For the rest of the theorem — assuming set theory is consistent — one begins with a model M of set theory and $A_{\aleph_2}$ (this is defined in [4]). One can check that $D^{\aleph_1}$ is strongly sequentially compact in M (see, for example, Theorem 4.10 of [1]) therefore M contains a $P(\mathfrak{c})$ point. Carry out a Cohen extension of M obtaining a new model A in which $2^{\aleph_1} = \aleph_3$ and $S(N)^M = S(N)^A$. The $P(\mathfrak{c})$ point P of M is still an ultrafilter and a $P(\mathfrak{c})$ point. Let $\{a_\alpha: A \in \aleph_1\}$ be a collection of infinite sets, in order to meet the conditions of Theorem 2 we may as well suppose that $N \sim a_\alpha$ is infinite too. Either a_α or $N \sim a_\alpha$ is in P; let a'_α be a_α if $a_\alpha \in P$ and $N \sim a_\alpha$ otherwise; since P is a $P(\mathfrak{c})$ point there is a $b \subseteq N$ such that for each α, $b < a'_\alpha$. This b satisfies condition 2 of Theorem 2.

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Completely regular proximities and RC-proximities

by

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1. The author recently introduced the theory of RC-proximities to characterize the spaces that can be embedded in a regular-closed space. The present results are concerned with the manner in which RC-proximities are made up from other types of proximities.

The theory of RC-proximities was developed in our paper [HS]; this paper is a continuation of [HS], and terms, notations, and techniques introduced therein will be used herein without further reference.

2. LO-proximities and R-proximities. A proximity δ such that the induced closure operator is topological and such that from $A \ non\delta\ B$ there follows $\mathrm{cl}A \ non\delta\ \mathrm{cl}B$ is called a *LO-proximity*. An *R-proximity* is defined in [HS] as a proximity satisfying Axioms P1-P5 of [HS]; a *LR-proximity* is a proximity that is simultaneously a LO-proximity and an R-proximity.

There are three proximities that can be defined on any T_1 space and that will be useful later in forming examples. These proximities are considered in [CH]; it is appropriate here to observe that the proximities considered in [CH] are more general than those that we consider, since Axiom P4, which requires that distinct points be far, need not be satisfied by the proximities of [CH].

The proximities considered below do satisfy P4, however, since the associated topologies are T_1.

2.1 [CH, 25A. 18(a)]. *For any T_1 space X the relation $A\ \delta_s\ B$ if $(A \cap \mathrm{cl}_X B) \cup (\mathrm{cl}_X A \cap B) \neq \emptyset$ is the finest proximity that induces the topology of X.*

2.2 [CH, 25A. 18(b)]. *For any T_1 space X the relation $A\ \delta_c\ B$ if $A\ \delta_s\ B$ or both A and B are infinite is the coarsest proximity that induces the topology of X.*

2.3 [CH, 25A. 18(c)]. *For any T_1 space X the relation $A\ \delta_w\ B$ if $\mathrm{cl}_X A \cap \mathrm{cl}_X B \neq \emptyset$ is a proximity that induces the topology of X.*

The following results are readily established from the definitions.

2.4. *If X is a T_1 space then $A\,\delta_c\,B$ if and only if $A\,\delta_w\,B$ or both A and B are infinite.*

2.5. *If X is a T_1 space then the proximity δ_w is the finest LO-proximity that induces the topology of X.*

The proximity δ_w is called the *Wallman-proximity* on X. If X is regular then δ_w is a LR-proximity by [HS, Lemma 2].

2.6. *Let X be a T_1 space with no isolated point. Then the LO-proximity δ_c induces the topology of X and is not an R-proximity.*

Proof. Suppose δ_c is an R-proximity; then the topology of X is regular. If X has the cofinite topology then it is finite and so every point is an isolated point. If X does not have the cofinite topology then there is $x \in X$ and a neighborhood V of x such that $X-V$ is infinite. Now there is a neighborhood W of x such that $V > W$, and it follows that W is finite, and therefore x is an isolated point.

COROLLARY. *There is a compact Hausdorff space X such that δ_c is not an R-proximity and thus $\delta_c \neq \delta_w$.*

The interest of the corollary lies in the fact that it shows we can have two distinct LO-proximities inducing the topology of a compact Hausdorff space, although according to 2.9 below, δ_w is the unique LR-proximity that induces the topology.

The following lemma is often useful in constructing examples.

2.7. *Any proximity finer than an R-proximity and inducing the same topology is also an R-proximity.*

Proof. Let δ be an R-proximity, suppose ϱ is a finer proximity giving the same topology, and write $>$, $\geqslant$ for the corresponding proximal neighborhood relations. Now if $V \geqslant x$ then since both topologies are the same we have also $V > x$, and since δ is an R-proximity there is a W with $V > W > x$. It now follows since ϱ is finer than δ that $V \geqslant W \geqslant x$.

COROLLARY. *The proximity δ_s on a regular T_1 space is an R-proximity.*

Proof. It is finer than the R-proximity δ_w, and induces the same topology.

2.8. *The proximity δ_s on a normal but not hereditarily normal (= completely normal) Hausdorff space is an R-proximity that is not a LO-proximity.*

The proof is immediate.

An example of a compact Hausdorff space for which the three proximities δ_c, δ_w, and δ_s are all distinct is provided by an uncountable product of unit intervals, since such a space is not hereditarily normal (see [ST]) and has no isolated points. This shows also that two distinct R-proximities can induce the topology of a compact Hausdorff space.

The following result is now quite interesting; although LO-proximities and R-proximities need not be unique on a compact Hausdorff space, the combined property is unique.

2.9. *The proximity δ_w is the only LR-proximity on a compact Hausdorff space.*

Proof. By 2.5 such a proximity is certainly coarser than δ_w, so we need only show that it is also finer than δ_w. This can be shown by a device similar to the one usually used to show that a compact Hausdorff space is normal. This is a generalization of the usual theorem [CH, 41C.10] that a compact Hausdorff space has only one completely regular proximity.

3. Maximally full and maximally saturated proximities. To analyse the makeup of RC-proximities we break the axiom P6 of RC-proximities, stated in [HS], into two parts.

A proximity satisfying

P7. If $A > B$ then A surrounds B is called a *maximally full* proximity.

A proximity satisfying

P8. If A surrounds B then $A > B$ is called a *maximally saturated* proximity.

We first turn our attention to showing that maximally full and maximally saturated are independent conditions.

The following result provides a general method of obtaining maximally full proximities on RC-regular spaces.

3.1. *Let Z be a regular-closed space with topology induced by LR-proximity δ. If X is any dense subspace of Z then the proximity ϱ induced on X by δ is a maximally full LR-proximity.*

Proof. It is easy to see that the induced proximity is a LR-proximity. Now if $A, B \subset X$ and $A \geqslant B$, writing $\geqslant$ for the proximal neighborhood relation of ϱ, then it follows readily that $Z-\mathrm{cl}_Z(X-B) \subset \mathrm{cl}_Z B \subset Z-\mathrm{cl}_Z(X-A)$; therefore any round filter γ on X generates a regular filter λ on Z with the base $\{\mathrm{cl}_Z B\colon B \in \gamma\}$. Since Z is regular-closed the filter λ has a cluster point $p \in Z$, and since δ is an R-proximity on Z the neighborhood filter $\eta(p)$ is maximal round, and it follows that γ must be contained in the trace on X of the filter $\eta(p)$. Now suppose γ is a maximal round filter (on X) that intersects B; from the preceding argument it follows that γ is the trace on X of $\eta(p)$ for some $p \in Z$, and therefore $p \in \mathrm{cl}_Z B$, from which it follows that $A \in \gamma$.

3.2. *Let Z be a regular-closed but not compact space and let X be any dense subspace of Z. Then there are subsets C and D of X such that $\mathrm{cl}_Z C \cap \mathrm{cl}_Z D = \emptyset$, but $\mathrm{cl}_Z C$ and $\mathrm{cl}_Z D$ do not have disjoint neighborhoods in Z.*

Proof. We shall show that if the conclusion does not hold, then Z is compact.

Suppose λ is an ultrafilter on Z. Let $\varrho_0 = \{V \subset X: V$ is open, and for some $B \subset X$ $\mathrm{cl}_Z B \in \lambda$ and $\mathrm{cl}_Z(X-V) \cap \mathrm{cl}_Z B = \emptyset\}$. Let $\gamma_0 = \{\mathrm{cl}_Z V: V \in \varrho_0\}$. We shall show that γ_0 is a subbase for a regular filter γ on Z, and that if $z \in Z$ is any cluster point of γ, then λ converges to z. It will then follow that λ does converge, since the regular filter γ on the regular-closed space Z must have a cluster point.

To see that any finite intersection of members of γ_0 is nonempty, observe that if $V_1, \ldots, V_n \in \varrho_0$ then there are sets $B_i \subset X$ such that $\mathrm{cl}_Z B_i \in \lambda$ and $\mathrm{cl}_Z(X-V_i) \cap \mathrm{cl}_Z B_i = \emptyset$, for $i = 1, \ldots, n$. Now since λ is a filter, then

$$\begin{aligned} \emptyset \neq \bigcap \mathrm{cl}_Z B_i \subset Z - \bigcup \mathrm{cl}_Z(X-V_i) \\ = Z - \mathrm{cl}_Z\big(\bigcup (X-V_i)\big) = Z - \mathrm{cl}_Z(X - \bigcap V_i)\,, \end{aligned}$$

and thus $\mathrm{cl}_Z(X-\bigcap V_i) \neq Z$, and so $\bigcap V_i \neq \emptyset$, from which there follows $\bigcap \mathrm{cl}_Z V_i \neq \emptyset$.

To see that γ_0 generates a regular filter it will suffice to show that if $\mathrm{cl}_Z V \in \gamma_0$ then there is an open $W \subset X$ with $\mathrm{cl}_Z W \subset Z - \mathrm{cl}_Z(X-V) \subset \mathrm{cl}_Z V$, and $\mathrm{cl}_Z W \in \gamma_0$. To this end, if $V \in \varrho_0$ then there is $B \subset X$ with $\mathrm{cl}_Z B \in \lambda$ and $\mathrm{cl}_Z(X-V) \cap \mathrm{cl}_Z B = \emptyset$. By hypothesis there are open sets $T, S \subset Z$ with $\mathrm{cl}_Z(X-V) \subset T \subset Z - S \subset Z - \mathrm{cl}_Z B$. Setting $W = S \cap X$ we have $X - W \subset Z - S$, so $\mathrm{cl}_Z(X-W) \subset Z - \mathrm{cl}_Z B$, giving $W \in \varrho_0$. Now $\mathrm{cl}_Z W = \mathrm{cl}_Z(S \cap X) \subset \mathrm{cl}_Z S \subset Z - T \subset Z - \mathrm{cl}_Z(X-V) \subset \mathrm{cl}_Z V$, the last inclusion following from the sequence $\mathrm{cl}_Z(X-V) \cup \mathrm{cl}_Z V = \mathrm{cl}_Z\big((X-V) \cup V\big) = \mathrm{cl}_Z X = Z$.

Now let $z \in Z$ be a cluster point of the regular filter γ. Suppose U is an open neighborhood of z. Choose open sets T, S with $z \in T \subset \mathrm{cl}_Z T \subset S \subset \mathrm{cl}_Z S \subset U$. Set $V = X - \mathrm{cl}_Z T$. Since $V \cap T = \emptyset$ then $z \notin \mathrm{cl}_Z V$. It follows that $\mathrm{cl}_Z V \notin \lambda_0$, so $V \notin \varrho_0$ and thus if $B \subset X$ with $\mathrm{cl}_Z B \cap \mathrm{cl}_Z(X-V) = \emptyset$ then $\mathrm{cl}_Z B \notin \lambda$. Taking $B = X - S$ we have

$$\mathrm{cl}_Z B \subset X - S \subset X - \mathrm{cl}_Z T \subset Z - \mathrm{cl}_Z(X-V)\,,$$

and thus $\mathrm{cl}_Z B \notin \lambda$. Now $X \subset B \cup S$, thus $\mathrm{cl}_Z B \cup \mathrm{cl}_Z S = Z$, and since λ is an ultrafilter we have $\mathrm{cl}_Z S \in \lambda$, and thus $U \in \lambda$.

We now give a general method for finding maximally full proximities that are not maximally saturated.

3.3. *Let Z be a regular-closed but not compact space and let X be any dense subspace of Z. Then there is a LR-proximity on X that induces its topology and is maximally full but not maximally saturated.*

Proof. Under the present hypotheses there are subsets C and D of X such that $\mathrm{cl}_Z C \cap \mathrm{cl}_Z D = \emptyset$ but the two sets do not have disjoint neighborhoods in Z. It follows (from the regularity of Z) that the filter γ generated by $\{V: V$ is open in Z and $\mathrm{cl}_Z C \subset V$ or $\mathrm{cl}_Z D \subset V\}$ is an open filter on Z with no cluster point. Now define a relation π on Z by setting $A\,\pi\,B$ if $A\,\delta_w\,B$ or if every member of the filter γ intersects every neighborhood of $\mathrm{cl}_Z A$ and every neighborhood of $\mathrm{cl}_Z B$. This is a proximity for Z, and it is clearly coarser than the Wallman proximity δ_w for Z. To see that π is an R-proximity and that it induces the topology of Z, observe that if $z \in Z$ there is $U \in \eta(z)$ such that U does not intersect some member of γ, from which it follows that if V is a neighborhood of z then there is W with $V > W > z$, found by choosing $z \in W \subset \mathrm{cl}_Z W \subset V \cap U$.

To see that π is a LO-proximity note that if A *non*π B then A *non*δ_w B and there is an open set $V \in \gamma$ and a neighborhood W of $\mathrm{cl}_Z A$ (or of $\mathrm{cl} B$) that is disjoint from V. Now since A *non*δ_w B then $\mathrm{cl}_Z A$ *non*δ_w $\mathrm{cl}_Z B$, and $X - \mathrm{cl}_Z V$ is a neighborhood of $\mathrm{cl}_Z A$ that fails to intersect $V \in \gamma$; thus $\mathrm{cl}_Z A$ *non*π $\mathrm{cl}_Z B$. It has now been shown that π is a LR-proximity.

From 3.1 it follows that the proximity ϱ induced on the dense subspace X by π is a maximally full LR-proximity. It is also clear from the proof of 3.1 that the maximal round filters on X are precisely the traces of the neighborhood filters of points of Z, and thus if G and H are subsets of X then G surrounds H if and only if $X - G$ and H as subsets of Z are far in the Wallman proximity of Z. To see that ϱ is not maximally saturated merely observe that $X - C$ surrounds D but $C\,\varrho\,D$.

We now turn our attention to LR-proximities that are maximally saturated but are not maximally full. The ground space for such a proximity cannot be regular closed in view of 3.1.

3.4. *The round filters with respect to the Wallman proximity on any space are precisely the regular filters.*

3.5. *The Wallman proximity on any regular space is a LR-proximity that is maximally saturated.*

Proof. By 2.5 the Wallman proximity is a LO-proximity and for a regular space X it is an R-proximity by [HS, Lemma 2]. Now suppose $A, B \subset X$ and A surrounds B. Since by [HS, 3.2] every neighborhood filter is maximal round, it follows that every neighborhood filter that intersects B contains A, that is, $\mathrm{cl} B \subset \mathrm{int} A$, which means $B < A$ in the Wallman proximity.

It is shown (in § 5) that the Wallman proximity on the well-known Tychonoff plank is not maximally full. Another perhaps more interesting example is the subspace $W = Z - \{p\}$ of the minimal regular non-compact space Z constructed in [BS, § 3]. We shall not give the details here but merely remark that the Wallman proximity δ_w for W is not maximally full. However, it is finer than the RC-proximity δ induced on W by the Wallman proximity on Z. Further, by the proof of 3.3 the proximity δ is finer than a maximally full but not maximally saturated proximity ϱ that also induces the topology of W.

The following results give new criteria for the compactness of a regular closed space.

3.6. *A regular closed space is compact if and only if every LR-proximity that determines its topology is maximally saturated.*

Proof. This follows from 2.9, 3.5 and 3.3.

Using 2.9, 3.6, and Theorem F from [HS] we obtain the following criterion:

THEOREM A. *A regular-closed space is compact if and only if there is only one LR-proximity that determines its topology.*

It is possible that this criterion will be useful in examining the unsolved problem of whether a regular-closed space in which every closed subspace is regular-closed is compact.

4. RC-normal spaces. This class of spaces will be seen to form a natural generalization of the normal spaces with respect to many of their properties.

The space X is *RC-normal* if no maximal regular filter intersects both of two disjoint closed sets. Clearly a normal space is RC-normal, since the filter of neighborhoods of a closed set will be a regular filter. It is also clear that every regular-closed space is RC-normal, since the maximal regular filters are just the neighborhood filters in this case.

The following result is readily obtained using the methods of Section 3.

THEOREM B. *The following are equivalent for a regular space X.*

(a) *X is RC-normal.*

(b) *A maximal regular filter on X contains a member of every binary open cover.*

(c) *The Wallman proximity on X is maximally full.*

(d) *The Wallman proximity on X is an RC-proximity.*

(e) *There is a regular closed space Z into which X is densely ambedded in such a way that disjoint closed subsets of X have disjoint closures in Z.*

Condition (e) is the analogue of the familiar condition for normality of a completely regular space expressed in terms of its compactifications. Just as there can be only one compactification with this property, so there can be only one regular-closed extension with this property.

The Tychonoff plank is an example of an RC-regular but not RC-normal space, as we shall see in the next section.

5. Maximally bounded and completely regular proximities. A *maximal cover* is a cover of X such that every maximal round filter contains

a member of the cover. A proximity is *maximally bounded* if every maximal cover has a finite maximal subcover. A maximally bounded RC-proximity is called a *BRC-proximity*.

5.1. *A maximally bounded R-proximity is maximally saturated.*

Proof. Suppose A surrounds B; then if γ is a maximal round filter we have $A \in \gamma$ or $X-B \in \gamma$, and so there is $W_\gamma \in \gamma$ with $W_\gamma < A$ or $W_\gamma < X-B$. Now $\{W_\gamma\}$ is a maximal cover and so contains a finite maximal cover $\{W_i\}$. Setting $C = \bigcup\{W_i\colon W_i < A\}$ and $D = \bigcup\{W_i\colon W_i < X-B\}$, it follows that $C \cup D = X$, $C < A$ and $D < X-B$ which gives $B < X-D \subset A$ and thus $B < A$.

It is interesting to ask if the analogue of 5.1 for maximal fullness holds, that is, if a maximally bounded proximity is maximally full. An example will be given at the end of this section of a maximally bounded LR-proximity that is not maximally full.

We now introduce the well-known axiom of complete regularity for proximities, and show that the proximities satisfying this axiom are precisely the BRC-proximities.

P9 (*Axiom of complete regularity*). If $A < B$ then there is C such that $A < C < B$.

A *completely regular proximity* is a proximity that satisfies P1-P4 of [HS] and P9 above. We shall see that such a proximity also satisfies P5 and P6, that is, it is an RC-proximity, and moreover that the completely regular proximities are precisely the BRC-proximities.

5.2. *A completely regular proximity is a maximally full LR-proximity.*

Proof. Clearly P9 implies P5. Now if δ is a completely regular proximity and $A \operatorname{non}\delta B$ then by P9 there are $C, D \subset X$ with $A < C$, $B < D$, and $C \operatorname{non}\delta D$. By [HS, 2.6] $\mathrm{cl}_X A \subset C$ and $\mathrm{cl}_X B \subset D$, and by [HS, 2.4] $\mathrm{cl}_X A \operatorname{non}\delta \mathrm{cl}_X B$. Thus a completely regular proximity is a LR-proximity.

Using P9 it is immediate that for any $B \subset X$ the collection $\lambda = \{A\colon A > B\}$ is a round filter. Therefore if $A > B$ and γ is a maximal round filter that intersects B, then $\lambda \subset \gamma$ and thus $A \in \gamma$.

5.3. *A completely regular proximity is maximally bounded.*

Proof. Suppose α is a cover that contains no finite maximal cover. Define $\lambda = \{A\colon \text{there is } F \in \alpha \text{ such that } A > X-F\}$.

It follows from 5.2 and the assumption on α that λ is a filter subbase. It follows from P9 that the filter generated by λ is round, and thus it is contained in a maximal round filter γ. Now if $F \in \gamma$ there is $G \in \gamma$ with $F > G$; then $X-F < X-G$ and since $\lambda \subset \gamma$ then $F \notin \alpha$. It follows that α is not a maximal cover.

The main result of this section is the following:

THEOREM C. *A proximity is completely regular if and only if it is a BRC-proximity.*

Proof. It follows from 5.2, 5.3, and 5.1 that a completely regular proximity is a BRC-proximity.

Conversely suppose δ is a BRC-proximity. If $A > B$ then $\mathcal{A} = \{A, X-B\}$ is a maximal cover. Thus for each maximal round filter γ there is $F_\gamma \in \mathcal{A}$ with $F_\gamma \in \gamma$, and there is $G_\gamma \in \gamma$ with $F_\gamma > G_\gamma$. Then $\mathcal{B} = \{G_\gamma\}$ is a maximal cover, and so it contains a finite maximal cover $\{G_i\}$. Now for each G_i there is $F_i \in \{F_\gamma\}$ with $G_i < F_i$, and for each i either $F_i = A$ or $F_i = X-B$. Setting $C = \bigcup \{G_i\colon F_i = A\}$ and $D = \bigcup \{G_i\colon F_i = X-B\}$ it follows that $C < A$, $D < X-B$, and $C \cup D = X$. Thus, $B < X-D \subset C < A$, and so there is a set C with $B < C < A$.

We now give the previously mentioned example of a maximally bounded LR-proximity that is not maximally full. Let T be the Tychonoff plank, that is, the space $\omega' \times \Omega' - \{(\omega, \Omega)\}$ where ω' and Ω' are the one point compactifications of the space of countable ordinals and the space of uncountable ordinals, respectively. Let λ be the filter on T generated by the sets with compact complement. Given $x, v \in \omega'$, $y, w \in \Omega'$, write $(x, y) < (v, w)$ if $x < v$ and $y < w$. Also given $A, B \subset T$ write $A < B$ for A *non* $\delta_w (X-B)$.

The subset U of T *gets into the corner* if for each $(x, y) < (\omega, \Omega)$ there is $(v, w) > (x, y)$ with $(v, w) \in U$. Clearly a subset U gets into the corner if and only if the filter λ intersects U. The subset W of T *contains a tail* if there is $y < \Omega$ such that if $y < z < \Omega$ then $(\omega, z) \in W$. It can be shown just as in the proof of [BS, 3.3] that if U, V and W are open sets and $U < V < W$ then if U gets into the corner the set W contains a tail.

Now suppose γ is a free regular filter on T. Then γ cannot intersect any compact subset of T, and thus $\lambda \subset \gamma$. From the preceding paragraph it follows immediately that every member of γ contains a tail. Next suppose there is $V \in \gamma$ such that V is closed and $V \notin \lambda$. Since γ is a regular filter, then γ is round. Hence there exist closed sets $R, W \in \gamma$ such that $R < W < V$.

Since $V \notin \lambda$, then for each $F \in \lambda$, $F \cap (T-V) \neq \emptyset$. Thus $T-V$ gets into the corner. Now $T-V$, $T-W$, $T-R$ are open sets and $T-V < T-W < T-R$. By the above paragraph, $T-R$ contains a tail.

Since $R \in \gamma$ and $\lambda \subset \gamma$, then R contains a tail. Thus $R \cap (T-R) \neq \emptyset$, which is a contradiction. It follows that every member of γ is a member of λ, and therefore finally it follows that $\gamma = \lambda$, that is, the only free regular filter on T is the filter.

Consider the Wallman proximity δ_w on T. By 3.5 the round filters are just the regular filters, so the maximal round filters are the neighborhood filters along with the unique free maximal round filter λ. It follows that if A and B are subsets of T then A surrounds B if and only if A and B have disjoint closures in the (unique) compactification $\omega' \times \Omega'$ of T. Since T is not normal it follows that δ_w is not maximally full. To see that δ_w is maximally bounded observe that if W is an open member of the unique free maximal round filter λ, then $T-W$ is compact; thus every maximal cover has a finite maximal subcover.

Since the Wallman proximity on T is maximally saturated (3.6), it is an example of a maximally saturated proximity that is not maximally full.

Since the Wallman proximity is not maximally full it is not an RC-proximity, and thus by Theorem B the space T is an example of an RC-regular space that is not RC-normal.

It is interesting to observe that T has a generalization of the property of almost-compactness, that is, having only one compactification; it has just been shown that the space T has only one regular-closed extension. Clearly any completely regular space having only one regular-closed extension is almost-compact.

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