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On the existence of a density

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We shall give the details which demonstrate a formula for a number theoretical density which played a vital role in our paper [2], but doubts about existence and correctness of the formula have been expressed by A. Garsia, H. Moeller, and the editors. In the meantime Everett [1] has used our encoding idea to derive a new proof for one of our assertions.

We shall recall some of the conventions and symbolisms in our paper. We considered a function T, mapping the positive integers into themselves, given by

$$Tn = (3^{X(n)}n + X(n))/2, \tag{1}$$

where $X(n) = 1$ when n is odd and $X(n) = 0$ when n is even.

Given an integer n we considered iterated partities $n, Tn, T^2n, \ldots \ldots, T^kn$ and we agreed to stop the iteration at the very first instance when $T^kn < n$. This *stopping time* was denoted by $\chi(n) = k$. Infinite values for the stopping time were permitted. We also introduced a second stopping time $\tau(n)$ which had a periodicity property. The quantity $\mathrm{P}[\tau = k]$ was defined to be the proportion of integers in $[1, 2^k]$ which satisfy the relation $\tau(n) = k$. The quantities $\mathrm{P}[\tau < k]$ and $\mathrm{P}[\tau \geqslant k]$ were defined similarly in the same block of integers.

If A is a set of positive integers then the *density* of A is defined in terms of the counting function μ to be

$$\delta(A) = \lim_{m\to\infty} (1/m)\mu\{n \leqslant m \mid n \in A\} \tag{2}$$

provided this limit exists. We now set $[\chi = k] = \{n \geqslant 0 \mid \chi(n) = k\}$, and we define $[\tau < k]$ and $[\tau \geqslant k]$ in a similar manner.

THEOREM. *The denisty of the set $[\chi \geqslant k]$ exists and is given by*

$$\delta[\chi \geqslant k] = \mathrm{P}[\tau \geqslant k]. \tag{3}$$

Proof. The trick involved is to get this formula without forming any infinite sums. In [2] we established the formula $\delta[\chi = k] = \mathrm{P}[\tau = k]$. Finite additivity of density gives $\delta[\chi < k] = \mathrm{P}[\tau < k]$. Since the sets

$[\chi < k]$ and $[\chi \geqslant k]$ are complementary sets of positive integers one has that $\delta[\chi \geqslant k]$ exists and that

(4) $$\delta[\chi < k] + \delta[\chi \geqslant k] = 1.$$

One also has the relation

(5) $$P[\tau < k] + P[\tau \geqslant k] = 1,$$

which holds because we have defined the quantities involved in terms of a finite block of integers $[1, 2^k]$. The assertion of the theorem follows.

The relation $P[\tau \geqslant k] = \delta[\chi \geqslant k]$ enables one to compute explicit values of $\delta[\chi \geqslant k]$ for quite large values of k. In [3] we consider two quite distinct general algorithms in a probabilistic context which enables one to perform such a computation. A table of these density values already appeared in [2].

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