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Added in proof (June 1986). Theorem 2 and its version for more general domains in C^d have been proved by Prof. Lars Hörmander.

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The boundary of Taylor's joint spectrum for two commuting Banach space operators

by

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Abstract. In this note it is shown that the boundary $\partial\sigma$ of Taylor's joint spectrum for a pair of commuting operators on an arbitrary Banach space is contained in the union of the joint approximate point spectrum $\mathrm{AP}\sigma$ and the joint approximate compression spectrum $\mathrm{AC}\sigma$, but neither $\partial\sigma \subset \mathrm{AP}\sigma$ nor $\partial\sigma \subset \mathrm{AC}\sigma$ is true in general. This is in strict contrast to the case of a single operator where $\partial\sigma \subset \mathrm{AP}\sigma \cap \mathrm{AC}\sigma$.

1. Introduction. In [5] and [6] F.-H. Vasilescu characterized Taylor's joint spectrum [3] for commuting operators on Hilbert spaces by means of the noninvertability of a certain operator acting on a direct sum of copies of the initial space. In this way he succeeded in giving a characterization of Taylor's joint spectrum in terms of classical spectral theory.

Based on Vasilescu's characterization C. Muneo and M. Takaguchi [2] proved that the boundary of Taylor's joint spectrum for a pair of commuting Hilbert space operators is contained in the union of the joint approximate point spectrum and the joint approximate compression spectrum in the sense of A. T. Dash [1]. Since this union is of course contained in Taylor's joint spectrum, the result of Muneo and Takaguchi gives an easy characterization of at least an important part of the spectrum. The method of proof in [2] heavily relies on the Hilbert space setting. It is the purpose of this note to show that the above-mentioned result holds true in the Banach space setting, too. As it seems our proof is completely elementary.

Moreover, we shall show that in general neither $\partial\sigma \subset \mathrm{AP}\sigma$ nor $\partial\sigma \subset \mathrm{AC}\sigma$, but $\mathrm{AP}\sigma \cap \mathrm{AC}\sigma$ is nonempty for two commuting operators.

Let X, Y, Z denote complex Banach spaces and let $L(X, Y)$ denote the space of all continuous linear operators from X into Y, writing $L(X)$ for $L(X, X)$ and X' for the dual space $L(X, C)$ instead. Given $S \in L(X, Y)$ we let $S' \in L(Y', X')$ denote the dual operator.

Let $T = (T_1, T_2)$ $(T_i \in L(X),\ i = 1, 2)$ denote a pair of commuting operators. Consider the sequence

$$0 \to X \xrightarrow{\delta_T^0} X \oplus X \xrightarrow{\delta_T^1} X \to 0 \tag{1.1}$$

where $\delta_T^0(x) := T_1 x \oplus T_2 x$ $(x \in X)$ and $\delta_T^1(x_1 \oplus x_2) := T_1 x_2 - T_2 x_1$ $(x_1, x_2 \in X)$. By definition [3], T is said to be *nonsingular* if the sequence (1.1) is exact. Taylor's joint spectrum $\sigma(T; X)$ is the complement of the set of all $z = (z_1, z_2) \in C^2$ such that $z - T := (z_1 - T_1, z_2 - T_2)$ is nonsingular. An element $z = (z_1, z_2) \in C^2$ belongs to the *joint approximate point spectrum* $\mathrm{AP}\sigma(T; X)$ resp. *joint approximate compression spectrum* $\mathrm{AC}\sigma(T; X)$ (Dash [1]) if there exists a sequence $(x_n)_{n \in N} \subset X$ resp. $(x'_n)_{n \in N} \subset X'$ such that

$$\|x_n\| = 1 \quad \text{and} \quad \|(z_i - T_i) x_n\| \to 0 \quad \text{as } n \to \infty \quad (i = 1, 2)$$

resp.

$$\|x'_n\| = 1 \quad \text{and} \quad \|(z_i - T'_i) x'_n\| \to 0 \quad \text{as } n \to \infty \quad (i = 1, 2).$$

Finally, given $T \in L(X, Y)$ let $\ker(T)$ and $\mathrm{im}(T)$ denote the kernel and the range space of T, respectively.

2. Main result. The following is our main result:

2.1. THEOREM. *Let $T = (T_1, T_2) \in L(X)^2$ denote a pair of commuting operators on a complex Banach space X. Then*

$$\partial\sigma(T; X) \subset \mathrm{AP}\sigma(T; X) \cup \mathrm{AC}\sigma(T; X),$$

where $\partial\sigma(T; X)$ denotes the boundary of Taylor's joint spectrum.

We repeat again that in contrast to the case of a single operator the left-hand side is in general neither contained in $\mathrm{AP}\sigma(T; X)$ nor in $\mathrm{AC}\sigma(T; X)$.

The proof requires some preparations which we state in some elementary lemmas.

2.2. LEMMA. *Let $(S_n)_{n \in N}$ $(S_n \in L(X, Y))$ denote a sequence of operators between Banach spaces X and Y. Assume that $(S_n)_{n \in N}$ tends to a topological monomorphism $S \in L(X, Y)$ in the uniform operator topology. Then there exist $c > 0$ and $n_0 \in N$ such that*

$$\|S_n x\| \geqslant c\|x\| \quad (x \in X,\ n \geqslant n_0).$$

This follows easily from the triangle inequality and the fact that S is assumed to be a topological monomorphism.

By duality we obtain

2.3. COROLLARY. *Let $(T_n)_{n \in N}$ $(T_n \in L(Y, Z))$ denote a sequence of operators between Banach spaces Y and Z. Assume that $(T_n)_{n \in N}$ tends to a surjection $T \in L(Y, Z)$ in the uniform operator topology. Then there exist $c > 0$ and $n_0 \in N$ such that*

$$T_n(K_Y(0, 1)) \supset K_Z(0, c) \quad (n \geqslant n_0),$$

where $K_Y(0, 1)$ denotes the open 1-ball in Y centered at 0.

Putting together we obtain

2.4. LEMMA. *Let $(S_n)_{n \in N}$ $(S_n \in L(X, Y))$, $(T_n)_{n \in N}$ $(T_n \in L(Y, Z))$ denote sequences of operators between Banach spaces X, Y, Z such that $\ker(T_n) \subseteq \mathrm{im}(S_n)$ $(n \in N)$. Assume that $(S_n)_{n \in N}$ resp. $(T_n)_{n \in N}$ tends to a topological monomorphism $S \in L(X, Y)$ resp. a surjection $T \in L(Y, Z)$ in the uniform operator topology. Then*

$$\ker(T) \subseteq \mathrm{im}(S).$$

Proof. Let $y \in \ker(T)$. Then we have

$$\|T_n y\| = \|T_n y - Ty\| \leqslant \|T_n - T\| \cdot \|y\| \to 0 \quad \text{as } n \to \infty.$$

By 2.3 we know that T_n is a surjection for n sufficiently large. Consequently we find $y_n \in \ker(T_n)$ $(n \in N)$ such that

$$\|y - y_n\| \to 0 \quad \text{as } n \to \infty.$$

But as $\ker(T_n) \subseteq \mathrm{im}(S_n)$ by assumption, we find $x_n \in X$ such that $y_n = S_n x_n$ and $\|y_n\| = \|S_n x_n\| \geqslant c\|x_n\|$ by 2.2 at least for n sufficiently large. As $\{y_n \colon n \in N\}$ is bounded, so is $\{x_n \colon n \in N\}$. Moreover, $y = \lim_{n \to \infty} S_n x_n$ and

$$\|S x_n - S_n x_n\| \leqslant \|S - S_n\| \cdot \|x_n\| \to 0 \quad \text{as } n \to \infty.$$

Therefore $y \in \mathrm{im}(S)$, because S was assumed to be a topological monomorphism. Hence $\ker(T) \subseteq \mathrm{im}(S)$. ∎

Proof of 2.1. By doing a translation, it is enough to show that $(0, 0) \in \partial\sigma(T; X)$ implies $(0, 0) \in \mathrm{AP}\sigma(T; X) \cup \mathrm{AC}\sigma(T; X)$. Thus take a sequence $z_n \notin \sigma(T; X)$ $(n \in N)$ tending to $(0, 0)$, and let $T_n := z_n - T$. Then evidently $\delta^i_{T_n} \to \delta^i_T$ as $n \to \infty$ in the uniform operator topology for $i = 0, 1$.

So let us first assume $(0, 0) \in \partial\sigma(T; X) \setminus \mathrm{AP}\sigma(T; X)$. This in particular implies that $\ker(\delta_T^0) = \{0\}$ and that $\delta_T^0 \in L(X, X \oplus X)$ is a topological monomorphism, for otherwise we would find a sequence $(x_n)_{n \in N} \subset X$ such that $\|x_n\| = 1$ and $\|T_i x_n\| \to 0$ as $n \to \infty$ for $i = 1, 2$, contradicting the assumption $(0, 0) \notin \mathrm{AP}\sigma(T; X)$.

If $\mathrm{im}(\delta_T^1) \neq X$, we find $x'_n \in X'$ such that $\|x'_n\| = 1$ $(n \in N)$ and $x'_n \circ \delta_T^1 \to 0$ in $(X \oplus X)'$ as $n \to \infty$. But this in particular implies that $x'_n \circ T_i \to 0$ in X' as $n \to \infty$ for $i = 1, 2$, and hence $(0, 0) \in \mathrm{AC}\sigma(T; X)$. So we are done. Therefore assume that $\mathrm{im}(\delta_T^1) = X$. But then the sequences $(\delta^i_{T_n})_{n \in N}$ $(i = 1, 2)$ fulfill the assumptions of 2.4. Consequently, $\ker(\delta_T^1) \subset \mathrm{im}(\delta_T^0)$, and hence we have the exactness of (1.1) because $\mathrm{im}(\delta_T^0) \subset \ker(\delta_T^1)$ is always true. This contradicts our assumptions. Therefore $(0, 0) \in \mathrm{AC}\sigma(T; X)$.

On the other hand, let $(0, 0) \in \partial\sigma(T; X) \setminus \mathrm{AC}\sigma(T; X)$. Thus $\mathrm{im}(\delta_T^1) = X$. But by the argument of the first part of the proof, δ_T^0 cannot be a topological monomorphism, for this together with $\mathrm{im}(\delta_T^1) = X$ implies the exactness of

(1.1) contradicting our assumptions. But if δ_T^0 is not a topological monomorphism, then $(0,0)\in \mathrm{AP}\sigma(T;X)$ and we are done. ∎

We now give an example showing that in general neither $\partial\sigma(T;X)\subset \mathrm{AP}\sigma(T;X)$ nor $\partial\sigma(T;X)\subset \mathrm{AC}\sigma(T;X)$ is true for two commuting operators. This is in strict contrast to the case of a single operator where $\partial\sigma(T;X)\subset \mathrm{AP}\sigma(T;X)\cap \mathrm{AC}\sigma(T;X)$ holds true.

2.5. Example. Let X denote the complete Hilbert space tensor product of two copies of $l^2(N)$. Moreover, let $S_1, S_2\in L(l^2(N))$ denote the left shift $S_1(x_n)_{n\in N} := (x_{n+1})_{n\in N}$ and the right shift $S_2(x_n)_{n\in N} := (0, x_1, x_2, \ldots)$. Then it is well known that

$$\sigma(S_i;\, l^2(N)) = D = \mathrm{AP}\sigma(S_1;\, l^2(N)) = \mathrm{AC}\sigma(S_2;\, l^2(N)) \quad (i=1,2),$$
$$\mathrm{AC}\sigma(S_1;\, l^2(N)) = \mathrm{AP}\sigma(S_2;\, l^2(N)) = \partial D,$$

where D denotes the closed unit disc in C.

Letting $T_1 := S_1\hat{\otimes} I$, $T_2 := I\hat{\otimes} S_2$, we have a commuting pair $T=(T_1, T_2)$ on the Hilbert space tensor product X. By a result of Vasilescu [6], we have

$$\sigma(T;X) = \sigma(S_1;\, l^2(N))\times\sigma(S_2;\, l^2(N)).$$

But

$$\partial\sigma(S_1;\, l^2(N))\times\sigma(S_2;\, l^2(N)) = \partial D\times D \not\subset \mathrm{AP}\sigma(T;X)\subset D\times\partial D,$$
$$\sigma(S_1;\, l^2(N))\times\partial\sigma(S_2;\, l^2(N)) = D\times\partial D \not\subset \mathrm{AC}\sigma(T;X)\subset \partial D\times D,$$

and thus $\partial\sigma(T;X)$ is contained neither in $\mathrm{AP}\sigma(T;X)$ nor in $\mathrm{AC}\sigma(T;X)$.

Remark. As $\sigma(T;X)$ has the projection property (Taylor [3]), i.e. $\pi_i\sigma(T;X)=\sigma(T_i;X)$, where $\pi_i\colon C^2\to C$ denotes the canonical projection onto the ith complex coordinate $(i=1,2)$, we have

$$\emptyset \neq \big(\partial(\sigma(T_1;X)\times\sigma(T_2;X))\big)\cap\sigma(T;X)\subset\partial\sigma(T;X)$$
$$\subset \mathrm{AP}\sigma(T;X)\cup \mathrm{AC}\sigma(T;X)$$

and all inclusions are proper in general.

Next we are going to prove that $\mathrm{AP}\sigma(T;X)\cap \mathrm{AC}\sigma(T;X)\cap\partial\sigma(T;X)$ is nonempty for a pair of commuting operators. This constitutes a refined answer to Dash's [1] question whether $\mathrm{AP}\sigma(T;X)$ is always nonempty. Of course we should mention that the nonemptiness of the joint approximate point spectrum for an arbitrary n-tuple of commuting Banach space operators has been proved by W. Żelazko [7].

In order to make things work easier, we apply ultrapower techniques.

2.6. Proposition. *Let $T=(T_1,T_2)\in L(X)^2$ be as in 2.1. Let $\mathfrak{U}$ denote a nontrivial ultrafilter on N and $(X)_\mathfrak{U}$ the corresponding ultrapower of X, let $T_{i\mathfrak{U}}$*

denote the extension of T_i $(i=1,2)$ upon $(X)_\mathfrak{U}$ and $T_\mathfrak{U}=(T_{1\mathfrak{U}}, T_{2\mathfrak{U}})$. Then

$$\sigma(T_\mathfrak{U};(X)_\mathfrak{U}) = \sigma(T;X).$$

Proof. Let $(0,0)\notin\sigma(T;X)$. Since δ_T^0 is a topological monomorphism, so is $\delta^0_{T_\mathfrak{U}}$ and $\mathrm{im}(\delta^0_{T_\mathfrak{U}}) = (\mathrm{im}(\delta_T^0))_\mathfrak{U}$. On the other hand, $\delta^1_{T_\mathfrak{U}}$ is a surjection, because δ_T^1 is. Finally

$$\begin{aligned}\ker(\delta^1_{T_\mathfrak{U}}) &= (\ker(\delta^1_T))_\mathfrak{U}\\ &= (\mathrm{im}(\delta^0_T))_\mathfrak{U} \quad \text{by the exactness of (1.1)}\\ &= \mathrm{im}(\delta^0_{T_\mathfrak{U}})\end{aligned}$$

and thus $(0,0)\notin\sigma(T_\mathfrak{U};(X)_\mathfrak{U})$.

Conversely, let $(0,0)\notin\sigma(T_\mathfrak{U};(X)_\mathfrak{U})$. Then δ_T^0 is a topological monomorphism as a restriction of the topological monomorphism $\delta^0_{T_\mathfrak{U}}$. If $\delta_T^1(x\oplus y)=0$, then $\delta^1_{T_\mathfrak{U}}(jx\oplus jy)=0$, where j denotes the canonical embedding of X into $(X)_\mathfrak{U}$. Consequently we find $(x_n)_\mathfrak{U}\in(X)_\mathfrak{U}$ such that $jx\oplus jy = \delta^0_{T_\mathfrak{U}}((x_n)_\mathfrak{U})$. But this in particular implies that $x\oplus y$ is an accumulation point of $\mathrm{im}(\delta_T^0)$, hence $\ker(\delta_T^1)=\mathrm{im}(\delta_T^0)$, as δ_T^0 is a topological monomorphism.

As a final step we have to show $X=\mathrm{im}(\delta_T^1)$. By assumption we have

$$\delta^1_{T_\mathfrak{U}}(K_{(X)^2_\mathfrak{U}}(0,1)) \supset K_{(X)_\mathfrak{U}}(0,c)$$

for some $c>0$: For every $y\in K_X(0,c)$ there exists $(x_n)_\mathfrak{U}\in K_{(X)^2_\mathfrak{U}}(0,1)$ such that $\delta^1_{T_\mathfrak{U}}((x_n)_\mathfrak{U}) = jy$. Thus δ_T^1 is nearly open, and hence open by the open mapping theorem. Thus $(0,0)\notin\sigma(T;X)$. ∎

2.7. Corollary. *Let $T=(T_1,T_2)\in L(X)^2$ be as in 2.1. Then for every $z_1\in\partial\sigma(T_1;X)$ there exist $z_2, z_3\in\sigma(T_2;X)$ such that*

$$z := (z_1, z_2)\in \mathrm{AP}\sigma(T;X)\cap\partial\sigma(T;X),$$
$$w := (z_1, z_3)\in \mathrm{AC}\sigma(T;X)\cap\partial\sigma(T;X).$$

Proof. Let $z_1\in\partial\sigma(T_1;X)$. Then z_1 is an approximate eigenvalue and by passing to the ultraproduct we may assume that z_1 is an ordinary eigenvalue. Let $X_0 := \ker(z_1 - T_1)$. Then T_1, T_2 leave X_0 invariant, since T_1 and T_2 commute. Take $z_2\in\partial\sigma(T_2;X_0)\subset\sigma(T_2;X)$. Then $(z_1,z_2)=z\in \mathrm{AP}\sigma(T;X)\cap\partial\sigma(T;X)$. By duality we find $w\in \mathrm{AC}\sigma(T;X)\cap\partial\sigma(T;X)$. ∎

Finally we state the following

2.8. Theorem. *Let $T=(T_1,T_2)\in L(X)^2$ be as in 2.1. Then*

$$\mathrm{AP}\sigma(T;X)\cap \mathrm{AC}\sigma(T;X)\cap\partial\sigma(T;X)\neq\emptyset.$$

Proof. First we state that there exists a pair $(\alpha, \beta) \in \operatorname{ext}\operatorname{conv} \sigma(T; X)$ such that $\alpha \in \operatorname{ext}\operatorname{conv} \sigma(T_1; X)$. Here conv K denotes the convex hull of a set K and ext L denotes the extreme points of a set L. Suppose that for all pairs (α, β) such that $\alpha \in \operatorname{ext}\operatorname{conv} \sigma(T_1; X)$ and $\beta \in \sigma(T_2; X)$ we had $(\alpha, \beta) \notin \operatorname{ext}\operatorname{conv} \sigma(T; X)$. Then by Carathéodory's lemma (α, β) is the convex combination of at most 5 extreme points $(\alpha_1^{(i)}, \alpha_2^{(i)}) \in \operatorname{ext}\operatorname{conv} \sigma(T; X)$ $(1 \leqslant i \leqslant 5)$. But as α is an extreme point of conv $\sigma(T_1; X)$ by assumption, and as $\sigma(T; X)$ has the projection property, we have $\alpha = \alpha_1^{(i)}$ $(1 \leqslant i \leqslant 5)$. Consequently each $(\alpha, \alpha_2^{(i)})$ is an extreme point of the desired form. Now fix an α with this property.

We next show that

$$\{\alpha\} \times C \cap \operatorname{int}\operatorname{conv} \sigma(T; X) = \emptyset,$$

where int K denotes the interior points of a set K. For if there were an $(\alpha, \mu) \in \operatorname{int}\operatorname{conv} \sigma(T; X)$, then we would find an open neighborhood of α in conv $\sigma(T_1; X)$. A contradiction, since α was assumed to be an extreme point.

By Corollary 2.7 we find β such that $(\alpha, \beta) \in \mathrm{AP}\sigma(T; X) \cap \partial\sigma(T; X)$. Let $C(\alpha, \beta)$ denote the connected component of (α, β) in $\sigma(T; X)$. We distinguish two cases:

1° If $\sigma := C(\alpha, \beta)$ is open relative to $\sigma(T; X)$, then by a result of Taylor [4], 4.9 we find a spectral projection π with the following properties:

(i) $\pi \in \{T_1, T_2\}^{cc}$ (the bicommutant algebra of T_1, T_2 in $L(X)$).

(ii) X has a direct decomposition $X = X_1 \oplus X_2$ with

$$X_1 = \pi X, \quad X_2 = (I - \pi) X.$$

(iii) $$\sigma(T; X_1) = \sigma, \; \sigma(T; X_2) = \sigma(T; X) \setminus \sigma.$$

So without loss of generality we may reduce our considerations to the case where $\sigma = \sigma(T; X)$ is connected. By 2.7 we find another $(\alpha, \mu) \in \partial\sigma(T; X) \cap \mathrm{AC}\sigma(T; X)$. But since $\{\alpha\} \times C$ lies in a supporting hyperplane of conv $\sigma(T; X)$ by construction, (α, β) and (α, μ) lie in the same component of $\partial\sigma(T; X)$. But as $\partial\sigma(T; X) \subset \mathrm{AP}\sigma(T; X) \cup \mathrm{AC}\sigma(T; X)$ by 2.1, we get the desired result, because $\mathrm{AP}\sigma(T; X)$ and $\mathrm{AC}\sigma(T; X)$ are compact sets.

2° Next assume that $C(\alpha, \beta)$ is not open relative to $\sigma(T; X)$. Then

$$C(\alpha, \beta) = \bigcap \{K\colon (\alpha, \beta) \in K,\ K \text{ closed and open in } \sigma(T; X)\}.$$

With respect to each such K we again have a spectral decomposition as in 1°. By 2.7 we find for each such K an (α, μ_K) in $K \cap \mathrm{AC}\sigma(T; X) \cap \partial\sigma(T; X)$. By the compactness of $\sigma(T; X)$ we find a cluster point (α, μ) in $C(\alpha, \beta) \cap \mathrm{AC}\sigma(T; X) \cap \partial\sigma(T; X)$. By the same argument as above, (α, β) and (α, μ) lie in the same component of $\partial\sigma(T; X)$. Hence we are done. ■

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