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Intersections of ANR's

by

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Abstract. Let $\{F_i\}$ be a sequence of compact ANR's such that each F_i is a retract of F_{i+1}. K. Borsuk conjectured that the intersection of this collection is a fundamental ANR. In this note, algebraic conditions which imply this conjecture are obtained. For example, the conjecture is verified if the fundamental group of some F_i is Abelian or finite. A partial converse is obtained if some F_i has the homotopy type of a 2-complex.

Let $\{F_i\}_{i=1}^{\infty}$ be a sequence of compact ANR's such that $F_i \supseteq F_{i+1}$ and there exist retractions $r_i\colon F_i \to F_{i+1}$ for all i. If F_1 is contractible, then Borsuk [2] has shown that $\bigcap F_i$ is a fundamental AR and conjectured that, in general, $\bigcap F_i$ is a fundamental ANR. In this note we attempt to reduce this conjecture to an algebraic problem and we solve the algebraic problem in many cases. If F is a compact ANR, then F *satisfies* FIR if whenever $\{F_i\}_{i=1}^{\infty}$ is a sequence of subspaces of F with $F_1 = F$ and there exist retractions $r_i\colon F_i \to F_{i+1}$ for all i, then $\bigcap F_i$ is a fundamental ANR. If G is a group (R-module), then G *satisfies* FIR if whenever $\{G_i\}_{i=1}^{\infty}$ is a sequence of subgroups (submodules) of G with $G_1 = G$ and there exist retractions $r_i\colon G_i \to G_{i+1}$ for all i, then there exists n such that for all $i \geqslant n$, $G_i = G_n$.

THEOREM 1. *Let $\{F_i\}_{i=1}^{\infty}$ be a sequence of compact connected* ANR*'s such that $F_i \supseteq F_{i+1}$ and there exist retractions $r_i\colon F_i \to F_{i+1}$ for all i. Let $x \in F_i$. The following are equivalent.*

(A) *$(\bigcap F_i, x)$ is a pointed fundamental* ANR [3].

(B) *For each $j \geqslant 1$, the induced system of groups $\{\pi_j(F_i, x)\}_{i=1}^{\infty}$ is equivalent in the category of pro-groups to a group.*

(C) *For each $j \geqslant 1$, there exists n_j such that if $i > n_j$, then the inclusion induced homomorphism $\pi_j(F_{i+1}, x) \to \pi_j(F_i, x)$ is an isomorphism.*

Proof. By West [12], F_1 has the homotopy type of a finite n-dimensional complex. By Theorem F of Wall [11], each F_i has the homotopy type of a finite complex of dimension $\leqslant \max\{3, n\}$. It follows that F_i has finite fundamental dimen-

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sion [3]. Now the equivalence of (A) and (B) follows from the stability theorem of Edwards and Geoghegan [4]. To prove the equivalence of (B) and (C) we need the tools of pro-category theory; the reader is referred to [7] for basic definitions. We will suppress the use of the basepoint in the notation.

(B)$\Rightarrow$(C). Fix $j\geqslant 1$ and suppose that the system $\{\pi_j(F_i)\}$ is equivalent in pro-groups to the group H. Thus there exist systems of homomorphisms $\{\varphi_i\}$: $\{\pi_j(F_i)\}\to\{H\}$ and $\{\lambda_i\}$: $\{H\}\to\{\pi_j(F_i)\}$ such that $\{\lambda_i\}\circ\{\varphi_i\} = \{\mathrm{id}\}$ and $\{\varphi_i\}\circ\{\lambda_i\} = \{\mathrm{id}\}$ (id = identity function). Note that $\{\varphi_i\}$ consists of a single homomorphism φ_k: $\pi_j(F_k)\to H$. Since $\varphi_k\lambda_k = \mathrm{id}$, $(\lambda_k\varphi_k)^2 = \lambda_k\varphi_k$ and, thus, $\lambda_k\varphi_k$: $\pi_j(F_k)\to\pi_j(F_k)$ is a retraction. Let μ_{in}: $\pi_j(F_n)\to\pi_j(F_i)$ denote the homomorphism induced by inclusion. Since $\{\lambda_i\}$ is a mapping of systems, μ_{ki} (image λ_i) = image λ_k for all $i\geqslant k$. Since $\{\lambda_i\}\circ\{\varphi_k\} = \mathrm{id}$, there exists $n_j\geqslant k$ such that for all $i\geqslant n_j$, $\lambda_k\varphi_k\mu_{ki} = \mu_{ki}$. Thus $\mu_{ki}(\pi_j(F_i))\subseteq$image $\lambda_k = \mu_{ki}$ (image λ_i). Since F_i is a retract of F_k, μ_{ki} is one-to-one and, hence, image $\lambda_i = \pi_j(F_i)$ for all $i\geqslant n_j$. (C) follows. The implication (C)$\Rightarrow$(B) is trivial.

Remark. In our applications of Theorem 1, we need to note that a pointed fundamental ANR is a fundamental ANR.

COROLLARY 2. *If F is a compact connected* ANR *such that for all $j\geqslant 1$, $\pi_j(F)$ has property* FIR, *then F has property* FIR.

In general, we would not expect that the converse be true since the homotopy groups of F need not be finitely generated even as modules over $Z\pi_1(F)$, the integral group ring of $\pi_1(F)$ [10].

PROPOSITION 3. *If the group G satisfies the maximum condition on subgroups, then G satisfies* FIR. *If the R-module M is Noetherian, then M satisfies* FIR.

Proof. Let $\{G_i\}_{i=1}^{\infty}$ be a sequence of subgroups of G with $G_1 = G$ and with retractions r_i: $G_i\to G_{i+1}$ for all i. Let K_i be the kernel of $r_ir_{i-1}\ldots r_1$; note that $K_i\subseteq K_{i+1}$ for all i. By the maximum condition, there exists n such that for $i\geqslant n$, $K_i = K_n$. It follows then that for $i\geqslant n$ that $G_i = G_n$. The proof of the second part is the same.

COROLLARY 4. *If G is a finitely generated Abelian group, then G satisfies* FIR.

The following theorem is essentially Theorem 10.4 of M. Moszyńska's [8]; we include the proof for completeness.

THEOREM 5. *If F is a compact simply-connected* ANR, *then F satisfies* FIR.

Proof. Let $\{F_i\}_{i=1}^{\infty}$ be a sequence of subspaces with retractions r_i: $F_i\to F_{i+1}$ and $F_1 = F$. Since F_1 is simply-connected, each F_i is also simply-connected. Since F_1 is homotopy equivalent to a compact polyhedron, the homology groups $H_i(F_1)$ are all finitely generated. For each $j\geqslant 0$, by Corollary 4 there exists n_j such that if $i\geqslant n_j$, the inclusion induced homomorphism $H_j(F_i)\to H_j(F_{i-1})$ is an isomorphism. Hence, if $i>n_j$, then the relative group $H_j(F_{i-1}, F_i) = 0$. By the relative Hurewicz Theorem, if $i>m_j = \max\{n_k: 1\leqslant k\leqslant j\}$, then the relative homotopy groups $\pi_j(F_{i-1}, F_i) = 0$.

Hence if $i>m_j$, then the inclusion induced homomorphism $\pi_j(F_i)\to\pi_j(F_{i-1})$ is an isomorphism and condition (C) of Theorem 1 is satisfied.

COROLLARY 6. *If F is a compact connected* ANR *such that $\pi_1(F)$ is finite, then F satisfies* FIR.

Proof. Let $\{F_i\}_{i=1}^{\infty}$ be a sequence of subspaces of F with $F_1 = F$ and with retractions r_i: $F_i\to F_{i+1}$ for all i. Consider the universal covering ϱ: $\tilde F_1\to F_1$. Fix a base point $x\in\tilde F_1$ such that $\varrho(x)\in\bigcap F_i$. Let $\tilde F_i$ be the component of $\varrho^{-1}(F_i)$ which contains x. Consider $r_i\varrho|\tilde F_i$: $\tilde F_i\to\tilde F_{i+1}$; by [9; p. 76] there exists a lifting $\tilde r_i$: $\tilde F_i\to\tilde F_{i+1}$ such that $\tilde r_i(x) = x$. By using standard arguments from covering space theory, it is straightforward to show that $\tilde r_i$ is a retraction for each i. Since $\pi_1(F)$ is finite, $\tilde F_1$ is a compact connected ANR and Theorem 5 applies; hence $\bigcap\tilde F_i$ is a pointed fundamental ANR. By Theorem 1, for each j there exists n_j such that for $i>n_j$ the inclusion induced homomorphism $\pi_j(\tilde F_i)\to\pi_j(\tilde F_{i-1})$ is an isomorphism. For $j\geqslant 2$, $\pi_j(\tilde F_i)$ is naturally isomorphic to $\pi_j(F_i)$ and, hence, $\pi_j(F_i)\to\pi_j(F_{i-1})$ is also an isomorphism. Since a finite group satisfies FIR, condition (C) of Theorem 1 is satisfied.

PROPOSITION 7. *Let F be a compact connected* ANR *such that the homology groups $H_n(\tilde F)$ of the universal covering space considered as $Z\pi_1(F)$-modules satisfy* FIR. *Then if $\{F_i\}_{i=1}^{\infty}$ is a sequence of subspaces of F such that $F_1 = F$ and there exist retractions r_i: $F_i\to F_{i+1}$ which induce isomorphisms $\pi_1(F_i)\to\pi_1(F_{i+1})$ for all i, then $\bigcap F_i$ is a fundamental* ANR.

Proof. Let $\tilde F_i$ be the universal covering space of F_i and lift r_i to retractions $\tilde r_i$: $\tilde F_i\to\tilde F_{i+1}$ as in the proof of Corollary 6. By hypothesis, for each j, there exists m_j such that if $i\geqslant m_j$, then the inclusion induced homomorphisms $H_j(\tilde F_i)\to H_j(\tilde F_{i-1})$ are isomorphisms. For each j, let $n_j = \max\{m_k|\ 1\leqslant k\leqslant j\}$; hence if $i>n_j$, the relative groups $H_k(\tilde F_i, \tilde F_{i+1}) = 0$ for $k\leqslant j$. By using the relative Hurewicz Theorem and covering space theory as in the previous proof, we get that condition (C) of Theorem 1 is again satisfied.

THEOREM 8. *Let F be a compact connected* ANR *such that the integral group ring of $\pi_1(F)$ is Noetherian, then F satisfies* FIR.

Proof. Let $\{F_i\}_{i=1}^{\infty}$ be a sequence of subspaces of F with $F_1 = F$ and retractions r_i: $F_i\to F_{i+1}$ for all i. Since $Z\pi_1(F)$ is Noetherian, then $\pi_1(F)$ satisfies the maximum condition on subgroups [5] [11, p. 61] and, hence, by Proposition 3, there exists n such that for all $i\geqslant n$, the induced homomorphism $\pi_1(F_i)\to\pi_1(F_{i+1})$ are isomorphisms. The retraction $r_{n-1}r_{n-2}\ldots r_1$: $F\to F_n$ induces an epimorphism $Z\pi_1(F)\to Z\pi_1(F_n)$. Since $Z\pi_1(F)$ is Noetherian, $Z\pi_1(F_n)$ is likewise.

Let P be a finite complex which is homotopy equivalent to F_n. By considering simplicial homology theory, the chain groups of the universal covering space of P, $C_i(\tilde P)$, are finitely generated $Z\pi_1(P)$-modules. Since $Z\pi_1(P)$ is Noetherian, $C_i(\tilde P)$ is also Noetherian. Thus the submodule of cycles and, hence, the homology groups $H_i(\tilde P)$ are finitely generated Noetherian modules. By Proposition 3, $H_i(\tilde F_n)$ satisfies FIR for all i and the theorem now follows from Proposition 7.

Since the integral group ring of a commutative group is Noetherian [5], we have the following result.

COROLLARY 9. *Let F be a compact connected* ANR *such that $\pi_1(F)$ is commutative, then F satisfies* FIR.

Let G be a group and consider the integral groupring ZG of G. Consider the augmentation homomorphism α: $ZG\to Z$ which is induced by the trivial homomorphism of G to the trivial group. We can then consider Z as a ZG-module by defining $f\cdot n = \alpha(f)\cdot n$ where $f\in ZG$, $n\in Z$ and the latter multiplication is the usual multiplication in Z.

We say that G has *property* C if, whenever M is a finitely generated projective ZG-module such that M represents the trivial element in the projective class group, $\tilde{K}_0(ZG)$ [11] and the tensor product over ZG, $M\otimes_{ZG}Z = 0$, then $M = 0$ (1).

PROPOSITION 10. *If every finitely generated projective ZG-module is free, then G has property* C.

Proof. Let M be a finitely generated projective ZG-module such that $M\otimes_{ZG}Z = 0$. Then M is isomorphic to the direct sum of n copies of ZG where n is the rank of M. Since $ZG\otimes_{ZG}Z$ is isomorphic to Z and $\otimes$ is distributive over $\oplus$, $M\otimes_{ZG}Z$ is isomorphic to the direct sum of n copies of Z. Hence $n = 0$.

Bass [1] [11, p. 67] has shown that if G is a finitely generated free group, then every finitely generated projective ZG-module is free.

COROLLARY 11. *If G is a finitely generated free group, then G has property* C.

It is not difficult to show that a finitely generated free group also satisfies property FIR.

PROPOSITION 12. *Suppose that G is a group which satisfies condition* C. *If H is a subgroup of G which is a retract of G then H also satisfies condition* C.

Proof. Let M be a finitely generated projective ZH-module which represents the trivial element in $\tilde{K}_0(ZH)$ and such that $M\otimes_{ZH}Z = 0$. Since $ZH\subseteq ZG$, ZG can be considered as a ZH-module. $ZG\otimes_{ZH}M$ can be made into a ZG-module by defining $g(h\otimes m) = (gh)\otimes m$ for $g, h\in ZG$ and $m\in M$. Since M is projective, there exists a finitely generated ZG-module N such that $M\oplus N$ is a free ZH-module. Thus

$$(ZG\otimes_{ZH}M)\oplus(ZG\otimes_{ZH}N)\simeq ZG\otimes_{ZH}\Big(\bigoplus_{i=1}^{n} ZH\Big)\simeq \bigoplus_{i=1}^{n}(ZG\otimes_{ZH}ZH)\simeq \bigoplus_{i=1}^{n} ZG$$

for some integer n and, hence, $ZG\otimes_{ZH}M$ is a finitely generated projective ZG-module.

Since M represents the trivial element of $\tilde{K}_0(ZH)$, N can be chosen to be a free ZH-module. Since $ZG\otimes_{ZH}N$ is then a free ZG-module, $ZG\otimes_{ZH}M$ is also represents the trivial element of $\tilde{K}_0(ZG)$.

Let α: $ZH\to Z$ be the augmentation homomorphism and let A be the kernel of α. Consider the epimorphism $M\to M\otimes_{ZH}Z$ defined by $m\to m\otimes 1$; the kernel of this map is $AM = \{\sum a_i m_i \mid a_i\in A$ and $m_i\in M\}$. Since $M\otimes_{ZH}Z = 0$, if $m\in M$, then

(1) See "Added in proof".

$m = \sum a_i m_i$ where $a_i\in A$ and $m_i\in M$. Hence if $(g\otimes m)\otimes 1\in(ZG\otimes_{ZH}M)\otimes_{ZG}Z$, then

$$\begin{aligned}(g\otimes m)\otimes 1 &= \big(g\otimes \textstyle\sum a_i m_i\big)\otimes 1 = \textstyle\sum a_i(g\otimes m_i)\otimes 1\\ &= \textstyle\sum (g\otimes m_i)\otimes(a_i\cdot 1) = \sum (g\otimes m_i)\otimes 0 = 0\,.\end{aligned}$$

Thus $(ZG\otimes_{ZH}M)\otimes_{ZH}Z = 0$ and, by hypothesis, $ZG\otimes_{ZH}M = 0$.

The retraction $G\to H$ induces a retraction r: $ZG\to ZH$; note that if $h\in ZH$ and $g\in ZG$, then $r(hg) = h\cdot r(g)$ and, hence, r is a ZH-homomorphism. Let K = kernel r; then $ZG = ZH\otimes K$. Hence,

$$0 = ZG\otimes_{ZH}M = (ZH\oplus K)\otimes_{ZH}M = (ZH\otimes_{ZH}M)\oplus(K\otimes_{ZH}M)$$

and, thus, $M = ZH\otimes_{ZH}M = 0$.

PROPOSITION 13. *Let X be a compact connected* ANR *which is dominated by a 2 dimensional complex and let r: $X\to Y$ be a retraction onto a subspace. Suppose that the inclusion-induced homomorphism $\pi_1(Y)\to\pi_1(X)$ is an isomorphism and suppose that the relative homology groups $H_i(X, Y)$ are trivial for all i. If $\pi_1(X)$ satisfies condition* C, *then r is a homotopy equivalence.*

Proof. By West [12] and Theorem F of Wall [11], there exist homotopy equivalences φ: $K\to X$ and λ: $L\to Y$ where K and L are finite 3-complexes. Let φ_1 and λ_1 be homotopy inverses of φ and λ, respectively. Let $\alpha = \lambda_1 r\varphi$ and $\beta = \varphi_1\lambda$; note that $\alpha\beta$ is homotopic to the identity and that α induces an isomorphism $\pi_1(K)\to\pi_1(L)$. Hence, the homotopy groups of the map $\pi_1(\alpha) = \pi_2(\alpha) = 0$. Clearly L satisfies conditions D3 and F3 of Wall [11]; hence, $\pi_3(\alpha)$ is a finitely generated projective $Z\pi_1(L)$-module by [11]. By Theorem F of [11], $\pi_3(\alpha)$ represents the zero element of $\tilde{K}_0(Z\pi_1(L))$. By the relative Hurewicz theorem [9; p. 397], $H_3(\alpha)$ is isomorphic to $\pi_3(\alpha)\otimes_{Z\pi_1(L)}Z$. Since the inclusion $Y\subseteq X$ induces isomorphisms on homology groups, the homomorphisms $H_i(X)\to H_i(Y)$ induced by r are also isomorphisms for all i. Hence $H_i(r)$ and, thus, $H_i(\alpha)$ are isomorphic to the trivial group for all i. By Condition C, $\pi_3(\alpha)$ is also the trivial group. By repeating exactly the same argument, we get that $\pi_4(\alpha)$ is the trivial group. Hence α and, thus, r are homotopy equivalences.

THEOREM 14. *Let F be a compact connected* ANR *which is dominated by a 2 dimensional complex. If $\pi_1(F)$ satisfies conditions* FIR *and* C, *then F satisfies* FIR.

Proof. Let $\{F_i\}_{i=1}^{\infty}$ be a sequence of subspaces with $F_1 = F$ and retractions r_i: $F_i\to F_{i+1}$ for all i. Since $\pi_1(F)$ satisfies FIR, there exists n_1 such that for $i\geqslant n_1$, the inclusion induced homomorphism $\pi_1(F_{i+1})\to\pi_1(F_i)$ is an isomorphism. By Corollary 4, there exists $n_2>n_1$ such that if $i>n_2$, then the inclusion induced homomorphisms $H_j(F_{i+1})\to H_j(F_i)$ are isomorphisms for $j = 2, 3$. By Theorem F of [11], each F_i has the homotopy type of a complex of dimension 3 and, hence, for $i>n_2$, $H_n(F_i, F_{i+1}) = 0$ for all n. By Proposition 13, for each $i>n_2$, r_i is a homotopy equivalence. By Theorem 1, F_i is a fundamental ANR.

Remarks. The author is able to show that if G is a finitely presented group which does not satisfy condition C, then there exists a compact connected ANR which has the homotopy type of a 2 dimensional complex with fundamental group isomorphic to G and which does not satisfy FIR. If the group G does not satisfy FIR, then it appears to be much more difficult to construct such a space.

Added in proof. Martha Smith has shown the author a proof by using M. S. Montgomery's *Left and right inverses in group algebras*, Bull. Amer. Math. Soc. 75 (1969), pp. 539–540 that all groups have property C.

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Superextensions and Lefschetz fixed point structures

by

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Abstract. The superextension of a sufficiently connected normal T_1-space admits both a convexoid structure and a semi-complex structure, and it consequently satisfies the Lefschetz fixed point property.

0. Introduction. The concept of supercompactness — a strong compactness property — arose naturally from investigations of de Groot and Aarts on internal characterizations of complete regularity (cf. [1]).

Superextensions are supercompact extensions of topological spaces, constructed in much the same way as Wallman compactifications, but replacing "filters" by "linked systems". This results in a functorial procedure transformating topological spaces into surprisingly nice compact spaces (cf. e. g. A. Verbeek [21], J. van Mill [19]). We first recall some definitions (Verbeek [21], Van Mill [18]):

Let $\mathscr{S}$ be a collection of subsets of a set X. A *linked system in* $\mathscr{S}$ is a collection $\mathscr{M} \subset \mathscr{S}$ such that any two members of $\mathscr{M}$ meet. A *maximal linked system in* $\mathscr{S}$ — briefly: an *mls* — is a linked system not properly contained in another linked system in $\mathscr{S}$.

Let X now be a topological space and let $\mathscr{S}$ be a closed subbase of X. The *superextension of X relative to* $\mathscr{S}$, $\lambda_{\mathscr{S}}(X)$, is the topological space defined on the set of all mls's in $\mathscr{S}$ by the closed subbase

$$\mathscr{S}^+ = \{C^+ |\ C \in \mathscr{S}\},$$

where C^+ denotes the set of all mls's in $\mathscr{S}$ containing C as a member. If $\mathscr{S}$ equals the set of *all* closed subsets of X, then $\lambda_{\mathscr{S}}(X) = \lambda(X)$ is called the *superextension of* X. Each superextension of X is obviously a T_1-space.

A topological space X is *supercompact* provided that there is a closed subbase $\mathscr{S}$ of X such that each linked system $\mathscr{M}$ in $\mathscr{S}$ satisfies

$$\bigcap \{M |\ M \in \mathscr{M}\} \neq \emptyset .$$

In addition, this subbase $\mathscr{S}$ is said to be *binary*. If $\mathscr{S}$ is an arbitrary closed subbase of X, then the corresponding subbase $\mathscr{S}^+$ of $\lambda_{\mathscr{S}}(X)$ is binary, and $\lambda_{\mathscr{S}}(X)$ is supercompact. Observe that a supercompact space is compact by Alexander's lemma.