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On the existence of a density

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We shall give the details which demonstrate a formula for a number theoretical density which played a vital role in our paper [2], but doubts about existence and correctness of the formula have been expressed by A. Garsia, H. Moeller, and the editors. In the meantime Everett [1] has used our encoding idea to derive a new proof for one of our assertions.

We shall recall some of the conventions and symbolisms in our paper. We considered a function T , mapping the positive integers into themselves, given by

$$(1) \quad Tn = (3^{X(n)}n + X(n))/2,$$

where $X(n) = 1$ when n is odd and $X(n) = 0$ when n is even.

Given an integer n we considered iterated partities $n, Tn, T^2n, \dots, T^k n$ and we agreed to stop the iteration at the very first instance when $T^k n < n$. This *stopping time* was denoted by $\chi(n) = k$. Infinite values for the stopping time were permitted. We also introduced a second stopping time $\tau(n)$ which had a periodicity property. The quantity $P[\tau = k]$ was defined to be the proportion of integers in $[1, 2^k]$ which satisfy the relation $\tau(n) = k$. The quantities $P[\tau < k]$ and $P[\tau \geq k]$ were defined similarly in the same block of integers.

If A is a set of positive integers then the *density* of A is defined in terms of the counting function μ to be

$$(2) \quad \delta(A) = \lim_{m \rightarrow \infty} (1/m)\mu\{n \leq m \mid n \in A\}$$

provided this limit exists. We now set $[\chi = k] = \{n \geq 0 \mid \chi(n) = k\}$, and we define $[\tau < k]$ and $[\tau \geq k]$ in a similar manner.

THEOREM. *The density of the set $[\chi \geq k]$ exists and is given by*

$$(3) \quad \delta[\chi \geq k] = P[\tau \geq k].$$

Proof. The trick involved is to get this formula without forming any infinite sums. In [2] we established the formula $\delta[\chi = k] = P[\tau = k]$. Finite additivity of density gives $\delta[\chi < k] = P[\tau < k]$. Since the sets

$[\chi < k]$ and $[\chi \geq k]$ are complementary sets of positive integers one has that $\delta[\chi \geq k]$ exists and that

$$(4) \quad \delta[\chi < k] + \delta[\chi \geq k] = 1.$$

One also has the relation

$$(5) \quad P[\tau < k] + P[\tau \geq k] = 1,$$

which holds because we have defined the quantities involved in terms of a finite block of integers $[1, 2^k]$. The assertion of the theorem follows

The relation $P[\tau \geq k] = \delta[\chi \geq k]$ enables one to compute explicit values of $\delta[\chi \geq k]$ for quite large values of k . In [3] we consider two quite distinct general algorithms in a probabilistic context which enable one to perform such a computation. A table of these density values already appeared in [2].

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