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A result on the isomorphic embeddability of $l^1(\Gamma)$

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Abstract. It is proven that the isomorphic embeddability of $l^1(\Gamma)$ into a Banach space X is equivalent to the existence of a certain operator from X to $L^\infty\{-1, 1\}^\Gamma$.

Applying the theorem we get some results for the isomorphic structure of Banach spaces related to $L^1(\mu)$ spaces.

Introduction. In this paper we prove a criterion on the isomorphic embeddability of $l^1(\Gamma)$ into B-spaces and using this we get some results on the isomorphic structure of non-separable B-spaces. So we prove the following:

THEOREM A. *Assume Martin's axiom and the negation of the continuum hypothesis. Then for any B-space X and any infinite cardinal α the following are equivalent*:

(a) *$l^1(\Gamma)$ is isomorphic to a subspace of X for some set Γ with cardinality of Γ equal to α.*

(b) *There is a bounded linear operator T: $X \to L^\infty(\mu)$ for a finite measure μ such that for some uniformly bounded family $\{x_\xi: \xi < \alpha\}$ of elements of X we have $\|Tx_{\xi_1} - Tx_{\xi_2}\|_1 > \theta > 0$ for $\xi_1 < \xi_2 < \alpha$.*

(c) *There is a bounded linear operator T: $X \to L^\infty\{-1, 1\}^\alpha$ such that for some uniformly bounded family $\{x_\xi: \xi < \alpha\}$ of elements of X we have $\|Tx_{\xi_1} - Tx_{\xi_2}\|_1 > \theta > 0$ for all $\xi_1 < \xi_2 < \alpha$.*

In cases (b) and (c) $\|\cdot\|_1$ denotes the norm in the spaces $L^1(\mu)$, $L^1\{-1, 1\}^\alpha$, respectively, and $L^\infty\{-1, 1\}^\alpha$ denotes the space $L^\infty(\mu_\alpha)$ where μ_α is the Haar-measure on the compact group $\{0, 1\}^\alpha$.

The above theorem for the countable case ($\alpha = \omega$) is a well-known consequence of Rosenthal's criterion of isomorphic embeddings of l^1 into B-spaces [12].

Haydon in [7] and [8] proved this result for certain categories of cardinals.

The essential part of our proof is devoted to the implication (c) $\Rightarrow$ (a). For this we distinguish two cases. The case of cardinals greater than ω^+ and the case of the cardinal ω^+. In the last case we make use of the additional set-theoretical hypothesis, namely of Martin's axiom and the negation of continuum hypothesis (MA+$\neg$CH). The proofs of these cases are given in Sections 1 and 2, respectively. Finally, Section 4 contains two consequences of Theorem A.

THEOREM B (4.1). *Assume* MA+$\neg$CH. *Let X be a B-space and $W = (\mathrm{ba}[X^*], w^*)$. Then for cardinals α with uncountable* $\mathrm{cf}(\alpha)$ *the following are equivalent*:

(a) *l^1_α is isomorphic to a subspace of X.*

(b) *There is a map $f\colon W \to [0, 1]^\alpha$ continuous and onto.*

(c) *l^1_α is isomorphic to a subspace of $C(W)$.*

Talagrand in [16] has proved that (a) is-equivalent to (b) for regular uncountable cardinals without any set-theoretical assumption*. Our result extends Talagrand's result into the class of singular cardinals with uncountable cofinality.

THEOREM C (4.4). *Assume* MA+$\neg$CH. *If α is an uncountable cardinal and μ an α-homogeneous probability measure, then the existence of a semi-embedding of $L^1(\mu)$ into a conjugate B-space X^* implies the existence of an isomorphic embedding of $L^1(\mu)$ into X^*.*

For the case $\alpha = \omega$, this result has been proved by Bourgain and Rosenthal [13].

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0. Preliminaries. We consider an ordinal as the set of smaller ordinals. A cardinal is an ordinal not in one-to-one correspondence with any smaller ordinal. We denote by ω the first infinite cardinal and by ω^+ the first uncountable cardinal. The cofinality of a cardinal α is denoted by $\mathrm{cf}(\alpha)$ and it is the smaller cardinal β such that there is a cofinal subset of α with cardinality β. A cardinal α is said to be *regular* if $\alpha = \mathrm{cf}(\alpha)$, otherwise it is singular. For a set Λ we denote by $|\Lambda|$ the cardinality of Λ and by $\mathscr{P}_\omega(\Lambda)$ the set of all finite subsets of Λ.

0.1. LEMMA (Shanin [15]). *If α is an uncountable regular cardinal and f is a function from α to $\mathscr{P}_\omega(\alpha)$, then there is Λ – a subset of α with $|\Lambda| = \alpha$ and N – a finite set such that $f(\xi_1) \cap f(\xi_2) = N$ for all distinct $\xi_1, \xi_2 \in \Lambda$.*

Next lemma is an application of Hajnal's theorem [6] and a proof of it can be found in [2].

* In a revised version of his paper M. Talagrand also proves the non-regular case. An alternative proof of Talagrand's result is given in [1].

0.2. LEMMA. *Let α be cardinal greater than ω^+ and let $\{N_\xi\colon \xi < \alpha\}$, $\{A_\xi\colon \xi < \alpha\}$ be families of countable and finite sets, respectively, with $A_{\xi_1} \cap A_{\xi_2} = \emptyset$ for $\xi_1 < \xi_2 < \alpha$. Then there is Λ – a subset of α, with $|\Lambda| = \alpha$, such that for all distinct ξ_1, ξ_2 elements of Λ, $A_{\xi_1} \cap N_{\xi_2} = \emptyset$.*

Let $(P, <)$ be a partially ordered set. Two elements $p_1, p_2 \in (P, <)$ are said to be *compatible* if there is $q \in P$ with $q < p_1$ and $q < p_2$. A subset $\mathscr{F}$ of P is a *filter* if any finite subset of $\mathscr{F}$ is compatible. The partially ordered set $(P, <)$ satisfies c.c.c. if every uncountable family of elements of it has at least two compatible elements.

The following proposition is a consequence of Martin's axiom. A proof of it can be found in [14].

0.3. PROPOSITION. *Assume* MA *and let α be regular cardinal with $\omega < \alpha < 2^\omega$. Then for each $(P, <)$ partially ordered set satisfying* c.c.c. *and each family $H = \{p_\xi\colon \xi < \alpha\}$ of elements of P there is $\mathscr{F}$ – a filter of elements of P containing a subfamily of the family H with cardinality equal to α.*

For a set Γ let $l^1(\Gamma)$ be the Banach space of all real-valued functions f satisfying the condition

$$\|f\| = \sum_{\gamma \in \Gamma} |f(\gamma)| < \infty$$

and we denote by $l^\infty(\Gamma)$ the conjugate of $l^1(\Gamma)$.

In the rest of the paper μ always denotes a finite measure and $L^p(\mu)$ is the corresponding B-space of all p-integrable functions. We also connect $L^\infty(\mu)$ with the conjugate of $L^1(\mu)$.

Let I be a non-empty set; for finite A being a subset of I, let

$$\Pi_A\colon \{-1, 1\}^I \to \{-1, 1\}$$

be the function defined by

(a) $\Pi_A(x) = 1$ for all $x \in \{-1, 1\}^I$ in the case $A = \emptyset$, and

(b) $\Pi_A = \prod_{\xi \in A} \Pi_\xi$,

i.e. Π_A is the function product of the projections Π_ξ for all $\xi \in A$.

The set $\{\Pi_A\colon A \in \mathscr{P}_\omega(I)\}$ is said to be the *Walsh functions* of the group $\{0, 1\}^I$ and it consists of a biorthogonal system for the space $L^2(\mu_I)$ where μ_I is the Haar measure on the group $\{0, 1\}^I$. In the rest of the paper we will connect $\{0, 1\}^I$ with $\{-1, 1\}^I$ by the usual way and by $L^p\{-1, 1\}^I$ we will denote the space $L^p(\mu_I)$.

Let $1 \leqslant p \leqslant \infty$, $f \in L^p\{-1, 1\}^I$, $\xi \in I$. We say that the function f *depends on the element* ξ of the set I iff there is the set A – a finite subset of I, such that

$$\int f \cdot \Pi_A \, d\mu \neq 0 \quad \text{and} \quad \xi \in A.$$

It is a consequence of the Stone–Weierstrass Theorem that for each $1 \leqslant p \leqslant \infty$ and each $f \in L^p\{-1, 1\}^I$, f depends on a countable subset of I.

Let I be a set and Δ a non-empty subset of I; then for each $1 \leqslant p \leqslant \infty$ by $\mathscr{E}^p$: $L^p\{-1, 1\}^I \to L^p\{-1, 1\}^\Delta$ we denote the corresponding conditional expectation. This is a linear contractive projection and we have that, for $1 \leqslant p_1 \leqslant p_2 \leqslant \infty$,

$$\mathscr{E}_\Delta^{p_1}(f) = \mathscr{E}_\Delta^{p_2}(f) \quad \text{for all } f \in L^{p_2}\{-1, 1\}.$$

0.4. LEMMA [2]. *Let I be a set and Δ a subset of I. If $f \in L^p\{-1, 1\}^I$ depends on the set N – a subset of I, then the function $\mathscr{E}_\Delta^\infty(f)$ is depending on a subset of the set $\Delta \cap N$.*

1. 1.1. LEMMA. *Let $f \in L^\infty\{-1, 1\}^I$ and $\varepsilon > 0$. Then there is $A \subset I$, with A finite, such that*

$$\|\mathscr{E}_A^\infty(f) - f\|_1 < \varepsilon.$$

Proof. There is $f \in L^1\{-1, 1\}^I$, depending on a finite set N_g of coordinates, such that $\|f-g\|_1 < \varepsilon/2$. Then

$$\|\mathscr{E}_{N_g}^1(f) - g\|_1 = \|\mathscr{E}_{N_g}^1(f) - \mathscr{E}_{N_g}^1(g)\|_1 \leqslant \|f-g\|_1 < \varepsilon/2;$$

hence

$$\|\mathscr{E}_{N_g}^\infty(f) - f\|_1 = \|\mathscr{E}_{N_g}^1(f) - f\|_1 \leqslant \|\mathscr{E}_{N_g}^1(f) - g\|_1 + \|g-f\|_1 < \varepsilon.$$

1.2. THEOREM. *Let α be a cardinal number, $\alpha > \omega^+$, let $\{f_\xi: \xi < \alpha\}$ be subset of $L^\infty\{-1, 1\}^\alpha$ such that*

$$\|f_\xi\|_\infty \leqslant M \quad \textit{for all } \xi < \alpha,$$

and there is $\theta > 0$, with $\|f_\xi - f_\zeta\|_1 > \theta$ for $\xi < \zeta < \alpha$.

Then there is $A \subset \alpha$, with $|A| = \alpha$, such that the family $\{f_\xi: \xi \in A\}$ is equivalent to the l_α^1 – a basis in $L^\infty\{-1, 1\}^\alpha$.

Proof.

CLAIM 1. *There are $A_1 \subset \alpha$, with $|A_1| = \alpha$, and a family $\{I_\xi: \xi \in A_1\}$ of finite subsets of α such that, setting $J_\xi = \bigcup_{\substack{\zeta<\xi \\ \zeta \in A_1}} I_\zeta$, we have*

$$\|\mathscr{E}_{I_\xi}^\infty(f_\xi) - \mathscr{E}_{I_\xi \cap J_\xi}^\infty(f_\xi)\|_1 > \theta/5.$$

Proof of Claim 1. We proceed by transfinite induction. Let $\xi < \alpha$, and suppose that we have defined $\{i_\zeta, \zeta < \xi\}$, $\{I_\zeta, \zeta < \xi\}$ to be finite subsets of α, such that

$$\|\mathscr{E}_{I_\zeta}^\infty(f_{i_\zeta}) - \mathscr{E}_{I_\zeta \cap J_\zeta}^\infty(f_{i_\zeta})\|_1 > \theta/5,$$

where we have set $J_\zeta = \bigcup_{n<\zeta} I_n$. We define i_ξ, I_ξ. We set $J_\xi = \bigcup_{\zeta<\xi} I_\zeta$ and $\bar{i}_\xi = \sup\{i_\zeta: \zeta < \xi\}$. Assume that for every $\zeta > \bar{i}_\xi$ and every finite subset A of I we have

$$\|\mathscr{E}_A^\infty(f_\zeta) - \mathscr{E}_{A \cap J_\xi}^\infty(f_\zeta)\|_1 \leqslant \theta/5.$$

From Lemma 1.1 for every $\zeta > \bar{i}_\xi$ there is a finite set $A_\zeta \subset I$ such that

$$\|\mathscr{E}_{A_\zeta}^\infty(f_\zeta) - f_\zeta\|_1 < \theta/5.$$

Hence, for $\zeta_1, \zeta_2 > \bar{i}_\xi$, $\zeta_1 \neq \zeta_2$, we have

$$\|\mathscr{E}_{A_{\zeta_1}}^\infty(f_{\zeta_1}) - \mathscr{E}_{A_{\zeta_2}}^\infty(f_{\zeta_2})\|_1 > \theta/2,$$

and thus

$$\|\mathscr{E}_{A_{\zeta_1} \cap J_\xi}^\infty(f_{\zeta_1}) - \mathscr{E}_{A_{\zeta_2} \cap J_\xi}^\infty(f_{\zeta_2})\|_1 > \theta/8,$$

a contradiction to the facts that $|J_\xi| < \alpha$, and that $L^1\{-1, 1\}^{J_\xi}$ has (functional) dimension less than α. The proof of the claim is complete.

CLAIM 2. *For every $\xi \in A_1$ there is $d_\xi \in L^\infty\{-1, 1\}^{I_\xi}$ with*

$$\|d_\xi\|_\infty \leqslant 1, \quad \int f_\xi \cdot d_\xi > \theta/10, \quad \int d_\xi \Pi_S = 0$$

for all finite subsets $S \subset J_\xi$.

Proof of Claim 2. By Claim 1, there is $g_\xi \in L^\infty\{-1, 1\}^{I_\xi}$, such that $\|g_\xi\|_\infty \leqslant 1$, and

$$|\mathscr{E}_{I_\xi}^\infty(f_\xi)(g_\xi) - \mathscr{E}_{I_\xi \cap J_\xi}^\infty(f_\xi)(g_\xi)| > \theta/5.$$

We have that

$$g_\xi = \sum_{S \subset I_\xi} \alpha_S \Pi_S \quad \text{with} \quad \alpha_S = \int g_\xi \cdot \Pi_S$$

and thus, setting

$$d_1^\xi \sum_{S \subset I_\xi \cap J_\xi} \alpha_S \Pi_S, \quad d_2^\xi = \sum_{\substack{S \subset I_\xi \\ S \not\subset J_\xi}} \alpha_S \Pi_S,$$

we have $g_\xi = d_1^\xi + d_2^\xi$ and

$$\theta/5 < |\mathscr{E}_{I_\xi}^\infty(f_\xi)(d_1^\xi) + \mathscr{E}_{I_\xi}^\infty(f_\xi)(d_2^\xi) - \mathscr{E}_{I_\xi \cap J_\xi}^\infty(f_\xi)(d_1^\xi) - \mathscr{E}_{I_\xi \cap J_\xi}^\infty(f_\xi)(d_2^\xi)|.$$

Observing that (inside the absolute value) the first and third terms are equal while the fourth is zero, we conclude that

$$\left|\int f_\xi d_2^\xi\right| = |\mathscr{E}_{I_\xi}^\infty(f_\xi)(d_2^\xi)| > \theta/5 \quad \text{with} \quad \|d_2^\xi\| \leqslant 2.$$

We finally set $d_\xi = \pm d_2^\xi/2$ where the sign is chosen so that

$$\int f_\xi d_\xi > \theta/10.$$

It is clear that $\int d_\xi \Pi_S = 0$ if S is finite and $S \subset J_\xi$.

We set

$$A_\xi = I_\xi \setminus J_\xi \quad \text{for} \quad \xi \in A_1$$

and we have that $\{A_\xi\colon \xi \in A_1\}$ is a pairwise disjoint family of finite subsets of α. We also choose a countable set N_ξ of coordinates so that f_ξ depends on N_ξ and $I_\xi \subset N_\xi$ for $\xi \in A_1$.

It follows from Lemma 0.2 that there is $A \subset A_1$, with $|A| = \alpha$, such that

$$(*) \qquad N_\xi \cap A_\zeta = \emptyset \quad \text{for} \quad \xi, \zeta \in A, \ \xi \neq \zeta.$$

CLAIM 3. *For all $\xi_1, \ldots, \xi_r \in A$, $\xi_1 < \ldots < \xi_r$,*

(i) $\int d_{\xi_1} \ldots d_{\xi_r} = 0$,

(ii) *if $\xi \in A$ and $\int f_\xi d_{\xi_1} \ldots d_{\xi_r} \neq 0$, then $r = 1$ and $\xi_1 = \xi$.*

Proof of Claim 3. (i): The function $d_{\xi_1}, \ldots, d_{\xi_{r-1}}$ depends on the set J_{ξ_r}, and $d_{\xi_r} = \sum \{\alpha_S \Pi_S\colon S \subset I_{\xi_r}, S \subset J_{\xi_r}\}$. By Claim 2, we have that $\int d_{\xi_1} \ldots d_{\xi_r} = 0$.

(ii): *Case* 1. $\xi_r \neq \xi$. Let N_ξ be a countable set such that f_ξ depends on N_ξ. Then $f_\xi \cdot d_\xi, \ldots, d_{\xi_{r-1}}$ depends on $N_\xi \cup J_{\xi_r}$. If $S \subset I_{\xi_r}$ and $S \not\subset J_{\xi_r}$, then $S \not\subset N_\xi \cup J_{\xi_r}$ (since $S \not\subset J_{\xi_r}$ and by property $(*)$). We now use Claim 2.

Case 2. $\xi_r = \xi$, $r > 1$. Since $I_\xi \subset N_\xi, f_\xi \cdot d_{\xi_1}, \ldots, d_{\xi_{r-2}} \cdot d_{\xi_r}$ depends on $N_\xi \cup J_{\xi_{r-1}}$, and $d_{\xi_{r-1}}$ plays the same role that d_{ξ_r} plays in Case 1.

CLAIM 4. *The family $\{f_\xi\colon \xi \in A\}$ is equivalent to the usual l^1_α-basis.*

Proof of Claim 4. Let $\xi_1 < \ldots < \xi_r < \alpha$, $\xi_1, \ldots, \xi_r \in A$ and $c_1, \ldots, c_r \in \mathbf{R}$ for some natural number $r \geqslant 1$. We set $\varepsilon_i = 1$ if $c_i \geqslant 0$ and $\varepsilon_i = -1$ if $c_i < 0$ for $1 \leqslant i \leqslant r$, and set

$$g = \prod_{i=1}^{r} (1 - \varepsilon_i d_{\xi_i}) - 1.$$

Thus $g \in L^1\{-1, 1\}^\alpha$ and g depends on a finite set of coordinates. We note that

$$\|g\|_1 \leqslant 2$$

since $1 - \varepsilon_i d_{\xi_i} \geqslant 0$ for $1 \leqslant i \leqslant r$ and $\int \prod_{i=1}^{r} (1 - \varepsilon_i d_{\xi_i}) = 1$, using Claim 3 (i).

Hence

$$\begin{aligned} \|c_1 f_{\xi_1} + \ldots + c_\xi f_{\xi_r}\|_\infty &\geqslant \tfrac{1}{2} |c_1 f_{\xi_1}(g) + \ldots + c_r f_{\xi_r}(g)| \quad \text{(using Claim 3)} \\ &= \tfrac{1}{2} (\varepsilon_1 c_1 f_{\xi_1}(d_{\xi_1}) + \ldots + \varepsilon_r c_r f_{\xi_r}(d_{\xi_r}) \quad \text{(by Claim 2)} \\ &\geqslant \theta/20 (\varepsilon_1 c_1 + \ldots + \varepsilon_r c_r) = (\theta/20)(|c_1| + \ldots + |c_r|). \end{aligned}$$

Since in addition $\|f_\xi\|_\infty \leqslant M$ for all $\xi \in A$, the claim follows.

2. 2.1. THEOREM. *Assume* MA $+ \neg$CH. *If $\{f_\xi\colon \xi < \omega^+\}$ is a (uniformly bounded) family of elements of $L^\infty\{-1, 1\}^{\omega^+}$ satisfying the property: there is $\varepsilon > 0$ such that $\|f_\xi - f_\zeta\|_1 > \varepsilon$, for all $\xi < \zeta < \omega^+$, then it contains a subfamily with the same cardinality equivalent to the usual basis of $l^1_{\omega^+}$.*

Proof. The proof goes along similar steps as the proof of Theorem 6.11 in [2]. Namely, a partially ordered set $(P, <)$ will be constructed, with uncountable elements satisfying c.c.c. Then using MA we will get a filter of elements of $(P, <)$, with uncountable elements and this will give us immediately the desired family.

We start with some refinements of the original family $\{f_\xi\colon \xi < \omega^+\}$.

Step 1. There is B_1 – a subset of ω^+, with $|B_1| = \omega^+$, and a family $\{A_\xi\colon \xi \in B_1\}$ of finite subsets of ω^+ such that:

(i) $\int f_\xi \Pi_{A_\xi} \neq 0$ for all $\xi \in B_1$,

(ii) if for $\xi \in B_1$, N_ξ is the countable subset of ω^+ on which the function f_ξ depends and $M_\xi = \bigcup_{\substack{\zeta < \xi \\ \zeta \in B_1}} N_\zeta$, then $A_\xi \setminus M_\xi$ is non-empty.

The existence of the set B_1 follows from the fact that the family $\{f_\xi\colon \xi < \omega^+\}$ forms a non-separable set in the L^1-norm.

Without the loss of generality, passing if necessary to a subfamily with the same cardinality, we assume that there is a finite set A with $A_{\xi_1} \cap A_{\xi_2} = A$ for all $\xi_1, \xi_2 \in B_1$, $\xi_1 \neq \xi_2$.

We assume that $A \neq \emptyset$. Also, because of property (ii), $A \neq A_\xi$ for all $\xi \in B_1$. (It may be necessary to except one element of B_1.) Now $\int f_\xi \Pi_{A_\xi} d\mu \neq 0$ for all $\xi \in B_1$. Therefore for each $\xi \in B_1$ the function f_ξ depends essentially on the whole set A_ξ; namely, if we consider the function $\mathcal{E}^\infty_{A_\xi}(f_\xi)$, we can find $e_\xi \in \{-1, 1\}^A$, r_ξ, δ_ξ rationals with $\delta_\xi > 0$ such that

$$[\mathcal{E}^\infty_{A_\xi}(f_\xi)^{-1}(-\infty, r_\xi)] \cap (\{e_\xi\} \times \{-1, 1\}^{A_\xi \setminus A}) \neq \emptyset,$$

$$[\mathcal{E}^\infty_{A_\xi}(f_\xi)^{-1}(r_\xi + \delta_\xi, \infty)] \cap (\{e_\xi\} \times \{-1, 1\}^{A_\xi \setminus A}) \neq \emptyset.$$

Since cardinal ω^+ is regular uncountable, we can find a subset B_2 of B_1 with $|B_2| = \omega^+$ and r, δ, e such that

$$e = e_\xi, \qquad r = r_\xi, \qquad \delta = \delta_\xi$$

for all $\xi \in B_2$.

Step 2. Definition of partially ordered $(P, <)$. Our partially ordered set will consist of all finite sets F satisfying the following properties:

(i) F is a subset of B_2,

(ii) if $A_F = \bigcup_{\xi \in F} A_\xi$, then for all $\xi \in F$ there are Δ^F_ξ, E^F_ξ such that

$$\text{(a)} \qquad \begin{aligned} \Delta^F_\xi &\subset [\mathcal{E}^\infty_{A_F}(f_\xi)]^{-1}(-\infty, r), \\ E^F_\xi &\subset [\mathcal{E}^\infty_{A_F}(f_\xi)]^{-1}(r + \delta, \infty), \end{aligned}$$

(b) sets Δ_ξ^F, E_ξ^F depend on the set A_ξ and

$$p_A(E_\xi^F) = p_A(\Delta_\xi^F) = e.$$

We notice that from properties of the family B_2 it follows that for all $\xi \in B_2$, $\{\xi\}$ belongs to the set P.

Step 3. Partially ordered set $(P, <)$ satisfies c.c.c. We consider P ordered by the usual set-theoretical inclusion. Let $\{F_\sigma: \sigma < \omega^+\}$. be a given family of elements of $(P, <)$. Using Erdös–Rado theorem we find Δ – a subset of ω^+ and F – a finite set such that $|\Delta| = \omega^+$ and for σ_1, σ_2 – distinct elements of Δ, $F_{\sigma_1} \cap F_{\sigma_2} = F$. We consider the case $F \neq \emptyset$. (In case $F = \emptyset$ the proof is similar.)

We set

$$A_F = \bigcup_{\xi \in F} A_\xi, \quad A_{F_\sigma} = \bigcup_{\xi \in F_\sigma} A_\xi \quad \text{for all } \sigma \in \Delta,$$

$$N_F = \bigcup_{\xi \in F} N_\xi, \quad N_{F_\sigma} = \bigcup_{\xi \in F_\sigma} N_\xi.$$

We choose, inductively, a set Δ_1 – a subset of Δ such that

(i) there is natural number $v \in N$ such that

$$|A_{F_\sigma}| = v \quad \text{for all } \sigma \in \Delta,$$

(ii) $N_F \cap A_{F_\sigma} = A_F$ for all $\sigma \in \Delta$,

(iii) for every $\sigma_1 < \sigma_2$, $\sigma_1, \sigma_2 \in \Delta_1$,

$$N_{F_{\sigma_1}} \cap A_{F_{\sigma_2}} \subset A_F.$$

We choose $\sigma_0 \in \Delta_1$ with the property that the set $\Lambda = \{\sigma \in \Delta_1: \sigma < \sigma_0\}$ is infinite.

Properties (ii) and (iii) and Lemma 0.6 imply that

$$(*) \qquad \mathscr{E}^\infty_{F_\sigma \cup F_{\sigma_0}}(f_\xi) = \mathscr{E}^\infty_{F_\sigma}(f_\xi)$$

holds for all $\sigma \in \Delta$ and $\xi \in F_\sigma$. Here $\mathscr{E}^\infty_{F_\sigma}(f_\xi)$ is regarded as an element of $L^\infty\{-1, 1\}^{F_\sigma \cup F_{\sigma_0}}$ by the usual way.

We will prove now that for every $\varepsilon > 0$ there is $I(\varepsilon)$ – a finite subset of Λ such that for every $\sigma \in \Lambda \setminus I(\varepsilon)$ we have

$$(**) \qquad \|\mathscr{E}^\infty_{F_{\sigma_0}}(f_\xi) - \mathscr{E}^\infty_{F_{\sigma_0} \cup F_\sigma}(f_\xi)\|_\infty < \varepsilon$$

for all $\xi \in F_{\sigma_0}$.

If this has been proved setting

$$\varepsilon_{\sigma_0} = \min\{\varepsilon_\xi: \xi \in F_{\sigma_0}\}$$

where

$$\varepsilon_\xi = \min\{\inf(\mathscr{E}^\infty_{F_{\sigma_0}}(f_\xi)[E_\xi^{F_{\sigma_0}}] - (r+\delta), r - \sup \mathscr{E}^\infty_{F_{\sigma_0}}(f_\xi)[\Delta_\xi^{F_{\sigma_0}}]\}$$

and choosing $\sigma \in \Lambda \setminus I(\varepsilon_{\sigma_0})$, then from $(*)$ and $(**)$ immediately follows that $F_\sigma \cup F_{\sigma_0}$ belongs to the partially ordered set and so $(P, <)$ satisfies c.c.c. Therefore we have to prove $(**)$.

Let $\varepsilon > 0$ be given. For each $\xi \in F_{\sigma_0}$ the function f_ξ is bounded, consequently L^2-integrable, and so we can express it in L^2-norm as

$$f_\xi = \sum\{\alpha_M^\xi \Pi_M: M \in \mathscr{P}_\omega(\omega^+)\}.$$

Also, for every Λ – a subset of ω^+, the function $\mathscr{E}^\infty_\Lambda(f_\xi)$ has in L^2-norm the expression $\mathscr{E}^\infty_\Lambda(f_\xi) = \sum\{\alpha_M^\xi \Pi_M: M \in \mathscr{P}_\omega(\Lambda)\}$. From this and the fact that $F_{\sigma_1} \cap F_{\sigma_2} \subset F$, for $\sigma_1, \sigma_2 \in \Lambda$, it follows that vectors

$$\{\mathscr{E}^\infty_{F_\sigma \cup F_{\sigma_0}}(f_\xi) - \mathscr{E}^\infty_{F_{\sigma_0}}(f_\xi): \sigma \in \Lambda\}$$

are orthogonal. This implies that there is finite $I(\varepsilon) \subset \Lambda$ such that we have

$$\|\mathscr{E}^\infty_{F_\sigma \cup F_{\sigma_0}}(f_\xi) - \mathscr{E}^\infty_{F_{\sigma_0}}(f_\xi)\|_2 < \varepsilon/2^v$$

for every $\xi \in F_{\sigma_0}$ and for all $\sigma \in \Lambda \setminus I(\varepsilon)$. Since $|F_\sigma \cup F_{\sigma_0}| \leqslant 2v$, it follows that

$$\|\mathscr{E}^\infty_{F_\sigma \cup F_{\sigma_0}}(f_\xi) - \mathscr{E}^\infty_{F_{\sigma_0}}(f_\xi)\| \leqslant 2^{1/2 \cdot |F_\sigma \cup F_{\sigma_0}|} \cdot \|\mathscr{E}^\infty_{F_\sigma \cup F_{\sigma_0}}(f_\xi) - \mathscr{E}^\infty_{F_{\sigma_0}}(f_\xi)\|_2 < \varepsilon$$

for all $\xi \in F_{\sigma_0}$ and $\sigma \in \Lambda \setminus I(\varepsilon)$.

Proof of theorem completed. Since the partially ordered set $(P, <)$ satisfies c.c.c., it follows from MA that there is a filter $\mathscr{F}$ containing uncountable elements of the family $\{\{\xi\}: \xi \in B_2\}$. It is easy to check that the family $\{f_\xi: \{\xi\} \in \mathscr{F}\}$ is equivalent to the usual basis of $l^1_{\omega^+}$. The proof is now complete.

3. Proof of theorem A. (c) ⇒ (a): Theorems 1.2, 2.1 and 3.1 that have been proved above give essentially the proof of the implication (c) ⇒ (a) for all uncountable cardinals α. Case $\alpha = \omega$ is well known and follows from the fact that a uniformly bounded sequence $\{f_n: n < \omega\}$ of elements of $L^\infty\{-1, 1\}^\omega$ satisfying property $\|f_n - f_m\|_1 > \delta > 0$ does not contain weak Cauchy subsequence. This holds because of Dunford–Pettis property of $L^\infty\{-1, 1\}^\omega$. Therefore, by Rosenthal's criterion [12] it contains a subsequence equivalent to the usual basis of l^1.

(a) ⇒ (c): If $\{x_\xi: \xi < \alpha\}$ is a family of a Banach space X equivalent to the usual basis of l^1_α, then since $L^\infty\{-1, 1\}^\alpha$ is an injective B-space, it follows that there is bounded linear operator

$$T: X \to L^\infty\{-1, 1\}^\alpha$$

such that $T(x_\xi) = \Pi_\xi$ for all $\xi < \alpha$. This proves the implication since $\|\Pi_\xi - \Pi_\zeta\|_1 = \sqrt{2}$ for all $\xi < \zeta < \alpha$.

(c) ⇒ (b): It is obvious.

(b) ⇒ (c): This is an immediate consequence of the next proposition.

3.1. PROPOSITION. *Let $(X, \mathscr{S}, \mu)$ be a probability measure space and α an infinite cardinal. Then for every uniformly bounded family $\{g_\xi\colon \xi < \alpha\}$ of elements of $L^\infty(\mu)$ with $\|g_\xi - g_\zeta\|_1 > \delta > 0$ for all $\xi < \zeta < \alpha$ there is a bounded linear operator from $L^\infty(\mu)$ to $L^\infty\{-1, 1\}^\alpha$ and $\delta' > 0$ such that $\|T_{g_\xi} - T_{g_\zeta}\|_1 > \delta' > 0$ holds for all distinct ξ, ζ elements of a set Δ_1 — a subset of α with $|\Delta| = \alpha$.*

Proof. By Maharan's decomposition theorem [9] there is sequence $\{V_1, \ldots, V_n, \ldots\}$ (finite or infinite) of elements of the sigma algebra $\mathscr{S}$ pairwise disjoint, with $\sum_n \mu(V_n) = 1$, and the restriction $\mu|V_n$ is α_n-homogeneous for some cardinal α_n.

We choose n such that $\sum_{k>n} \mu(V_k) < \delta/4$. We may assume that $\|g_\xi\|_\infty \leqslant 1$ for all $\xi < \alpha$; therefore if $V = \bigcup_{i=1}^{n} V_i$, then

$$\|g_\xi|V - g_\zeta|V\|_1 > \delta/2.$$

Let $D = \{i < n\colon \alpha_i \geqslant \alpha\}$. By passing (if necessary) to a subfamily we can assume that

$$\|g_\xi|W - g_\zeta|W\|_1 > \delta/2$$

for all distinct ξ, ζ. Here W denotes the set $\bigcup_{i\in D} V_i$. For each $i \in D$ we choose $\mathscr{S}_i$, sigma subalgebra of $\mathscr{S}|V_i$, such that $g_\xi|V_i$ be $\mathscr{S}_i$ measurable function and $\mathscr{S}_i$ be α-homogeneous. Then it is immediate that the sigma algebra $\mathscr{S}_1 = \bigcup_{i\in D} \mathscr{S}_i$ is a sigma algebra α-homogeneous on the set W. Therefore the probability measure space

$$(W, \mathscr{S}_1, \mu|W/\mu(W) = \lambda)$$

is homeomorphic to $(\{-1, 1\}^\alpha, \mathscr{B}, \mu_\alpha)$ [10]. Now if we consider the induced linear bounded operators T, L

$$L^\infty(\mu) \xrightarrow{T} L^\infty(\lambda) \xrightarrow{L} L^\infty\{-1, 1\}^\alpha,$$

it is easy to check that the composition of them defines the desired linear operator.

3.2. PROPOSITION. *Assume MA + $\neg$CH. Let α be a cardinal with uncountable cofinality and Z a closed subspace of $L^\infty\{-1, 1\}^I$ such that the set*

$$\Delta = \{\xi\colon \exists z \in Z \text{ depending on } \xi\}$$

has cardinality greater than or equal to α. Then Z contains isomorphically a copy of l^1_α.

Proof. We will prove that there is $\delta > 0$ and family $\{z_\xi\colon \xi \in \Delta\}$ of norm-one elements of Z with $|\Lambda| = \alpha$ such that $\|z_\xi - z_\zeta\|_1 > \delta$ for all ξ, ζ — distinct elements of α. If this has been proved, then we get the result from Theorems 1.2 and 2.1.

Using transfinite induction we choose family $\{z_\xi\colon \xi < \alpha\}$ of norm-one elements of Z satisfying the property: for every $\xi < \alpha$ there is M_ξ — a finite subset of I with $\int z_\xi \cdot \Pi_{M_\xi} d\mu \neq 0$ and M_ξ is not contained in the set $W_\xi = \bigcup_{\zeta<\xi} N_\zeta$ where N_ζ denotes the countable set on which the function z_ζ depends.

Since $\mathrm{cf}(\alpha) > \omega$, there is $\delta > 0$ and Λ — a subset of α with $|\Lambda| = \alpha$ such that $|\int z_\xi \Pi_{M_\xi} d\mu| > \delta$ for all $\xi \in \Lambda$. By our construction it follows that if $\zeta < \xi$, then $\int z_\zeta \Pi_{M_\xi} d\mu = 0$, therefore for ζ, ξ — elements of Λ, $\xi \neq \zeta$

$$\|z_\xi - z_\zeta\|_1 > \delta$$

as we required.

3.3. THEOREM. *Assume MA + $\neg$CH. If α is a cardinal with uncountable cofinality and X is a Banach space generated by a family $\{x_i\colon i \in I\}$ and l^1_α is isomorphic to a subspace of X, then there is a subfamily $\{x_{i_\sigma}\colon \sigma \in J\}$ equivalent to the usual l^1_α basis.*

Proof. Since l^1_α is isomorphic to a subspace of X, there is $T\colon X \to L^\infty\{-1, 1\}^\alpha$ — a linear bounded operator such that $T(X)$, regarded as a subspace of $L^1\{-1, 1\}^\alpha$, has dimension α. Since the family $\{x_i\colon i \in I\}$ generates the space X, it follows that the set

$$\Delta = \{\xi < \alpha\colon \exists i \in I \text{ with } Tx_i \text{ depending on } \xi\}$$

has cardinality equal to α.

Therefore $\{x_i\colon i \in I\}$ has a subfamily equivalent to the usual basis of l^1_α (Proposition 4.2).

4. 4.1. THEOREM. *Assume MA + $\neg$CH. Let X be a Banach space and $W = (b\alpha[X^*], w^*)$. Then for cardinals α with $\mathrm{cf}(\alpha) > \omega$ the following are equivalent:*

(a) *l^1_α is isomorphic to a subspace of X,*

(b) *there is a map $f\colon W \to [0, 1]^\alpha$ continuous and onto,*

(c) *l^1_α is isomorphic to a subspace of $C(W)$.*

Proof. (a)$\Rightarrow$(b): If $T\colon l^1_\alpha \to X$ is an isomorphism, then $T^*\colon X^* \to l^\infty_\alpha$ is w^*-continuous and onto. The result follows from the fact that $(b\alpha[l^\infty_\alpha], w^*] = [-1, 1]^\alpha$.

(b)$\Rightarrow$(c): Obvious.

(c)$\Rightarrow$(a): Since l^1_α is isomorphic to a subspace of $C(W)$, it follows that $L^1\{-1, 1\}^\alpha$ is isomorphic to a subspace of $M(W)$ [11]. Therefore there is $\mu \in M(W)$ α-homogeneous. That means that the algebra of Borel subsets of W modulo μ-null sets is homeomorphic to the algebra of Borel sets of $\{-1, 1\}^\alpha$

modulo μ_α-null sets. Therefore there is $T\colon C(W)\to L^\infty\{-1,1\}^\alpha$ such that $T(C(W))$ is a w^*-dense subalgebra of $L^\infty\{-1,1)^\alpha$. Consequently, the set

$$D=\{\xi\colon \exists x\in X \text{ such that } x \text{ depends on } \xi\}$$

has cardinality α since $X\cup\{1\}$ generates a dense subalgebra of $C(W)$. Now the result follows from Proposition 4.2.

4.2. Remark. Implication (a)$\Leftrightarrow$(b) has been proved by Talagrand in [16] for all uncountable regular cardinals without any additional set-theoretical assumption. Our proof makes use of Martin's axiom only for the cardinal ω^+. Therefore our result extends Talagrand's result into the class of cardinals α greater than ω^+ and with uncountable cofinality.

4.3. Definition. A B-space X *semi-embeds* into a B-space Y if there is a linear bounded one-to-one operator T from X to Y with $T[b\alpha[X]]$ being a closed subset of Y.

We will prove that

4.4. Theorem. *Assume* MA$+\neg$CH. *If α is an uncountable cardinal and μ an α-homogeneous probability measure, then semi-embedding of $L^1(\mu)$ into a conjugate B-space X^* implies actually isomorphic embedding of $L^1(\mu)$ into X^*.*

4.5. Remark. This result extends an analoguous one for $\alpha=\omega$ that have been proved by Bourgain–Rosenthal [13]. Some techniques which will be used in the proof of the above theorem are closely related to methods developed by Bourgain and Rosenthal in the study of semi-embeddings of $L^1[0,1]$ into B-spaces.

4.6. Remark. For uncountable cardinals α with uncountable cofinality the assumption "μ is an α-homogeneous measure" can be replaced by "μ is a probability measure and $\dim L^1(\mu)=\alpha$". However, in the case of uncountable cardinals α with countable cofinality, the weaker condition does not imply the corresponding result. Indeed, if $\{\alpha_n\colon n<\omega\}$ is a strictly increasing sequence of cardinals with $\sup\alpha_n=\alpha$, then there is semi-embedding of $(\sum_{n=1}^{\omega}\oplus L^1\{-1,1\}^{\alpha_n})_1$ into the conjugate of the space $(\sum_{n=1}^{\omega}\oplus(C\{-1,1\}^{\alpha_n})_2$ but no isomorphic embedding can be found.

First we give some auxiliary results.

4.7. Proposition [8]. *Let X be a Polish space and Y a metric space. If $f\colon X\to Y$ is an F_σ-function (i.e. $f^{-1}(V)$ is F_σ-set for all V — open subsets of Y), then, for each K — a closed subset of X, $T|K\colon K\to Y$ has a point of continuity.*

4.8. Proposition. *Let X be a subspace of a weakly compactly generated B-space and $T\colon X\to Y$ a linear bounded one-to-one operator. Then*

$$\dim T(X)=\dim X.$$

Proof. Assume the contrary. Then letting Z be the closure of $T(X)$, we have that $\dim Z$ is smaller than $\dim X$. Therefore $(b\alpha[Z^*], w^*)$ has a dense subset with cardinality less than $\dim X$, and since $T^*(Z^*)$ is w^*-dense into X^*, it follows that $(b\alpha[X^*], w^*)$ has a dense subset with cardinality less than $\dim X$. But this contradicts to the fact that $(b\alpha[X^*], w^*)$ is Eberlein-compact set and so every dense subset must have cardinality equal to $\dim X$.

4.9. Corollary. *Let X be a subspace of a weakly compactly generated B-space and $T\colon X\to Y$ be a semi-embedding of X. Then for each K — a closed separable subset of the set $b\alpha[X]$,*

$$T^{-1}\colon T(K)\to K$$

has a point of continuity.

Proof. Let K be a separable closed subset of the $b\alpha[X]$. Then letting Z be the B — a space generated by the set K, it follows that Z is separable. Let $W=T^{-1}(T(b\alpha[Z]))$. This is a subset of the unit ball of X. Also W is closed and by the previous proposition W is separable.

We prove now that $T^{-1}\colon T(W)\to W$ is an F_σ-function. Indeed, if V is open subset of W, then there is a sequence $\{B_n\colon n<\omega\}$ of closed balls such that $V=\bigcup(B_n\cap W)$ and since T is one-to-one, $T(V)=\bigcup_{n=1}^{\omega}T(B_n)\cap T(W)$. Therefore for each Λ — a closed subset of $T(W)$,

$$T^{-1}|_\Lambda\colon \Lambda\to X$$

has a point of continuity. Now setting $\Lambda=\overline{T(K)}$ and choosing a point of continuity Tx_0 in Λ, we have that $x_0\in K$ since K is a closed set and $T(K)$ is dense into the set Λ. Therefore

$$T^{-1}|_{T(K)}\colon T(K)\to K$$

has a point of continuity.

4.10. Proposition. [3]. *If $(X,d),(Y,\varrho)$ are metric spaces with (X,d) complete and $f\colon X\to Y$ a function satisfying the property: for every compact subset K of X, $f|_K\colon K\to Y$ has a point of continuity, then f has a point of continuity.*

Next corollary follows from the above proposition and Corollary 5.8.

4.11. Corollary. *Let X be a subspace of a W.C.G. B-space Z and $T\colon X\to Y$ be a semi-embedding of X into a B-space Y. Then*

$$T^{-1}\colon T(b\alpha[X])\to b\alpha[X]$$

has a point of continuity

Proof of Theorem 4.4. By Maharam's theorem follows that if μ is a probability α-homogeneous measure, then $L^1(\mu)$ is isometric to $L^1\{-1,1\}^\alpha$. So we will deal with $L^1\{-1,1\}^\alpha$.

Case 1. $\mathrm{cf}(\alpha) > \omega$. Since $L^1\{-1, 1\}^\alpha$ is mapped one-to-one into X^*, it follows that X is mapped by the conjugate map w^*-dense into $L^\infty\{-1, 1\}^\alpha$. Consequently the set

$$D = \{\xi\colon \exists x \in X \text{ with } T^* x \text{ depending on } \xi\}$$

has cardinality equal to α. So Proposition 3.2 implies that l^1_α is isomorphic to a subspace of X and by a known result of Pełczyński [11], $L^1\{-1, 1\}^\alpha$ is isomorphic to a subspace of X^*.

Case 2. $\mathrm{cf}(\alpha) = \omega$.

CLAIM. *There is function φ in $L^\infty\{-1, 1\}$ such that $\|\varphi\|_1 \leqslant 1$, and $\|\varphi\|_\infty \leqslant \lambda$ and $\|T(\varphi \cdot \Pi_\xi)\| > \varepsilon > 0$ for some $\varepsilon > 0$ and for all $\xi \in \alpha - N$ for some finite set N.*

Proof. By Corollary 4.11 there is f — an element of the unit ball of $L^1\{-1, 1\}^\alpha$, such that $T(f)$ be a point of continuity for the map T^{-1}: $T(b\alpha[L^1\{-1, 1\}^\alpha]) \to b\alpha[L^1\{-1, 1\}^\alpha]$. So there is $1 \geqslant \delta > 0$ such that $\|f-g\| > 1/2$ implies $\|T(f) - T(g)\| > \delta$.

Let $\varphi \in L^1\{-1, 1\}^\alpha$ depending on a finite set N be such that $\|\varphi\| \leqslant 1$, $\|f - \varphi\| < \varepsilon_1 = \min(\delta/4(1+\|T\|))$. Since φ depends on a finite set N, φ is bounded and let $\|\varphi\|_\infty = \lambda$.

Let $\xi \in \alpha - N$. We will prove now that

$$\|T(\varphi \cdot \Pi_\xi)\| > \varepsilon = \delta/2.$$

Indeed, since Π_ξ does not depend on the set N, it follows that

$$\|\varphi - \varphi \cdot \Pi_\xi\| = \|\varphi\| \cdot \|1 - \Pi_\xi\| = 1,$$

and so

$$\|T(f) - T(\varphi) + T(\varphi \cdot \Pi_\xi)\| \geqslant \delta.$$

So

$$\|T(\varphi \cdot \Pi_\xi)\| \geqslant \delta - \|T(f) - T(\varphi)\| \geqslant \delta - \delta/4 \geqslant \delta/2,$$

as we required, and that is the proof of the claim.

Inductively we choose a family $\{x_\xi\colon \xi \in \Lambda\}$ of elements of the unit ball of X such that

(i) $|\Lambda| = \alpha$,

(ii) if N_ξ is the countable set on which the function Tx^*_ξ depends, then $\xi \notin \bigcup\{N_\zeta\colon \zeta \in \Lambda,\ \zeta < \xi\}$,

(iii) $T(\varphi \cdot \Pi_\xi)(x_\xi) \geqslant \varepsilon/2$.

Now it is easy to establish that for ξ_1, ξ_2 — elements of

$$\Lambda\ \|T^* x_{\xi_1} - T^* x_{\xi_2}\|_1 > \delta_1 = \varepsilon/2\lambda,$$

and the result follows from Theorem A.

References

[1] S. Argyros, *A short topological proof of a combinatorial result*, (to appear).

[2] — *On non-separable Banach spaces*, Trans. Amer. Math. Soc. 270 (1982), 193–216.

[3] S. Argyros, S. Negrepontis, *Universal embeddings of l^1_α into $C(X)$ and $L^\infty(\mu)$*, Coloq. Math. Soc. Janos Bolyai Topology (23) Budapest 1978.

[4] J. Bourgain, D. A. Fremlin, M. Talagrand, *Pointwise compact sets of Baire measurable functions*, Amer. J. Math.

[5] W. W. Comfort, S. Negrepontis, *Chain-conditions in Topology*, to appear in Cambridge University Press.

[6] A. Hajnal, *Proof of a conjecture of S. Ruziewicz*, Fund. Math. 50 (1961), 123–128.

[7] R. Haydon, *On Banach spaces which contain $l^1(\tau)$ and types of measures on compact spaces*, Israel J. Math. 28 (1977), 313–324.

[8] — *On dual L^1 spaces and injective bidual Banach spaces*, Israel J. Math. 31 (1978), 142–152.

[9] F. Hausdorff, *Set Theory*, Chelsea, New York 1962.

[10] D. Maharam, *On homogeneous measure algebras*, Proc. Nat. Acad. Sci. USA 28 (1942), 108–111.

[11] A. Pełczyński, *On Banach spaces containing $L^1(\mu)$*, Studia Math. 30 (1968), 238–246.

[12] H. P. Rosenthal, *A characterization of Banach spaces containing l^1*, Proc. Nat. Acad. Sci. USA 71 (1974), 2411–2413.

[13] — *Lecture in Functional Analysis Seminar, Spring* 1981, Austin, TX.

[14] M. E. Rudin, *Martin's axiom*, in: *Handbook of Mathematical Logic*, North-Holland, Amsterdam 1977, 481–502.

[15] N. A. Shanin, *A theorem from the general theory of sets*, Comptes Rendues (Doklady) de l'Academie des Sciences de l'URSS 53 (1946), 399–400.

[16] M. Talagrand, *Sur les espaces de Banach contenant $l^1(\tau)$*, Israel J. Math. 40 (1981), 324–330.

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