OPERATOR ALGEBRAS THAT ONE CAN SEE
BANACH CENTER PUBLICATIONS, VOLUME 130
INSTITUTE OF MATHEMATICS
POLISH ACADEMY OF SCIENCES
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PREFACE

Graph C*-algebras are analytical objects blessed with a tangible structure and classification theory derived from their combinatorial origins. Through the analysis of directed graphs, one can visualize and explore them in intuitive ways lacking elsewhere, which is why they are called *operator algebras that one can see*. They serve as strikingly efficient models for key open problems in noncommutative geometry and topology, as well as in C*-dynamical systems. They also provide a focal point for the much-needed extension of the celebrated Elliott classification program to non-simple C*-algebras.

Here the key observation is that graphs provide convenient models for C*-algebras yielding manageable K-theoretical invariants. Such invariants are crucial for the aforementioned Elliott classification program, and for studying the structure of corresponding operator algebras. This program was famously completed for simple C*-algebras in recent years, but the much-needed progress in extending the classification program to the non-simple cases relevant here requires deeper understanding of the relationship between combinatorics and invariants of operator algebras.

In this volume of the Banach Center Publications, we are delighted to present six lecture notes given by Simons Professors of the Banach Center Simons Semester *Operator Algebras That One Can See* that took place in the period 22 June – 21 September 2023. They can serve as a concise but comprehensible introduction to the state of the art of the above described semester's main topic and beyond. We very much hope that these lecture notes will become a handy basic reference both for newcomers and experts.

The first lecture notes *Notes on universal C^*-algebras* by Gilles G. de Castro (Universidade Federal de Santa Catarina, Florianópolis, Brazil) introduce the topic of C*-algebras. Then *Applications of C^*-classification* by Bhishan Jacelon (Matematický ústav Akademie věd České republiky, Praha, Czechia) digs deeper into the structure of C^*-algebras. Next come the pivotal lecture notes *Directed graphs, variations on directed graphs, and their C^*-algebras* by Jack Spielberg (Arizona State University, Tempe, USA) that are at the heart of the semester research theme. They are followed by *Compact quantum surfaces* by Elmar Wagner (Universidad Michoacana de San Nicolás de Hidalgo, Morelia, Mexico), where the graph C*-algebra of the quantum disc (Toeplitz algebra) is the main building block. Finally, quantum symmetries come into play, first in *Quantization, quantum groups, and quantum groupoids* by Byung-Jay Kahng (Canisius University, Buffalo, USA), and then in *Knots, operator algebras, and physics* by Adam S. Sikora (State University of New York at Buffalo, USA).

Our Banach Center Simons Semester was a milestone both in the EU Staff Exchange network project *Operator Algebras That One Can See* and in the University of Warsaw

Thematic Research Programme *Quantum Symmetries*. All but one of our Simons Professors are members of the network, and the audience of their lecture courses was largely sponsored by the thematic programme. The semester culminated in the Banach Center research conference *Graph Algebras*, which took place in Będlewo, 3–7 July 2023, with substantial financial support of the thematic programme.

While our biggest thanks go to the Simons Professors for leading the way, we are equally grateful for the indispensable administrative assistance to Justyna Gowin, Katarzyna Zdybel and Anna Maria Orłowska, who went beyond the call of duty to reconcile the Banach Center and the University of Warsaw bureaucracies, and thus ensured the smooth running of the semester.

Søren Eilers, Piotr M. Hajac, and Tomasz Maszczyk

Operator Algebras That One Can See

Simons Semester 22 June - 21 September 2023 Będlewo & Warsaw

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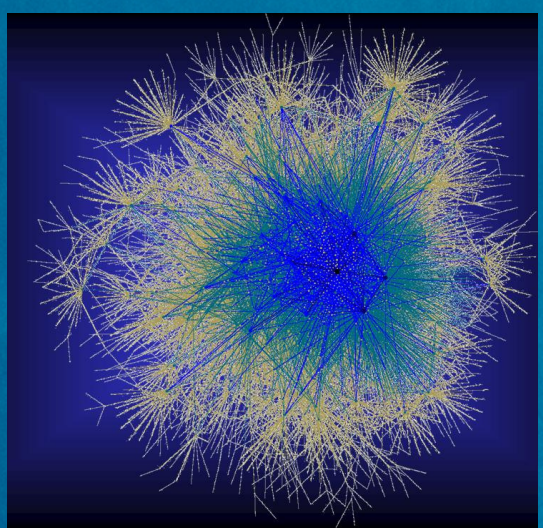

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SIMONS FOUNDATION

Graph Algebras

3 - 7 July 2023

Banach Center
Będlewo, Poland

Organizers: Søren Eilers, Piotr M. Hajac, Tomasz Maszczyk

SPEAKERS

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G. G. de Castro (UFSC, Florianópolis, Brazil)
G. Cortiñas (Univ. de Buenos Aires, Argentina)
F. D'Andrea (Univ. di Napoli Federico II, Italy)
S. Eilers (Københavns Universitet, Denmark)
A. Frei (University of Waterloo, Canada)
P. M. Hajac (IM PAN, Warszawa, Poland)
B. Jacelon (IM CAS, Praha, Czechia)
P. Kasprzak (Uniwersytet Warszawski, Poland)
J. Krajczok (University of Glasgow, Scotland)
T. Maszczyk (Uniw. Warszawski, Poland)
M. Schötz (IMPAN, Warszawa, Poland)
P. M. Sołtan (Uniw. Warszawski, Poland)
J. Spielberg (ASU, Tempe, USA)
M. Tobolski (Uniw. Wrocławski, Poland)
E. Wagner (UMSNH, Morelia, Mexico)
T. Weber (Università di Torino, Italy)
A. Wysoczańska-Kula (U. Wrocławski, Poland)
M. Ziembowski (Polit. Warszawska, Poland)

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Quantum Symmetries

Thematic Research Programme

10 March 2023 - 9 March 2024

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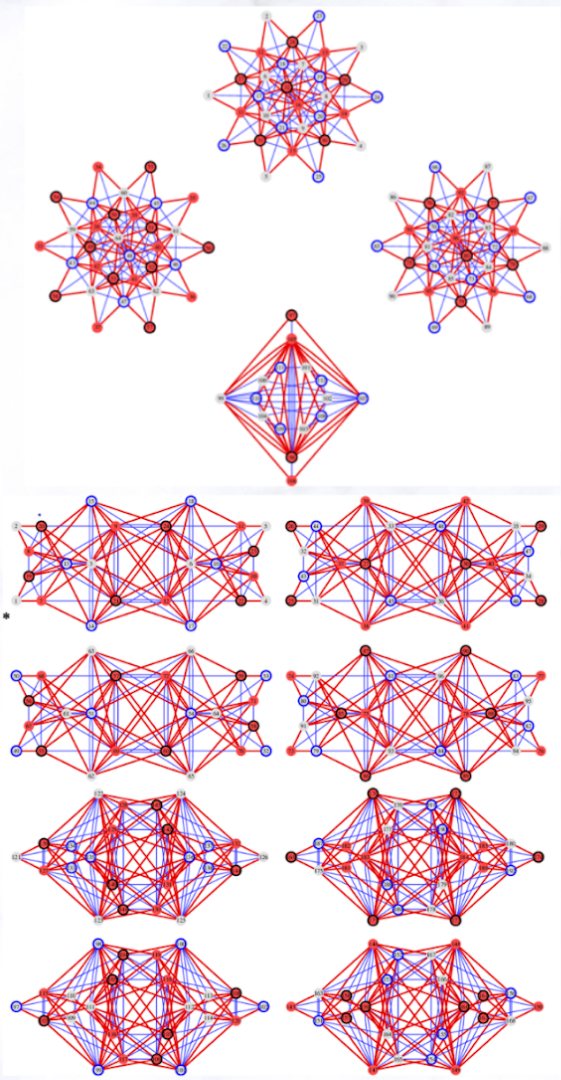

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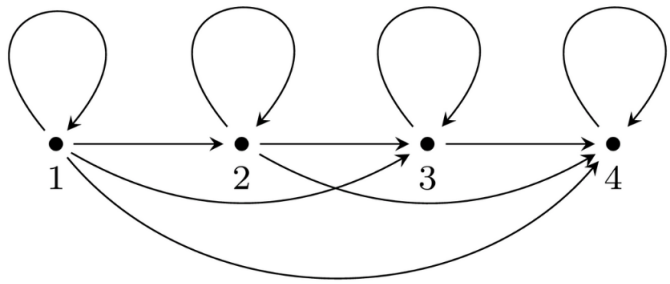
Uczelnia Badawcza

Operator Algebras That One Can See

1 Jan 23 - 31 Dec 26



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