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On a theorem of L. Schwartz and its applications to absolutely summing operators

by

S. KWAPIEŃ (Warszawa)

1. Introduction. In a recently published paper [12], L. Schwartz has presented a theory of p-radonifying operators. He has found numerous applications of its main theorem (the "duality theorem for radonifying operators"; cf. [13]). It has been also observed by him that p-radonifying operators and p-absolutely summing are "almost identical". This creates the possibility of applying the "duality theorem" to the theory of p-absolutely summing operators. The aim of this paper is to show how this theorem may be used to obtain in a simple way already known results as well as new ones in the theory of p-absolutely summing operators. To make this paper self-contained § 2 restates some of the results of L. Schwartz. We use here neither the theory of cylindrical measures nor of radonifying operators. All the theorems are formulated in the language of absolutely summing operators. The "duality theorem" is essentially the same as Theorem 1 of this paper.

Let us recall that if E, F are Banach spaces and $0 < p < +\infty$, then an operator $u\colon E \to F$ is said to be *p-absolutely summing* (we shall write $u \in \pi_p(E, F)$) if there exists a constant C such that for each $x_1, \ldots, x_n \in E$

$$\sum_{i=1}^{n} \|u(x_i)\|^p \leqslant C \sup_{x' \in E', \|x'\| \leqslant 1} \sum_{i=1}^{n} |\langle x_i, x'\rangle|^p.$$

u is said to be *0-absolutely summing* ($u \in \pi_0(E, F)$) if for each $\varepsilon > 0$ there exists a $\delta > 0$ such that if $x_1, x_2, \ldots, x_n \in E$ and

$$\sup_{x' \in E', \|x'\| \leqslant 1} \sum_{i=1}^{n} \frac{1}{n} \min\{1, |\langle x_i, x'\rangle|\} < \delta,$$

then

$$\sum_{i=1}^{n} \frac{1}{n} \min\{1, \|u(x_i)\|\} < \varepsilon.$$

Let us observe that the definition of a p-absolutely summing operator for $0 < p < +\infty$ is meaningful when F is a homogeneous quasi-normed linear space (cf. [3], p. 159), for instance the space L_s with $0 < s < 1$, and the definition of a 0-absolutely summing operator is meaningful when F is a metric linear space. The last definition is a reformulation of Schwartz definition of a radonifying operator in the terms of absolutely summing operators. It is known that if $0 \leqslant q < p$, then $\pi_q(E, F) \subset \pi_p(E, F)$.

2. If Ω is a topological Haussdorf space, μ a probability Radon measure on Ω, and $0 \leqslant p \leqslant \infty$, then by $L_p(\Omega, \mu)$ we shall denote the space of all μ-measurable functions on Ω, $f(\cdot)$, such that

$$\|f\| = \begin{cases} \left(\int_\Omega |f(\omega)|^p d\mu(\omega)\right)^{1/p} < \infty & \text{if } 0 < p < \infty, \\ \operatorname{sup\,ess} |f(\omega)| < \infty & \text{if } p = \infty, \\ \int_\Omega \min\{1, |f(\omega)|\} d\mu(\omega) & \text{if } p = 0. \end{cases}$$

If $\Omega = [0, 1]$ and μ is the Lebesgue measure, then we shall write L_p instead of $L_p(\Omega, \mu)$.

A linear operator γ from a Banach space E into $L_p(\Omega, \mu)$ is called *p-decomposable* if there exists a map $\varphi\colon \Omega \to E'$ such that

1° for each $x \in E$ the function on Ω, $\langle x, \varphi(\cdot)\rangle$, is μ-measurable and equal to $\gamma(x)$ μ-almost everywhere;

2° there exists an $f \in L_p(\Omega, \mu)$ such that $\|\varphi(\omega)\| \leqslant f(\omega)$ μ-almost everywhere.

Remark 1. If E' is separable, then 1° implies that φ is Bochner-measurable. If E is reflexive, 1° and 2° imply that there exists a $\bar{\varphi}$ which is Bochner-measurable and which satisfies 1° and 2°.

THEOREM 1. *If E is a Banach space, $0 \leqslant p < \infty$, and $\gamma\colon E \to L_p(\Omega, \mu)$ is p-decomposable, then γ is p-absolutely summing.*

Proof. If $p > 0$ and $x_1, x_2, \ldots, x_n \in E$, then

$$\sum_{i=1}^{n} \|\gamma(x_i)\|^p = \int_\Omega \left(\sum_{i=1}^{n} |\langle x_i, \varphi(\omega)\rangle|^p\right) d\mu(\omega)$$

$$\leqslant \left(\int_\Omega |f(\omega)|^p d\mu(\omega)\right) \sup_{x' \in E', \|x'\| \leqslant 1} \sum_{i=1}^{n} |\langle x_i, x'\rangle|^p.$$

If $p = 0$, $\varepsilon > 0$ and $x_1, x_2, \ldots, x_n \in E$, then for each $M > 0$

$$\sum_{i=1}^{n} \frac{1}{n} \min\{1, \|\gamma(x_i)\|\}$$

$$= \sum_{i=1}^{n} \frac{1}{n} \min\left\{1, \int_\Omega \min\{1, |\langle x_i, \varphi(\omega)\rangle|\, d\mu(\omega)\right\}$$

$$= \int_\Omega \left(\sum_{i=1}^{n} \frac{1}{n} \min\{1, |\langle x_i, \varphi(\omega)\rangle|\}\right) d\mu(\omega)$$

$$\leqslant \left(\int_{f(\omega) \leqslant M} \max\{1, f(\omega)\} d\mu(\omega)\right) \cdot \sup_{x' \in E', \|x'\| \leqslant 1} \sum_{i=1}^{n} \frac{1}{n} \min\{1, |\langle x_i, x'\rangle|\} +$$
$$+ \mu(\{\omega : f(\omega) > M\}).$$

Now it is enough to choose M so that

$$\mu(\{\omega : f(\omega) > M\}) < \frac{\varepsilon}{2}$$

and put

$$\delta = \frac{\varepsilon}{2} \left(\int_{f(\omega) \leqslant M} \max\{1, f(\omega)\} d\mu(\omega)\right)^{-1}.$$

THEOREM 2. *Let E, F be Banach spaces, $0 \leqslant p < \infty$, and let $u\colon E \to F$ be a continuous linear operator such that the adjoint $u^t \in \pi_p(F', E')$. If $\gamma\colon F \to L_p(\Omega, \mu)$ is a continuous linear operator and either $p \geqslant 1$ or F has the metric approximation property* (cf. [2]), *then γu is p-decomposable.*

Proof. We shall prove that the image of the unit ball of E is bounded in the lattice $L_p(\Omega, \mu)$. This is equivalent to the following statements: if $p > 0$, there exists a constant M such that if $A_1, A_2, \ldots, A_m$ are measurable mutually disjoint subsets of Ω and $x_1, x_2, \ldots, x_m \in E$ with $\|x_i\| \leqslant 1$ for $i = 1, 2, \ldots, m$, then

$$\sum_{j=1}^{m} \int_{A_j} |\gamma u(x_j)(\omega)|^p d\mu(\omega) \leqslant M;$$

if $p = 0$ for each $\varepsilon > 0$ there exists a $\varrho > 0$ such that if $A_1, A_2, \ldots, A_m$ are measurable mutually disjoint subsets of Ω and $x_1, x_2, \ldots, x_m \in E$ with $\|x_i\| \leqslant 1$ for $i = 1, 2, \ldots, m$, then

$$\sum_{j=1}^{m} \int_{A_j} \min\{1, \eta |\gamma u(x_j)(\omega)|\} d\mu(\omega) < \varepsilon \quad \text{for } \eta < \varrho.$$

Suppose that γ is of the form

$$(*) \qquad \gamma = \sum_{i=1}^{n} y_i' \otimes \chi_{B_i},$$

where $y_1', \ldots, y_n' \in F'$, $B_1, \ldots, B_n$ are measurable mutually disjoint subsets of Ω with $\mu(B_i) = 1/n$ $(i = 1, \ldots, n)$, and χ_B is the characteristic function of B. Then

$$\sum_{j=1}^{m} \int_{A_j} |\gamma u(x_j)(\omega)|^p d\mu(\omega)$$

$$= \sum_{j=1}^{m} \int_{A_j} \Big| \sum_{i=1}^{n} \langle ux_j, y_i' \rangle \chi_{B_i}(\omega) \Big|^p d\mu(\omega) = \sum_{j=1}^{m} \sum_{i=1}^{n} |\langle ux_j, y_i' \rangle|^p \mu(A_j \cap B_i)$$

$$\leqslant \frac{1}{n} \sum_{i=1}^{n} \|u^t y_i'\|^p \leqslant \frac{1}{n} C \sup_{y'' \in F'', \|y''\| \leqslant 1} \sum_{i=1}^{n} |\langle y_i', y'' \rangle|^p$$

$$= C \sup_{y \in F, \|y\| \leqslant 1} \sum_{i=1}^{n} |\langle y, y_i' \rangle|^p \mu(B_i) = C \sup_{y \in F, \|y\| \leqslant 1} \int_{\Omega} |\gamma(y) \cdot (\omega)|^p d\mu(\omega) = C\|\gamma\|^p,$$

where C is a constant, as in the definition of p-absolutely summing operators (resp. when $p = 0$, by a similar computation we get

$$\sum_{i=1}^{m} \int_{A_i} \min\{1, \eta\,|\gamma u(x_i)(\omega)\}\, d\mu(\omega) < \varepsilon,$$

whenever

$$\sup_{y \in F, \|y\| \leqslant 1} \|\gamma(\eta y)\| < \delta,$$

where δ is a constant corresponding to ε, as in the definition of a 0-absolutely summing operator). Let $\gamma\colon E \to L_p(\Omega, \mu)$ be a continuous linear operator. If either E has the metric approximation property or $p \geqslant 1$, then γ is in a closure in the pointwise convergence topology of an equicontinuous family of linear operators of the form $(*)$. It follows from the above formulas that there exists a constant M such that for each $x_1, x_2, \ldots, x_m \in E$ $\|x_1\| \leqslant 1, \ldots, \|x_m\| \leqslant 1$ and $A_1, \ldots, A_m$ mutually disjoint measurable subsets of Ω

$$\sum_{i=1}^{m} \int_{A_i} |\gamma u(x_i)(\omega)|^p d\mu(\omega) \leqslant M$$

(resp.

$$\sum_{i=1}^{m} \int_{A_i} \min\{1, \eta\,|\gamma u(x_i)(\omega)|\}\, d\mu(\omega) < \varepsilon,$$

whenever

$$\sup_{y \in F, \|y\| \leqslant M} \|\gamma(\eta y)\| < \delta,$$

and the existence of ϱ is now obvious).

Thus the image of the unit ball of E is bounded in the lattice $L_p(\Omega, \mu)$. Hence there exists a $g \in L_p(\Omega, \mu)$ such that $v(x) = u(x)(\cdot)/g(\cdot)$ is a linear continuous operator from E into $L_\infty(\Omega, \mu)$. There exists (by the famous „lifting" theorem) a map $\psi\colon \Omega \to \big(L_\infty(\Omega, \mu)\big)'$ such that for each $f \in L_\infty(\Omega, \mu)$ the function $\langle f, \psi(\cdot) \rangle$ is μ-measurable, μ-a.e. equal to $f(\cdot)$ and $\|\psi(\omega)\| = 1$ for each $\omega \in \Omega$. Now it is easy to see that the map $\varphi\colon \Omega \to E'$ given by $\varphi(\omega) = g(\omega) v^t\big(\psi(\omega)\big)$ has properties 1° and 2°. This completes the proof.

Let $0 < p \leqslant 2$ and let $x_t(\cdot)$ be a symmetric stable process on (Ω, μ) with the exponent p (cf. [1], p. 425). The stochastic integral

$$\gamma_p(f) = \int_0^1 f(t)\, dx_t$$

defines a linear operator $\gamma_p\colon L_p \to L_0(\Omega, \mu)$. For our purposes it is enough to know that, for each $0 \leqslant q < p$, γ_p is an isomorphic embedding of L_p into $L_q(\Omega, \mu)$, i.e. $\gamma_p(L_p) \subset L_q(\Omega, \mu)$ and $\gamma_p\colon L_p \to L_q(\Omega, \mu)$ is an isomorphic embedding. If $p = 2$, then the same is true for each $0 \leqslant q < \infty$ (cf. [13], Example P″ 4).

3. The following theorem has its origin in a paper of Vakhania [15] (cf. Remark 2):

THEOREM 3. *Let $1 < p \leqslant \infty$, and let $u\colon L_p \to L_2$ be a continuous linear operator. Then the following conditions are equivalent:*

1° $\gamma_2 u\colon L_p \to L_q(\Omega, \mu)$ is q-decomposable for each $q < \infty$;

2° $\gamma_2 u\colon L_p \to L_0(\Omega, \mu)$ is 0-decomposable;

3° $u\colon L_p \to L_2$ is 0-absolutely summing;

4° $u\colon L_p \to L_2$ is p'-absolutely summing, where $p' = p/(p-1)$.

Proof. Of course 1° ⇒ 2°. If 2° then, by Theorem 1, $\gamma_2 u\colon L_p \to L_0(\Omega, \mu)$ is 0-absolutely summing. Hence u is 0-absolutely summing because γ_2 is an isomorphic embedding of L_2 into $L_0(\Omega, \mu)$. Thus 2° ⇒ 3°. Since $\pi_0(E, F) \subset \pi_p(E, F)$, 3° ⇒ 4°. Let $u \in \pi_{p'}(L_p, L_2)$ and $p < \infty$. By Theorem 2, $u^t\colon L_2 \to L_{p'}$ is p'-decomposable; hence, by Theorem 1, it is p'-absolutely summing, and so $u^t\colon L_2 \to (L_p)'$ is q-absolutely summing for each $q \geqslant p'$. The last is also true when $p = \infty$, because then u^t is nuclear. Now, by

Theorem 2, $\gamma_2 u\colon L_p \to L_q(\Omega, \mu)$ is q-decomposable for each $q \geqslant p'$ and hence for each q. Thus $4^\circ \Rightarrow 1^\circ$, and this completes the proof.

Remark 2. Since l_p is a complemented subspace of L_p, it follows that the above theorem remains valid if L_p is replaced by l_p. Condition 4° is then equivalent to

$4^{\circ\prime}$ $\sum_{k=1}^{\infty} \|e_k\|^{p'} < \infty$, (e_k) *is the sequence of standard unit vectors in* l_p.

For the spaces l_p it was proved by N. Vakhania that $4^{\circ\prime}$ is equivalent to 1° and to 2°.

THEOREM 4. *Let either* $1 \leqslant p < 2$ *and* $0 \leqslant q < p$ *or* $p = 2$ *and* $0 \leqslant q < \infty$. *Let* $u\colon E \to L_p$ *be a continuous linear operator. If* $u^t \in \pi_q((L_p)', E')$, *then* $u \in \pi_0(E, L_p)$.

Proof. By Theorem 2 we infer that $\gamma_p u\colon E \to L_q(\Omega, \mu)$ is q-decomposable and hence $\gamma_p u\colon E \to L_0(\Omega, \mu)$ is q-decomposable. By Theorem 1, $\gamma_p u\colon E \to L_0(\Omega, \mu)$ is 0-absolutely summing. Since γ_p is an isomorphic embedding of L_p into $L_0(\Omega, \mu)$, u is 0-absolutely summing also. This completes the proof.

The following may be considered as a generalization of Theorem 1, [4]:

THEOREM 5. *Let E be a Banach space.*

1° *If* $0 \leqslant p \leqslant 2$, $1 < r \leqslant 2$, *then* $\pi_p(L_r, E) = \pi_0(L_r, E)$.

2° *If* $2 \leqslant p < \infty$, $1 \leqslant r \leqslant 2$, *then* $\pi_p(E, L_r) = \pi_2(E, L_r)$.

Proof. 1° It is enough to show that $\pi_2(L_r, E) \subset \pi_0(L_r, E)$. Let $u \in \pi_2(L_r, E)$; then u may be factorized, $u\colon L_r \xrightarrow{v} L_2 \xrightarrow{w} E$, where $v \in \pi_2(L_r, L_2)$ and $w\colon L_2 \to E$ is a linear continous operator (cf. [10]). Since $\pi_2(L_r, L_2) \subset \pi_r(L_r, L_2)$, by Theorem 3 ($4^\circ \Rightarrow 3^\circ$) we have $v \in \pi_0(L_r, L_2)$. Hence $u = wv \in \pi_0(L_r, E)$.

2° $u \in \pi_2(E, L_r)$ if and only if, for each continuous linear operator $v\colon L_2 \to E$, $uv \in \pi_2(L_2, L_r)$. Thus it remains to prove that if $u \in \pi_p(L_2 L_r)$, then $u \in \pi_2(L_2 L_r)$, but this follows from Theorem 4 applied to u^t and then to u. This completes the proof.

It is easy to see that in a Hilbert space 2-absolutely summing operators are the same as the operators of Hilbert-Schmidt type. Hence, by Theorem 5, we have

COROLLARY 1. *Let* $0 \leqslant p < \infty$. *A linear operator in a Hilbert space is p-absolutely summing if and only if it is of Hilbert Schmidt type.*

This Corollary has been proved by Pełczyński [8] for $p > 2$ and by Pietsch [10] for $1 \leqslant p \leqslant 2$. In terms of p-radonifying operators it has been proved by Sazanov, Minlos, and Schwartz.

COROLLARY 2. *Let E be a Banach space; then* $u \in \pi_0(L_2, E)$ *if and only if it is factorizable through a Hilbert-Schmidt operator, i.e.* $u = vw$, *where* $v\colon L_2 \to E$ *is a continuous linear operator and* $w\colon L_2 \to L_2$ *is an operator of Hilbert-Schmidt type.*

Remark 3. Theorem 5 may be generalized as follows:

Let F be a subspace of L_1 and let E be a Banach space. Then:

1° $\pi_1(F, E) = \pi_2(F, E)$;

2° *if* $p \geqslant 2$, *then* $\pi_p(E, F) = \pi_2(E, F)$.

The proof of 1° is identical with the proof of Theorem 1, a. of [4], and 2° follows from Theorem 5, 2° applied to L^1 and then to the subspace F of L_1.

We do not know whether for each two Banach spaces E, F and $0 \leqslant p < q < 1$, $\pi_p(E, F) = \pi_q(E, F)$. For the spaces L_r we have the following result:

THEOREM 6. *Let* $1 < r < \infty$ *and let E be a Banach space; then* $\pi_1(L_r, E) = \pi_0(L_r, E)$.

Proof. If $r \leqslant 2$, then the proof results from Theorem 5.

1° Let $r > 2$ and let $u \in \pi_1(L_r, E)$. Without loss of generality we may assume that E is separable. Let v be an isometric embedding of E into L_∞. It is enough to prove that $vu \in \pi_0(L_r, L_\infty)$. Since L_r is reflexive, vu is nuclear (cf. [2]). Hence vu may be factorized as follows: $vu\colon L_r \xrightarrow{w} L_1 \xrightarrow{z} L_\infty$, where $w \in \pi_1(L_r, L_1)$ and $z\colon L_1 \to L_\infty$ is a continuous linear operator. Now if we apply Theorem 4 to w^t and then to w, we infer that $w \in \pi_0(L_r, L_1)$ and thus $vu = zw \in \pi_0(L_r, L_\infty)$. This completes the proof.

The following theorem is an answer to Problem 8 of [6]:

THEOREM 7. *Let* $2 < p < \infty$. *Then*

1° $B(L_\infty, L_p) = \pi_q(L\ , L_p)$ *if* $q > p$;

2° $B(L_\infty, L_p) \neq \pi_p(L_\infty, L_p)$.

$B(E, F)$ *denotes the class of all linear continuous operators from E into F.*

Proof. Let $N_r(E, F)$ denote the class of all r-nuclear operator from E into F (u is *r-nuclear* if it admits a factorization $u\colon E \xrightarrow{v} L_\infty \xrightarrow{\Delta} l_r \xrightarrow{w} F$, where Δ is a diagonal operator, cf. [9]). According to the results of Persson and Pietsch (Satz 5.2 and Satz 5.3 of [9]) and Saphar [11], $\pi_s(E, L_r)$ is the dual of the space $N_{s'}(L_r, E)$ $\big(\pi_\infty(E, F) = B(E, F)\big)$. Hence, for any q, $B(L_\infty, L_p) = \pi_q(L_\infty, L_p)$ if and only if $N_1(L_p, L_\infty) = N_{q'}(L_p, L_\infty)$. The last equality is equivalent to the following: each operator $u\colon L_p \xrightarrow{v} l_\infty \xrightarrow{\Delta} l_{q'}$ is 1-absolutely summing (because an operator $u \in \pi_1(L_p, E)$ if and only if, for each $w \in B(E, L_\infty)$, $wu \in N_1(L_p, L_\infty)\big)$.

1° Let $q > p$ and consider an operator $\bar{u}\colon L_p \xrightarrow{v} l_\infty \xrightarrow{\Delta} l_{q'} \xrightarrow{z} L_{q'}$, where v, Δ are as before and z is an isomorphic embedding. $\bar{u}$ is q'-absolutely summing. Hence, by Theorem 4 applied to $\bar{u}^t$ and then to $\bar{u}$, we find that $\bar{u}$ is 1-absolutely summing and so $u\colon L_p \xrightarrow{v} l_\infty \xrightarrow{\Delta} l_{q'}$ is also 1-absolutely summing.

2° By the preceding it is enough to point out an operator $u\colon L_p \xrightarrow{v} l_\infty \xrightarrow{\Delta} l_{p'}$ which is not 1-absolutely summing or, as we know by Theorem 4, which is not 0-absolutely summing. In [14] L. Schwartz has proved that a diagonal operator $\Delta\colon l_p \to l_{p'}$ is radonifying or, which is the same, is 0-absolutely summing if and only if Δ is given by a sequence (λ_n) such that

$$\sum_{n=1}^{\infty} |\lambda_n|^{p'}\left(1+\left|\log\frac{1}{|\lambda_n|}\right|\right) < \infty.$$

Thus there exists a $\bar{u}\colon l_p \xrightarrow{\bar{w}} l_\infty \xrightarrow{\Delta} l_{p'}$ which is not 0-absolutely summing. Since l_p is a complemented subspace of L_p, there exists a $u\colon L_p \xrightarrow{w} l_\infty \xrightarrow{\Delta} l_{p'}$ which is not 1-absolutely summing. This completes the proof.

Remark 4. The assertions of Theorems 4-7 of the present paper remain valid if we replace the spaces L_p by the $\mathscr{L}_p$ in the sense of [6] (To this end combine Theorem 7.1 of [6] in the case $1 < p < \infty$ and the results of [7] in the case $p = 1, \infty$).

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