HOMOTOPY ALGEBRAS,
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PREFACE

The present volume contains three minicourses and four other contributions by the participants of the Conference *Homotopy Algebras, Deformation Theory and Quantization*, which took place between September 16 and 22, 2018, at the Conference Center in Będlewo near Poznań, Poland.

The Conference was a part of the program of the Stefan Banach International Mathematical Center. Its main idea was to get together mathematicians, mathematical physicists and physicists working in homotopy algebras, deformation theory, theory of operads and props and quantization, to exchange and share ideas and new results.

The focus of the Conference was the highly active field of homotopy algebras, the deformation of higher algebraic structures, the Grothendieck–Teichmüller group, and consequences in other fields in mathematics and physics. The foundations of this interaction were already laid in the 1960s in the works of M. Boardman, M. Gerstenhaber, G. Hochschild, J. May, J. Stasheff, R. Vogt and others. The novel development to apply ideas of strongly homotopy algebras and deformation theory to geometry and mathematical physics arose with the influential paper of M. Kontsevich *Deformation quantization of Poisson manifolds* (1997). In that paper he proved that the Hochschild complex of polydifferential operators on a structure sheaf of a smooth manifold and the Schouten algebra of polyvector fields are quasi-isomorphic as differential graded Lie algebras. This work attracted much attention, it was widely viewed as a kind of message for the future. Some of the recent developments are the works of Tamarkin, Willwacher and the discovery of the Grothendieck–Teichmüller group. The latter has found applications in number theory, Lie theory, and in the theory of quantizations of Lie bialgebras. Very recently, its crucial role in the classification of universal deformation quantizations of Poisson manifolds has been fully uncovered. This exciting development provided the core of our Conference.

The main topics were:

1. *Deformations of algebraic and geometric structures.* In recent decades, there have been numerous results in the deformation theory of algebras, both formal and global, including infinite-dimensional objects. Some important open questions were discussed.

2. *Higher algebraic structures — Grothendieck–Teichmüller group.* These higher structures are controlled by operads and props. A new development in this approach to deformation theory is the appearance of the Grothendieck–Teichmüller group. It is a universal object which unifies different fields in mathematics and mathematical physics and which brought a breakthrough in many theories. These new developments were discussed in detail.

3. *Deformation quantization — mathematical aspects and applications in physics.* In 1978 Bayen, Flato, Fronsdal, Lichnerowicz and Steinheimer opened up a systematic approach to quantization via deformation theory of algebras. With their work, the intuitive understanding that quantization should in some sense be a deformation of the classical system was made precise. Today we have complete existence and classification results on the formal level. But a number of open questions remained, all of which have relevance in physics (quantization of singular manifolds, orbifolds, superspaces, relation to perturbation theory in quantum field theory, infinite-dimensional spaces etc.) These open questions were analyzed.

4. *Deformations of higher structures and their applications in theoretical physics.* Recently, it has been discovered that higher structures play an important role in string/M-theory: Nambu–Poisson structures in the description of open membranes, Courant and AKSZ sigma models in the context of non-geometric backgrounds etc. The deformation quantization of these objects was also discussed at the Conference.

There were 45 participants at the Conference with 12 PhD students. The 36 foreign participants came from 17 countries.

There were 4 minicourses by

— Giovanni Felder: *Derived representation schemes and supersymmetric gauge theory*;
— Simone Gutt: *Deformation theory and group action*;
— Ping Xu: *Dg manifolds, formality theorem and Kontsevich–Shoikhet conjecture*;
— Sergei Merkulov: *Graph complexes in algebra and geometry — recent advances.*

Besides the minicourses there were 7 plenary lectures, one hour each, and 10 shorter contributions (30 min. each).

In this volume we present 3 minicourses and 4 contributions of the invited lecturers:

— G. Felder: *Derived representation schemes and supersymmetric gauge theory*;
— S. Gutt: *Group actions in deformation quantization*;
— M. Stiénon & P. Xu: *Atiyah classes and Kontsevich–Duflo type theorem for dg manifolds*;
— O. Sheinman: *Quantization of the Lax integrable systems and conformal field theory*;
— A. V. Kiselev & R. Buring: *The Kontsevich graph orientation morphism revisited*;
— J. Ecker & M. Schlichenmaier: *The low-dimensional algebraic cohomology of the Witt and the Virasoro algebra with values in natural modules*;
— A. Fialowski & K. Iohara: *On Lie algebras of generalized Jacobi matrices.*

We are really grateful to the local Organizing Committee and the administrative staff of the Stefan Banach International Mathematical Center and the Conference Center in Będlewo for the excellent work they did before and during the Conference, which made the Workshop a really successful event. We are also grateful for the financial support from the Banach Center and the Warsaw Center of Mathematics and Computer Science, to the University of Luxembourg, and to Janusz Grabowski for using his personal grant to make this Conference working.

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