas leichter zu handhabende Form als die von A. Rose und B. Rosser in [2] entwickelte gewählt. Man kann aber unschwer zeigen, daß sich zu jeder Formel des hier entwickelten Kalküls eine erfüllungsgleiche aus der eben erwähnten finden läßt.