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P R O B L È M E S

**P 21, R 1.** Dr. E. Grzegorek has informed us that under the assumption of the Continuum Hypothesis or only Martin's Axiom the answer is positive. The solution is in preparation to this journal.

I, p. 149.

**P 415, R 1.** K. K. Kubota et P. Liardet ont prouvé que la conjecture est fausse. Ils ont trouvé <sup>(1)</sup> un corps  $K$  tel que

1°  $K$  n'est pas engendré par des éléments dont les degrés par rapport à une extension transcendante pure de  $Q$  sont bornés;

2° tout polynôme qui transforme une partie infinie de  $K$  sur elle-même est linéaire.

Il reste le problème de caractériser la classe des corps satisfaisant la condition 2°.

X.1, p. 186.

<sup>(1)</sup> K. K. Kubota and P. Liardet, *Réfutation d'une conjecture de W. Narkiewicz*, Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences, Paris, 282 (1976), p. A1261-A1264.

**P 563, R 1.** The answer is negative. It follows from Theorem 5 of Mal'cev <sup>(2)</sup> that a theory based on the axioms

$$f(x, y) = f(u, v) \rightarrow x = u \wedge y = v, \quad \varphi \neq x,$$

where  $\varphi$  is any term different from  $x$  in which  $x$  occurs, is decidable.

It should be added that the theory based solely on the first axiom is undecidable, which follows from the following easy lemma:

For any binary relation  $R \subset A^2$ , where  $A$  is any non-empty set, there exist a set  $B$  disjoint with  $A$  and a function  $f: (A \cup B)^2 \rightarrow A \cup B$  such that the algebra  $(A \cup B, f)$  satisfies the first axiom and

$$\begin{aligned} x \in A &\leftrightarrow \forall u, v [f(u, v) \neq x], \\ (x, y) \in R &\leftrightarrow \{x, y \in A \wedge \exists z [f((x, y), z) = z]\}. \end{aligned}$$

XV.2, p. 320.

Letter of Jan Mycielski, May 1976.

<sup>(2)</sup> A. I. Mal'cev, *Axiomatizable classes of locally free algebras of various types*, In the collection *The metamathematics of algebraic systems*, North Holland 1971, p. 262-281.

**P 909, R 1.** The answer is negative as shown by the known example of Katzenelson and McGehee <sup>(3)</sup>.

XXXI.2, p. 318.

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<sup>(3)</sup> Y. Katzenelson and O. C. McGehee, *Measures and pseudomeasures on compact subsets of the line*, Mathematica Scandinavica 23 (1968), p. 57-68.

**P 923, R 3.** There appeared another proof of the conjecture <sup>(4)</sup>.

XXXII.1, p. 151, et XXXVI.2, p. 329.

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<sup>(4)</sup> I. Kátai, *On additive functions having a non-decreasing normal order*, this fascicle, p. 153-157.

**P 937, R 1.** The answer is negative. Two independent and different solutions, given by E. Grzegorek and A. R. D. Mathias, are in preparation to this journal.

XXXII.2, p. 310.

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A. LELEK (DETROIT, MICHIGAN)

**P 994 - P 997.** Formulés dans la communication *On the surjective span and semispan of connected metric spaces*.

Ce fascicule, p. 37 et 38.

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LOUIS F. McAULEY (BINGHAMTON, NEW YORK)

**P 998 et P 999.** Formulés dans la communication *A characterization of light open mappings and the existence of group actions*.

Ce fascicule, p. 57.

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C. BESSAGA (WARSZAWA)

**P 1000.** Formulé dans la communication *Central points of convex sets*.

Ce fascicule, p. 67.

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A. NOWAK (KATOWICE)

**P 1001.** Formulé dans la communication *On a general dynamic programming problem*.

Ce fascicule, p. 137.

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M. BRUNEAU (RABAT)

**P 1002 et P 1003.** Formulés dans la communication *Approximation sur le tore et irrégularité des fonctions*.

Ce fascicule, p. 146.

## I. KÁTAI (BUDAPEST)

**P 1004.** Formulé dans la communication *On additive functions having a non-decreasing normal order.*

Ce fascicule, p. 153.

## FLEMING TØPSOE (COPENHAGEN)

**P 1005.** Is it possible to find in  $Q$  (the space of rationals) a sequence  $\{P_n\}_{n \geq 1}$  of probability measures such that  $P_n$  converges in the topology of weak convergence to some measure  $P$  and such that the only compact subset  $K \subset Q$  for which  $P_n K \rightarrow PK$  holds is the empty set? (With  $Q$  replaced by a Polish space such examples do not exist.)

New Scottish Book, Probl. 922, 10. 10. 1975.

**P 1005, R 1.** Miss M. Łachwa gave an affirmative answer (in preparation to this journal).

## DAVID P. BELLAMY (WARSZAWA)

**P 1006.** Can every finite-dimensional metric hereditarily indecomposable continuum be embedded into a finite product of pseudo-arcs?

New Scottish Book, Probl. 923, 11. 12. 1975.

**P 1007.** If  $M$  is an open  $n$ -manifold for  $n \geq 2$  and  $\beta M - M$  is connected, is  $\beta M - M$  an aposyndetic continuum? (This is true if  $M = R^n$ .)

New Scottish Book, Probl. 924, 13. 12. 1975.

## S. HARTMAN (WROCŁAW)

**P 1008.** Trouver, si possible, deux compacts  $K_1 \subset R^-$  et  $K_2 \subset R^+$  satisfaisant à deux conditions suivantes:

(1) il existe une constante  $C > 0$  telle que

$$\|P_1 + P_2\|_\infty \geq C(\|P_1\|_\infty + \|P_2\|_\infty)$$

pour toute paire de polynômes trigonométriques  $P_1$  et  $P_2$  à spectre dans  $K_1$  et  $K_2$  respectivement;

(2) il existe une pseudomesure  $S$  à support dans  $K_1 \cup K_2$  non représentable sous la forme  $S = S_1 + S_2$ ,  $\text{supp } S_1 \subset K_1$ ,  $\text{supp } S_2 \subset K_2$ .

Le Nouveau Livre Ecossais, Probl. 928. 31. 5. 1976.

**P 1008, R 1.** Si  $K_1$  et  $K_2$  sont de synthèse, alors (1) et (2) sont incompatibles (P. Głowacki, à paraître dans Colloquium Mathematicum 40.2).