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# Corrigenda to "Differentiable mappings on topological vector spaces" (Studia Mathematica 45(1972), pp. 147–160)

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The statement in ([2], p. 149) that the definition of strong continuity there is a generalisation to topological vector spaces of the definition used in normed spaces by Vainberg ([3], p. 10) is incorrect. With the same notation as [2], let us say that a mapping $f\colon E \to F$ is *strongly sequentially continuous* if for each $x \in E$ and sequence $(x_n, n \in N)$ in $E$ such that $x_n \rightharpoonup x$, then $f(x_n) \to f(x)$. Then strong sequential continuity is the correct generalisation of Vainberg's concept. The following result gives the connection between strong sequential continuity and strong continuity as defined in [2].

THEOREM. *Let $f\colon E \to F$, where $E$ is a (Hausdorff) locally convex space. Then $f$ is strongly continuous implies $f$ is strongly sequentially continuous. Generally, the converse does not hold. However, if $E$ is a semireflexive metrizable or semireflexive strict $(LF)$-space, then the converse does hold.*

Proof. The first part follows since a weakly convergent sequence in a locally convex space is bounded. The counterexample to the converse is as follows. Let $f$ be the identity mapping from $l^1$ into $l^1$. Since weak and strong sequential convergence coincide in $l^1$, $f$ is strongly sequentially continuous. But $f$ is not strongly continuous. For let $D$ be a weak 0-neighbourhood base for $l^1$, directed by $U \geqslant V$, if $U \subset V$. For each $U \in D$, define $x_U$ to be an element in $U$ such that $1/2 \leqslant \|x_U\|_1 \leqslant 1$. Then $(x_U)_{U \in D}$ is a bounded net which converges weakly to 0, but $f(x_U) = x_U \nrightarrow 0$.

Now suppose $E$ is a semireflexive metrizable or semireflexive strict (LF)-space and $f$ is not strongly continuous. Hence there exists a bounded net $(x_\alpha, \alpha \in A)$ with $x_\alpha \rightharpoonup x$ and a 0-neighbourhood $U$ in $F$ such that $f(x_\alpha) - f(x) \notin U$, for each $\alpha \in A$. Now $B = \{x_\alpha\}_{\alpha \in A}$ is a weakly relatively compact subset of $E$ and $x$ is a weak closure point of $B$. Hence, by Köthe ([1], p. 313), there exists a sequence $(x_n, n \in N)$ in $B$ such that $x_n \rightharpoonup x$. But then $f(x_n) \nrightarrow f(x)$, and so $f$ is not strongly sequentially continuous.

One can define "sequential" versions of the other types of continuity defined in [2]. Then the results of Section 2 in [2] hold for the "sequential" versions of the various types of continuity with almost exactly the same proofs (except in Theorems 2.8, 2.11 and 2.12, the hypothesis "bounded sets are weakly relatively compact" should be replaced by "bounded sets are weakly relatively *sequentially* compact").

The statement of the generalisation of Schauder's theorem to topological vector spaces ([2], p. 153) is incorrect and needs to be modified slightly as follows:

THEOREM. *Suppose $F$ is a locally convex space. Then $\mathscr{F}$ is collectively precompact if and only if for each $B \in \mathscr{B}$ and $U \in \mathscr{U}$, there exists a family $\mathscr{F}^* = \{f^* \mid f \in \mathscr{F}\}$ of maps from $B$ into $F$ such that*

(I) *$f^*(x) - f(x) \in U$, for each $f \in \mathscr{F}$ and each $x \in B$;*

(II) *$\bigcup_{f \in \mathscr{F}} f^*(B)$ is a bounded subset of a finite dimensional subspace $F_m$ of $F$.*

In other words, we have to drop the reference to the continuity of the maps. If each $f \in \mathscr{F}$ is continuous then each $f^* \in \mathscr{F}^*$ will be continuous, since the map $S$ in Nagumo's theorem ([2], p. 153) is continuous. However, the converse is not true. That is, each $f^*$ is continuous does not imply each $f \in \mathscr{F}$ is continuous. For let $F = R^{(A)}$, the locally convex direct sum of infinitely many real lines (that is, $A$ is infinite) and let $E = R^{(A)}$, but given the weak topology. Let $f\colon E \to F$ be the identity mapping. Certainly $f$ is not continuous. However, bounded subsets of $E^{(A)}$ are finite dimensional and so $f$ is continuous on bounded sets. Then, for each $B \in \mathscr{B}$ and $U \in \mathscr{U}$, if we put $f^* = f|B$, $f^*$ is continuous and satisfies the conditions (I) and (II) above.

However, if $E$ is a $k$-space, or more generally a space in which to prove a mapping $f$ from $E$ into another space is continuous, it suffices to show the restriction of $f$ to each bounded subset of $E$ is continuous, then the converse holds. That is, in this case, each $f^* \in \mathscr{F}^*$ is continuous implies each $f \in \mathscr{F}$ is continuous.

**References**

[1] G. Köthe, *Topological Vector Spaces I*, Berlin–Heidelberg–New York 1969.
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[3] M. M. Vainberg, *Variatonal methods for the study of non-linear operators*, San Francisco–London–Amsterdam 1964.

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