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On the units of cyclotomic fields

by

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§ 1. Let $f > 1$ be a natural number with $\varphi(f) > 2$, φ being the Euler totient function. Let α be a primitive fth root of unity and $Q(\alpha)$ the cyclotomic field generated by α over the rational number field Q. It is clear that for $(s,f) = 1$, $1 < s < f/2$, the numbers

$$u_s = \frac{\alpha^s - 1}{\alpha - 1} \tag{1}$$

are units of $Q(\alpha)$. Some time back Professor J. Milnor(¹) asked the following question: Do the units u_s together with $\pm\alpha$ form a basis for the units of $Q(\alpha)$?

In this note we prove the following two theorems.

THEOREM 1. *Let $f = \prod_{i=1}^{k} p_i^{a_i}$ be the prime factor decomposition of f and for $1 < s < f/2$, $(s,f) = 1$ let*

$$v_s = \prod_{e_i}{}' \left(\frac{1 - \alpha^{s p_1^{a_1 e_1} \cdots p_k^{a_k e_k}}}{1 - \alpha^{p_1^{a_1 e_1} \cdots p_k^{a_k e_k}}} \right)$$

where the product is extended over all $e_i = 0$ or 1, $i = 1, 2, \ldots, k$, except $e_1 = e_2 = \ldots = e_k = 1$. Then the $\frac{1}{2}\varphi(f) - 1$ units v_s of $Q(\alpha)$ generate a subgroup of finite index in the group of units of $Q(\alpha)$.

THEOREM 2. *Let p and q be two odd primes dividing f, q having the property that the residue class group* $\bmod q$ *has a nonprincipal character χ with $\chi(-1) = 1$, and $p \equiv 1 \pmod q$. Then the units u_s defined in* (1) *are multiplicatively dependent.*

Theorem 1 shows that if, in particular, f is a power of a prime, then the units u_s in (1) are multiplicatively independent, and hence generate

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(¹) In a letter to Professor K. G. Ramanathan dated 6th February 1964.

a subgroup of finite index in the unit group of $Q(\alpha)$. Theorem 2, in addition shows that if f is composite and divisible by at least two distinct odd primes the units u_s need not be independent. In the case of the units v_s of Theorem 1, the index of the subgroup generated by v_s in the group of all units of $Q(\alpha)$, is intimately connected with the class number of the cyclotomic field $Q(\alpha)$. These results in a more general setting will appear elsewhere.

After this paper was written we came to know from Professor Hyman Bass that he had proved some theorems which gave a system of units generating a subgroup of finite index in the group of all units of $Q(\alpha)$, this system being in general bigger than the maximal set. He has also a fairly simple proof of Theorem 2. However, our point of view is different and Theorem 1 appears to be new.

We should also mention that in the case where $f = p$ is an odd prime greater than 3, Theorem 1 is proved in Siegel's [7] lectures.

§ 2. In this section we set our notations and terminology and prove three lemmas which lead to the theorems stated in § 1. We denote by $\mathscr{R}_f$ the multiplicative group of residue classes prime to f modulo the subgroup generated by the classes 1 and -1. *Let χ be a nonprincipal character of $\mathscr{R}_f$.* Now if $f_1 > 1$ is a divisor of f then we have a map from $\mathscr{R}_f$ to $\mathscr{R}_{f_1}$ which takes a class R of $\mathscr{R}_f$ to the class of $\mathscr{R}_{f_1}$ represented by a representative of R. This map is well defined and onto. It may happen that for some divisor f_1 of f, χ may pass to a character of $\mathscr{R}_{f_1}$. If it passes to a character of $\mathscr{R}_{f_1}$ and also to a character of $\mathscr{R}_{f_2}$, it also passes to a character of $\mathscr{R}_{f_3}$ where $f_3 = (f_1, f_2)$ is the g.c.d. of f_1 and f_2. In this way we arrive at the least divisor f_χ of f such that χ will not pass to a character of $\mathscr{R}_{f_4}$ for a divisor f_4 of $f_\chi, f_4 \neq f_\chi$. *The character of $\mathscr{R}_{f_\chi}$ derived from χ will be denoted by χ_0.*

Let g be a divisor of f, $1 \leqslant g < f$, and write

$$\varphi_{f,g}(R) = \log|1 - e^{2\pi igr/f}| \tag{3}$$

where r is a representative of the class R of $\mathscr{R}_f$.

LEMMA 1.

$$V_{f,g} = \sum_{R \in \mathscr{R}_f} \bar{\chi}(R)\varphi_{f,g}(R) \tag{4}$$

$$= \begin{cases} 0 \quad \textit{if} \quad g \nmid f/f_\chi, \\ -\tfrac{1}{2}T(f_\chi)L(1, \chi_0)g \prod_{p \mid f\dagger, p \nmid g} (1 - |\chi_0(p)|p^{-1}) \prod_{p \mid f/g} (1 - \bar{\chi}_0(p)) \ \textit{if} \ g \mid f/f_\chi \end{cases}$$

where $T(f_\chi)$ is a certain gaussian sum of absolute value $\sqrt{f_\chi}$ and $\bar{\chi}$ is the complex conjugate character of χ.

Proof. If r is a representative of R we write $\chi(R) = \chi(r)$ and $\chi(r) = 0$ if $(r, f) > 1$. Now

$$V_{f,g} = \tfrac{1}{2} \sum_{\substack{R \in \mathscr{R}_f \\ r \text{ a representative of } R}} \bar{\chi}(R)\{\log(1 - e^{2\pi igr/f}) + \log(1 - e^{-2\pi igr/f})\}$$

$$= \tfrac{1}{2} \sum_{r \bmod f} \bar{\chi}(r) \log(1 - e^{2\pi igr/f})$$

$$= -\tfrac{1}{2} \sum_{r \bmod f} \bar{\chi}(r) \Big\{ \sum_{n=1}^{\infty} \frac{e^{2\pi ingr/f}}{n} \Big\} \tag{5}$$

$$= -\tfrac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n} \Big\{ \sum_{r \bmod f} \bar{\chi}(r) e^{2\pi ingr/f} \Big\}, \tag{6}$$

such rearrangements being permissible since we could have started with the series in (5) with $n^{-\sigma} e^{2\pi ingr/f}$ $(\sigma > 1)$ in place of $n^{-1} e^{2\pi ingr/f}$ and then passed to the limit $\sigma \to 1$.

Let

$$\begin{cases} f = \prod_{j=1}^{k} p_j^{a_j} \quad (k \geqslant 1, a_j > 0, j = 1, \ldots, k), \\ f_\chi = \prod_{j=1}^{l} p_j^{r_j} \quad (1 \leqslant l \leqslant k, 0 < r_j \leqslant a_j; j = 1, \ldots, l), \\ h = h_\chi = \prod_{j=1}^{l} p_j^{a_j}. \end{cases} \tag{7}$$

Now as α runs through a complete system of coprime residues $\bmod f/h$ and β through a complete system of coprime residues $\bmod h$, the numbers $r = h\alpha + \frac{f}{h}\beta$ run through a complete system of coprime residues $\bmod f$ each only once. Thus we have

$$T_{f,g}(\chi) = \sum_{r \bmod f} \bar{\chi}(r) e^{2\pi ingr/f} = \sum_{\xi_0 \bmod f_\chi} \bar{\chi}(\xi_0) \sum_{\substack{\xi \equiv \xi_0 \bmod f_\chi \\ (\xi, f) = 1, \xi \bmod f}} e^{2\pi ing\xi/f}$$

$$= \sum_{\beta_0 \bmod f_\chi} \bar{\chi}_0\Big(\frac{f}{h}\beta_0\Big) \sum_{\substack{\beta \equiv \beta_0 \bmod f_\chi \\ \beta \bmod h}} \sum_{\substack{\alpha \bmod f/h \\ (\alpha, f/h) = 1}} e^{2\pi ing/f(h\alpha + f\beta/h)}$$

$$= \bar{\chi}_0\Big(\frac{f}{h}\Big) \Big\{ \sum_{\beta_0 \bmod f_\chi} \bar{\chi}_0(\beta_0) \sum_{\substack{\beta \equiv \beta_0 \bmod f_\chi \\ \beta \bmod h}} e^{2\pi ing\beta/h} \Big\} \Big\{ \sum_{\substack{\alpha \bmod f/h \\ (\alpha, f/h) = 1}} e^{2\pi ing\alpha h/f} \Big\}$$

$$= \bar{\chi}_0\Big(\frac{f}{h}\Big) \Big\{ \sum_{\beta_0 \bmod f_\chi} \bar{\chi}_0(\beta_0) e^{2\pi ing\beta_0/h} \Big\} \Big\{ \sum_{\substack{\beta \equiv 0 \bmod f_\chi \\ \beta \bmod h}} e^{2\pi ing\beta/h} \Big\} \Big\{ \sum_{\substack{\alpha \bmod f/h \\ (\alpha, f/h) = 1}} e^{2\pi ingh\alpha/f} \Big\}.$$

The sum in the second bracket vanishes unless $hf_\chi^{-1}|ng$. In this case the sum is h/f_χ, and further since ng/h will have an exact denominator which divides f_χ, the sum in the first bracket will vanish unless ng/h will have exact denominator f_χ. The sum in the third bracket is the Ramanujan sum $C_{f/h}(ng)$ (properties necessary will be stated below and are not hard to prove). Thus

$$T_{f,g}(\chi) = \begin{cases} 0 & \text{if} \quad (h, ng) \neq h/f_\chi, \\ \bar{\chi}_0\left(\dfrac{f}{h}\right)\dfrac{h}{f_\chi}C_{f/h}(ng) \displaystyle\sum_{\beta_0 \bmod f_\chi} \bar{\chi}_0(\beta_0)\, e^{2\pi i n g \beta_0/h} & \text{if} \quad (h, ng) = \dfrac{h}{f_\chi}. \end{cases}$$

It is a standard result that the sum

$$\sum_{\beta_0 \bmod f_\chi} \bar{\chi}_0\left(\frac{\beta_0 n g f_\chi}{h}\right) e^{2\pi i n g \beta_0/h} = T(f_\chi)$$

is independent of n and is of absolute value $\sqrt{f_\chi}$. Hence

$$T_{f,g}(\chi) = \begin{cases} 0 & \text{if} \quad (h, ng) \neq h/f_\chi, \\ \bar{\chi}_0\left(\dfrac{f}{h}\right)\dfrac{h}{f_\chi}C_{f/h}(ng)\,\chi_0\left(\dfrac{ngf_\chi}{h}\right)T(f_\chi) & \text{if} \quad (h, ng) = \dfrac{h}{f_\chi}. \end{cases} \tag{8}$$

Also

$$C_{f/h}(ng) = \prod_{j=l+1}^{k} C_{p_j^{a_j}}(ng) \tag{9}$$

by the multiplicative property of the Ramanujan sum and

$$C_{p_j^{a_j}}(ng) = \begin{cases} 0 & \text{if} \quad (ng, p_j^{a_j}) \mid p_j^{a_j-2}, \\ -p_j^{a_j-1} & \text{if} \quad (ng, p_j^{a_j}) = p_j^{a_j-1}, \\ \varphi(p_j^{a_j}) & \text{if} \quad (ng, p_j^{a_j}) = p_j^{a_j}. \end{cases} \tag{10}$$

We now go back to the series (5) for $V_{f,g}$. If $f_\chi \nmid f/g$, i.e. $g \nmid f/f_\chi$, then (5) vanishes identically by (8) or directly from the definition of f_χ, since the series in the curly brackets in (5) is an invariant of the classes of the quotient of $\mathscr{R}_f$ modulo the kernel of the map from $\mathscr{R}_f$ to $\mathscr{R}_{f/g}$. As for the series (6), we write

$$\begin{aligned} g &= \prod_{j=1}^{k} p_j^{t_j} \quad (0 \leqslant t_j \leqslant a_j - r_j;\ j = 1, \ldots, l, \\ &\qquad\qquad\qquad 0 \leqslant t_j \leqslant a_j; \quad j = l+1, \ldots, k), \\ g_1 &= \prod_{j=1}^{l} p_j^{t_j}, \quad g_2 = \prod_{\substack{j \geqslant l+1 \\ t_j = a_j}} p_j^{a_j}, \quad g_3 = \prod_{\substack{j \geqslant l+1 \\ t_j < a_j}} p_j^{t_j}, \\ h_1 &= \prod_{\substack{j \geqslant l+1 \\ t_j < a_j}} p_j^{a_j} = \frac{f}{hg_2}. \end{aligned} \tag{11}$$

The condition $(ng, h) = h/f_\chi$ for n now reads

$$\Big(\prod_{j=1}^{l} p_j^{a_j - t_j}, n\Big) = \prod_{j=1}^{l} p_j^{a_j - t_j - r_j},$$

and so all such n are given by

$$n = \Big(\prod_{j=1}^{l} p_j^{a_j - t_j - r_j}\Big) m_1 = \frac{hm_1}{f_\chi g_1} \tag{12}$$

where m_1 runs through integers prime to h, i.e. to f_χ. Inserting (12) into (6) and using (8) we get

$$V_{f,g} = \begin{cases} 0 & \text{if} \quad g \nmid \dfrac{f}{f_\chi} \\ -\dfrac{1}{2}\cdot\dfrac{h}{f_\chi}T(f_\chi) \displaystyle\sum_{n=1}^{\infty}{}' \frac{1}{n}\left\{C_{f/h}(ng)\,\chi_0\left(\frac{ngf_\chi}{f}\right)\right\} & \text{if} \quad g \,\Big|\, \dfrac{f}{f_\chi}, \end{cases} \tag{13}$$

where $\sum_{n=1}^{\infty}{}'$ denotes the sum over integers given by (12). We have

$$\chi_0\left(\frac{ngf_\chi}{f}\right) = \chi_0(m_1)\,\chi_0\left(\frac{hg}{g_1 f}\right).$$

By (9) and (10) we may restrict the sum only to those n for which

$$\left(ng, \frac{f}{h}\right) = \prod_{j=l+1}^{k} p_j^{a_j - e_j} \quad (e_j = 0 \text{ or } 1,\ j = l+1, \ldots, k). \tag{14}$$

Observing further that for those j for which $p_j | g_2$, e_j has necessarily to be zero, the summation may further be restricted only to those n for which

$$n = \Big(\prod_{p_j | h_1} p_j^{a_j - t_j - e_j}\Big) \frac{mh}{f_\chi g_1} \tag{15}$$

where m is coprime to $f_\chi \prod_{p_j | h_1} p_j^{e_j} = f_{g,e}$, say. Hence the summation may be split up into $2^{k-l-\mu}$ parts (where μ is the total number of prime factors of g_2) each with a different choice of the numbers e_j ($j \geqslant l+1$, $t_j < n_j$). With a particular choice of the numbers e_j we have in case $g | f/f_\chi$ the contribution

$$\sum_{n=1}^{\infty}{}' \chi_0\left(\frac{ngf_\chi}{f}\right) C_{f/h}(ng)\, n^{-1} \tag{16}$$

to the sum over n in (13); here the sum is over all numbers n of the form (15). We have

$$\chi_0\left(\frac{ngf_\chi}{f}\right) = \chi_0(m)\,\chi_0\left(\frac{gh}{fg_1}\right)\chi_0\left(\frac{h_1}{g_3}\right)\prod_{p_j|h_1}\chi_0(p_j^{-e_j}),$$

$$n^{-1} = m^{-1}\left(\frac{g_1g_3f_\chi}{hh_1}\right)\prod_{p_j|h_1}p_j^{e_j}, \tag{17}$$

$$C_{f/h}(ng) = \left\{\prod_{p_j|g_2}\varphi(p_j^{a_j})\right\}\prod_{p_j|h_1}[\{\varphi(p_j^{a_j})\}^{1-e_j}(-p_j^{a_j-1})^{e_j}] = \varphi\left(\frac{f}{h}\right)\prod_{p_j|h_1}(1-p_j)^{-e_j}.$$

Hence if $g|f/f_\chi$, we have, for $V_{f,g}$ from (13), the expression

$$-\tfrac{1}{2}\chi_0\left(\frac{gh}{fg_1}\right)\frac{h}{f_\chi}T(f_\chi)\varphi\left(\frac{f}{h}\right)\left(\frac{g_1g_3f_\chi}{hh_1}\right)\chi_0\left(\frac{h_1}{g_3}\right)\times$$

$$\times\sum_{e's=0,1}\left[\prod_{p_j|h_1}\{-\bar\chi_0(p_j)p_j(p_j-1)^{-1}\}^{e_j}\sum_{(m,f_{g,e})=1}\frac{\chi_0(m)}{m}\right]$$

$$= -\tfrac{1}{2}L(1,\chi_0)\chi_0\left(\frac{ghh_1}{fg_1g_3}\right)T(f_\chi)\varphi\left(\frac{f}{h}\right)\left(\frac{g_1g_3}{h_1}\right)\times$$

$$\times\sum_{e's=0,1}\prod_{p_j|h_1}\{(-\bar\chi_0(p_j))p_j(p_j-1)^{-1}(1-\chi_0(p_j)p_j^{-1})\}^{e_j}$$

$$= -\tfrac{1}{2}L(\,,\chi_0)\,\chi_0\left(\frac{ghh_1}{fg_1g_3}\right)T(f_\chi)g_1g_3\varphi(g_2)\times$$

$$\times\sum_{e's=0,1}\prod_{p_j|h_1}[\{-\bar\chi_0(p_j)(1-\chi_0(p_j)p_j^{-1})\}^{e_j}(1-p_j^{-1})^{1-e_j}]$$

$$= -\tfrac{1}{2}L(1,\chi_0)\,T(f_\chi)g\prod_{p|f,p\nmid f/g}\left(1-\frac{|\chi_0(p)|}{p}\right)\prod_{p_j|h_1}(1-\bar\chi_0(p_j))$$

since

$$ghh_1 = fg_1g_3,\qquad \sum_{e's=0,1}\prod_{r=1}^{n}a_r^{e_r}b_r^{1-e_r} = \prod_{r=1}^{n}(a_r+b_r),$$

and

$$g_1g_3\varphi(g_2) = g\prod_{p|f,p\nmid f/g}\left(1-\frac{|\chi_0(p)|}{p}\right).$$

Lemma 1 is completely proved.

We now define the function (constructed from (3))

$$\Psi(R) = \Psi_a(R) = \sum_{e's=0,1}{}' a^{e_1+\ldots+e_k}\varphi_{f,p_1^{a_1e_1}\ldots p_k^{a_ke_k}}(R) \tag{18}$$

the sum extended over all $(e_1, e_2, \ldots, e_k) \neq (1, 1, \ldots, 1)$. We then have

LEMMA 2.

$$U = \sum_{R\in\mathcal{R}_f}\bar\chi(R)\Psi(R) = -\tfrac{1}{2}T(f_\chi)L(1,\chi_0)\prod_{p_j|f/h}\{1+a\varphi(p_j^{a_j})-\bar\chi_0(p_j)\}. \tag{19}$$

Proof. Since we have to consider only those g for which $g|f/f_\chi$, we have

$$U = -\tfrac{1}{2}T(f_\chi)L(1,\chi_0)\sum_{e's=0,1}{}'\Big[a^{e_{l+1}+\ldots+e_k}p_{l+1}^{a_{l+1}e_{l+1}}\ldots p_k^{a_ke_k}\times$$

$$\times\prod_{i=l+1}^{k}(1-|\chi_0(p_i)|p_i^{-1})^{e_i}\prod_{i=l+1}^{k}(1-\bar\chi_0(p_i))^{1-e_i}\Big]$$

$$= -\tfrac{1}{2}T(f_\chi)L(1,\chi_0)\sum_{e's=0,1}{}'\Big[\prod_{p_j|f/h}\{(a\varphi(p_j^{a_j}))^{e_j}(1-\bar\chi_0(p_j))^{1-e_j}\}\Big]$$

$$= -\tfrac{1}{2}T(f_\chi)L(1,\chi_0)\prod_{p_j|f/h}\{1+a\varphi(p_j^{a_j})-\bar\chi_0(p_j)\},$$

because all our calculations fail when all the e's are equal to 1 (and to avoid this trouble) we could have taken continuous invariants

$$\varphi_{f,g}(R,\sigma) = -\tfrac{1}{2}\sum_{n=1}^{\infty}e^{2\pi i ngr/f}n^{-\sigma}\qquad(\sigma>1)$$

and come to the conlusion that the term for which $f = g$ is zero.

Next we prove

LEMMA 3. *Let a be a primitive f-th root of unity and u_s as defined in* (1). *Then for a nonprincipal character χ of $\mathcal{R}_f$*

$$\sum_{(s,f)=1,\,s<f/2}\bar\chi(s)\log|u_s| = -\tfrac{1}{2}T(f_\chi)L(1,\chi_0)\prod_{p|f}(1-\chi_0(p))\,\varrho_a \tag{20}$$

where ϱ_a is a certain root of unity depending on a and χ.

Proof. Let $\sigma = e^{2\pi i b/f}$, $(b,f) = 1$. Then $\log|1-e^{2\pi i bs/f}| = \varphi_{f,1}(RR_b)$ where R_b is the class of b. Hence

$$\sum_{(s,f)=1,\,s<f/2}\bar\chi(s)\log|u_s| = \sum_{R\in\mathcal{R}_f}\bar\chi(R)\varphi_{f,1}(RR_b) = \chi(R_b)V_{f,1}$$

and this proves Lemma 3.

§ 3. We now prove Theorems 1 and 2 stated in § 1.

Proof of Theorem 2. Let χ_0 be the real nonprincipal character mod q for which $\chi_0(-1)=1$. We extend it to a character χ of $\mathscr{R}_f$ in a natural way (since $\mathscr{R}_q$ is a quotient of $\mathscr{R}_f$). For this character χ, Lemma 3 at once gives

$$\Big|\prod_{\substack{(s,f)=1\\ 1<s<f/2}} u_s^{\chi(s)}\Big| = 1$$

whatever be the primitive root a with which we start. Hence the unit

$$\prod_{(s,f)=1,\ 1<s<f/2} u_s^{\chi(s)-1} \tag{21}$$

is a root of unity.

Proof of Theorem 1. Denote the elements of $\mathscr{R}_f$ by $R_0, R_1, R_2, \ldots$, R_0 being the unit element. Let $\boldsymbol{\Theta}(R) = \prod\nolimits'_{e's=0,1}(1-a^{p_1^{a_1e_1}\ldots p_k^{a_ke_k}r})$, the product being extended over all k-tuples except $(1,1,\ldots,1)$, and r being a representative of R. Now we have $v_s = \boldsymbol{\Theta}(R_s)/\boldsymbol{\Theta}(R_0)$ and if the units v_s are dependent, say $\prod_{(s,f)=1, s<f/2} v_s^{b_s}=1$, on applying the isomorphisms $\sigma(R_j^{-1})$ we have

$$\prod_{(s,f)=1,\ s<f/2} v_s^{b_s\sigma(R_j^{-1})} = 1 \qquad (j=0,1,2,\ldots),$$

i.e.

$$\sum_{i\neq 0} b_i \log\left|\frac{\boldsymbol{\Theta}(R_iR_j^{-1})}{\boldsymbol{\Theta}(R_j^{-1})}\right| = 0 \qquad (j=0,1,2,\ldots),$$

where we have changed s to i and replaced the expression for v_i in terms of $\boldsymbol{\Theta}(R_i)$. Since b_i are not all zero, we have

$$\text{determinant}\left|\log\left|\frac{\boldsymbol{\Theta}(R_iR_j^{-1})}{\boldsymbol{\Theta}(R_j^{-1})}\right|\right|_{i\neq 0, j\neq 0} = 0. \tag{22}$$

But by Dedekind-Frobenius group determinant formula the determinant on the left is nothing but $\prod'_\chi \sum_R \chi(R)\log|\boldsymbol{\Theta}(R)| = \prod'_\chi \sum_R \chi(R)\boldsymbol{\Psi}(R)$ by (19) and this contradicts (22). Hence Theorem 1 is proved.

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