A) Replace X by $X - f_{-1}$. Then all proofs remain valid as given.

B) Add hypothesis relating connectivity to f_{-1}. We shall do this just for Proposition 2.1 in order to indicate the procedure.

Proposition 2.1. *A subset W of X is convex iff for any flat f with $\dim(f) \leqslant 1$, $f \cap W$ is connected.*

In the proof we no longer need that $\{x, y\}$ is linearly independent, but only that $\dim(f(x, y)) \leqslant 1$. This follows immediately from 4).

PROPOSITION. *Axiom* 2) *is derivable from* A1 *and axioms* 1), 4), 5), 6).

Proof. Suppose $x \in f_{-1}$. Then $\{x\} \subset f_{-1}$ and we have a 0-flat contained in the (−1)-flat, which contradicts 6). Thus $F^{-1} = \{\emptyset\}$.

The Theory of Dependence as given in [2] together with the following definition:

DD. $f \in F^k$ *iff there exist elements* $a_0, \ldots, a_k$ *such that* $\langle a_0, \ldots, a_k \rangle$ *is independent and* $f = \{a \mid a$ *depends on* $\langle a_0, \ldots, a_k \rangle\}$

is inferentially equivalent to the system consisting of Gemignani's axioms 1), 4), 5), 6) together with the following definition:

DG. *An element a depends on the sequence $\langle a_0, \ldots, a_k \rangle$ iff a is an element of the flat determined by* $\{a_0, \ldots, a_k\}$.

We omit the proof.

Note. The interplay between the Theory of Dependence and Topological Geometry is as yet unexplored.

References

[1] M. C. Gemignani, *Topological geometry and a new characterization of* R^m, Notre Dame Journal of Formal Logic 7 (1966), pp. 57-100.

[2] A. Seidenberg, *Lectures in projective geometry*, D. Van Nostrand, Princeton 1962.

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Note on a paper of Wojdysławski

by

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Wojdysławski [1] gives the following characterization of an absolute retract (AR).

THEOREM. *A necessary and sufficient condition for $\mathfrak{X}$ to be an* AR *is the existence of a barycentric function for $\mathfrak{X}$.*

(The appropriate definitions are in [1]; all spaces there are separable metric.) To establish the necessity of the condition the author states (in Section 6) that if g is a barycentric function for $\mathfrak{Y}$ and r is a function retracting $\mathfrak{Y}$ onto $\mathfrak{X}$ then rg is a barycentric function for $\mathfrak{X}$. This statement is incorrect. The theorem is true, however, and the necessity of the condition is established below by a slight modification of the author's technique. The notation here is the same as in [1].

To see that the statement in Section 6 is wrong we take $\mathfrak{Y}$ to be the closed interval $[0:4]$ and $\mathfrak{X}$ to be the subinterval $[0:1]$. We construct a retraction of $\mathfrak{Y}$ onto $\mathfrak{X}$ as the composite of two maps. First let $f_1\colon [0:4] \to [0:2]$ be the map which is the identity on $[0:1]$ and sends $[2:3]$ linearly onto $[1:2]$ with $f_1([1:2]) = 1$ and $f_1([3:4]) = 2$. Then let $f_2\colon [0:2] \to [0:1]$ be the map which is the identity on $[0:1]$ and maps both $[1:3/2]$ and $[3/2:2]$ linearly onto $[1/2:1]$ so that $f_2(1) = f_2(2) = 1$ and $f_2(3/2) = 1/2$. Then $r = f_2 f_1$ is a retraction of $\mathfrak{Y}$ onto $\mathfrak{X}$. We shall use the fact that $r([1:2]) = r([3:4]) = 1$ and $r(5/2) = 1/2$.

Let $g\colon \mathfrak{L} \to \mathfrak{Y}$ be the barycentric mapping constructed in Section 5. Choose subsequences $\{\Delta_{k_n}\}$ and $\{\Delta_{h_n}\}$ from $\{\Delta_n\}$ so that for each n we have $g(\Delta_{k_n}) \in [1:2]$ and $g(\Delta_{h_n}) \in [3:4]$. For each integer n let $S_{p_n} = \{\Delta_{k_n}, \Delta_{h_n}\}$. Then since g is continuous on each Δ_{p_n} and maps the endpoints of each Δ_{p_n} into $[1:2]$ and $[3:4]$, we have $5/2 \in g(\Delta_{p_n})$ for every n. Then for each n we have $rg(S_{p_n}) = \{1\}$ and $1/2 = r(5/2) \in rg(\Delta_{p_n})$, so diameter $rg(\Delta_{p_n}) = 1/2$. Thus condition 3.3 for barycentric functions does not hold for rg.

This example can be slightly generalized to show that rg is not a barycentric mapping for $\mathfrak{X}$ even when $\mathfrak{X}$ and $\mathfrak{Y}$ are related as in Section 7—

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i.e., when $\mathfrak{Y}$ is the convex hull (in a normed linear space) of its closed subset $\mathfrak{X}$. (Take for $\mathfrak{X}$ a "bent" line segment so that $\mathfrak{Y}$ is then a 2-simplex, and let two disjoint closed discs in $\mathfrak{Y}\setminus\mathfrak{X}$ play the role of the intervals $[1:2]$ and $[3:4]$ in the above example.)

The existence of the barycentric function can be established in the following way. We can assume that $\mathfrak{X}$ is a relatively closed subset of a convex set $\mathfrak{Y}$ in some normed linear space. Let $\{x_n\}_{n=1}^{\infty}$ be a dense subset of $\mathfrak{X}$. Define $g(\Delta_m) = x_m$ for each integer m and extend linearly (as in Section 5) on each simplex Δ_n to get $g\colon \mathfrak{L}\to\mathfrak{Y}$. (Note that g is not necessarily a barycentric function for $\mathfrak{Y}$.) If $r\colon \mathfrak{Y}\to\mathfrak{X}$ is a retraction of $\mathfrak{Y}$ onto $\mathfrak{X}$, then rg is a barycentric function for $\mathfrak{X}$. Only condition 3.3 is in doubt, so assume that $\{S_{k_n}\}$ is a subsequence of $\{S_n\}$ such that $\lim\limits_{n\to\infty} rg(S_{k_n}) = x \in \mathfrak{X}$. Then, since $g(S_{k_n}) \subset \mathfrak{X}$, we have $rg(S_{k_n}) = g(S_{k_n})$ for each integer n. Then $\lim\limits_{n\to\infty} g(S_{k_n}) = x$ and g is linear on each Δ_{k_n}, so we have $\lim\limits_{n\to\infty} g(\Delta_{k_n}) = x$. Then it follows from the continuity of r that $\lim\limits_{n\to\infty} rg(\Delta_{k_n}) = x$.

Reference

[1] M. Wojdysławski, *Rétractes absolus et hyperspaces des continus*, Fund. Math. 32 (1939), pp. 184-192.

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Zu einem Satz von E. S. Wolk über die Vergleichbarkeitsgraphen von ordnungstheoretischen Bäumen

by

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$G = (E, K)$ sei Graph mit der Eckenmenge E und der Kantenmenge K (die $k \in K$ sind zweielementige Teilmengen von E). Eine antisymmetrische Relation $R \subseteq E\times E$ heißt *Richtung* von G, wenn

(1) $$\{e, e'\} \in K \Longleftrightarrow eRe' \quad \text{oder} \quad e'Re \qquad (e, e' \in E)\,.$$

Eine teilweise geordnete Menge $(E, <)$ heißt *ordnungstheoretischer Baum*, wenn

(2) $$e' < e\,,\ e'' < e \Rightarrow e' \leqslant e'' \quad \text{oder} \quad e'' \leqslant e' \qquad (e, e', e'' \in E)\,.$$

Die Richtung $<$ von $G = (E, K)$ heißt *Baumrichtung*, wenn $<$ einen ordnungstheoretischen Baum auf E definiert.

E. S. Wolk charakterisierte in [5] die Graphen, dei eine Baumrichtung zulassen. Wir führen in dieser Note eine Äquivalenzrelation auf der Eckenmenge eines (jeden) Graphen ein, die einen einfachen Beweis dieser Charakterisierung ermöglicht und zugleich eine völlige Übersicht über sämtliche Baumrichtung eines gegebenen Graphen liefert.

Def. 1. $G = (E, K)$ sei Graph und $k = \{e, e'\} \in K$.

a) e heißt *V-Scheitel von k*, wenn ein $e'' \in E$ existiert mit $e'' \neq e'$, $\{e, e''\} \in K$ und $\{e', e''\} \notin K$.

b) k heißt *ν-Kante*, wenn genau ν der Ecken e, e' V-Scheitel von k sind ($\nu = 0, 1, 2$).

Def. 2. $G = (E, K)$ sei Graph. Für $e, e' \in E$ setzen wir $e\sim e'$, wenn $e = e'$ oder wenn $\{e, e'\}$ eine 0-Kante von G ist.

Bemerkung 1. *$\sim$ definiert eine Äquivalenzrelation auf E.*

Beweis. Es ist nur die Transitivität zu zeigen. Sei $e_1\sim e_2$ und $e_2\sim e_3$; o.B.d.A. seien e_1, e_2, e_3 verschieden. Somit gilt $\{e_1, e_2\} \in K$ und $\{e_2, e_3\} \in K$. Da e_2 nicht V-Scheitel von $\{e_1, e_2\}$ ist, ergibt sich $\{e_1, e_3\} \in K$. Ist etwa e_1 V-Scheitel von $\{e_1, e_3\}$, so gibt es ein $e_4 \neq e_3$ mit $\{e_1, e_4\} \in K$ und $\{e_3, e_4\} \notin K$. Im Fall $\{e_2, e_4\} \notin K$ wäre e_1 V-Scheitel von $\{e_1, e_2\}$, im Fall $\{e_2, e_4\} \in K$