

coherent continuum M, contains also by already quoted Miller's theorem (see [10], p. 187, theorem 2.6) a subcontinuum  $N \subset M$  which has an upper semi-continuous decomposition (15) on mutually disjoint continua, such that the hyperspace of this decomposition is the circumference. Thus N contains no cut-point.

B. Knaster even asks (New Scottish Book, problem 526) whether the simultaneously hereditarily decomposable and hereditarily unicoherent continua (which he calls " $\lambda$ -dendroids") have fixed points under arbitrary continuous mappings.

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# A correction to my paper: "Continua meeting an orbit at a point"

(Fundamenta Mathematicae 52 (1963), pp. 319-321)

1

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As pointed out to me by L. W. Anderson, there is an error in the proof of lemma 2, and in fact the statement is false as it stands. While this can be partially rectified, the author is at present unable to retrieve the theorem in its stated generality. The extra condition we must impose is as follows: (We do not assume that X is connected.)

(\*) Let  $\varphi \colon X \to X/G$  be the natural projection. There is an invariant naighborhood V of G(p) in X such that if U is any neighborhood of  $\varphi(p)$  in X/G, there is a neighborhood  $W \subset U$  such that the component A of  $\varphi(p)$  in  $W^-$  meets X/G - W and if  $a \in A \cap X/G - W$ , the component of a in  $\varphi(V^-) - W$  meets  $X/G - \varphi(V)$ .

Condition (\*) is implied by condition (ii) of Anderson and Hunter, and its form is more suitable for the induction argument. Our proof now is in a similar vein to that of Anderson and Hunter, but uses a slice instead of a local cross section to eliminate condition (i) and a revised form of lemma 2 to overcome the finite induction in their argument. The proof of the *theorem* remains unchanged except in one detail.

Since (\*) is satisfied for any compact connected semigroup, the corollary remains valid.

We now describe how lemma 2 may be altered in order to give the desired result.

First, in place of (iii) of lemma 2 we must have

- (iii)  $M \cap (X-V) \neq \emptyset$  for a fixed invariant neighborhood V in X, and in place of (iv), we have
- (iv) M/K is connected and satisfies (\*) with respect to  $M \cap V$  and K. Now in the proof we proceed exactly as before, but taking  $U \subset \pi(V)$  and such that W satisfies (\*) with respect to U, up to the point where we define  $M_0$ . Let A be the component of  $W^-$  containing  $\pi(p)$ . Since M/K is connected, A meets  $U^- W$ . Let  $B_a$  be the component of M/K W containing a, where a is any point of  $A \cap (M/K W)$ . Then  $B_a$  meets



 $M/K - \pi(V)$  and  $A \cup B_a$  is connected. Since v is open on  $(M - \pi^{-1}(W))/K_1$ , there is a continuum containing each point  $a' \in (v|T)^{-1}(a)$  and covering  $B_a$ . Let  $B_1$  be the union of all such continua over all  $a \in A \cap (M/K - W)$ . The set  $B_1$  is closed, for if  $b \in B_1^-$ , there is a net  $\tilde{b} \to b$  where  $\tilde{b}(i) \in B'_{\tilde{a}(i)}$ ,  $B'_{\tilde{a}(i)}$  a continua covering  $B_{\tilde{a}(i)}$ , where  $\tilde{a}(i)$  is a point in  $A \cap (M/K - W)$ . In the space of compact subsets of  $(M - \pi^{-1}(W))/K_1$ , there is a subnet  $B'_{\tilde{a}'a}$  converging to a continua B', and then  $b \in B'$ . Choose a subnet  $\tilde{a} \circ a \circ \beta$  converging to a point  $a \in A \cap (M/K - W)$ , and let  $a' \in \pi^{-1}(a) \cap B'$ . Then B' is contained in the component of a' in  $(M - \pi^{-1}(W))/K_1$  and this set maps under v onto  $B_a$ . Hence,  $b \in B_1$ . Since  $A_1 = (v|T)^{-1}(A)$  is connected,  $A_1 \cup B_1$  is then connected and meets  $(M - V)/K_1$ . Let  $M_1 = \pi_1^{-1}(A_1 \cup B_1)$ . Then  $M_1 \subset M$ ,  $M_1 \cap V \neq \emptyset$ ,  $p \in M_1 \cap G(p) = K_1(p)$ , and  $M_1/K_1$  is connected. We must show that (\*) is satisfied with respect to  $M_1 \cap V$ .

Let Z be any neighborhood of  $\pi_1(p)$  in  $\pi_1(M_1)$ . Choose a neighborhood  $Y \subset W$  such that  $(\nu|T)^{-1}(Y \cap \pi(M_1)) \subset T \cap Z$  and Y satisfies (\*) relative to W. Let C be the component of  $\pi(p)$  in  $Y^-$ , and  $c \in C \cap (M/K-Y)$ . By (\*) the component of c in M/K-Y meets  $M/K-\pi(V)$  and in particular the component C' of c in  $W^--Y$  meets  $U^--W$ . Hence  $C \cup C'$  is connected and therefore contained in A. Thus  $(\nu|T)^{-1}(C)$  is connected and contained in the component of Z. Also,  $(\nu|T)^{-1}(C \cup C')$  is connected and meets  $\nu^{-1}(A \cap (U^--W))$  and so meets  $B_1$  whose every component meets  $\pi_1(M_1-V)$ , which then proves (\*).

In order to prove the theorem, we observe that (G(P), G) satisfies the conditions of the new lemma 2 and so starts the induction. The rest of the proof remains unchanged.

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## Subdirect representations of relational systems

by

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1. Introduction. The representation of algebras as direct or subdirect products has been extensively investigated by Birkhoff [3], Hashimoto [7], Krull [9], and many others. Birkhoff establishes a necessary and sufficient condition for a universal algebra to be representable as a subdirect product. In this paper we give a generalization of the concept of subdirect product suitable for relational systems, and obtain representation conditions containing Birkhoff's result as a special case.

Adam [1] presents a counter example of L. Fuchs and G. Szász showing the invalidity of Birkhoff's conditions for an algebra to be represented as a direct product of finitely many algebras. We append to the subdirect representation conditions a third condition and prove the set both necessary and sufficient for the representation of a relational system as a direct product of finitely or infinitely many factors. This theorem is similar to a theorem of Hashimoto on the infinite direct product representations of an algebra.

Birkhoff also establishes the following representability theorem:

Every algebra is representable as a subdirect product of subdirectly irreducible algebras.

In section 5 we present a representability theorem for relational systems which yields Birkhoff's result as one special case, but also other more precise specializations to the algebraic case. Hindsight shows these specializations could have been obtained directly for algebras by Birkhoff's arguments alone.

Lyndon [10] and Pickert [12] have also generalized the concept of subdirect product to relational systems. The definition used by Pickert is slightly weaker than the definition presented here, and yields Birkhoff's condition for a relational system to be represented as a subdirect product. Pickert gives no direct product representation conditions. But the unity given to the two representation theorems by our definition makes it seem more natural. Pickert also fails to establish a significant subdirect product representability theorem, but it is clear that such