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On uncountable unconditional bases in Banach spaces

by

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Abstract. If a Banach space with an uncountable unconditional basis (v_j) contains an isomorphic copy of $l_1(A)$ or $c_0(A)$ for some uncountable set A, then the basis (v_j) has "large" subbases of l_1-type, or c_0-type, respectively (Theorems 1 and 2). This generalizes the results obtained by S. L. Troyanski in 1975 for Banach spaces with symmetric bases.

In Theorems 1 and 2 below, we extend to Banach spaces with uncountable unconditional bases the following result of Troyanski [6, Corollaries 1 and 2]:

Let F be a Banach space with a symmetric basis $(v_j)_{j\in J}$. If F has a subspace isomorphic to the Banach space $l_1(A)$ [resp., $c_0(A)$] for some uncountable set A, then the basis $(v_j)_{j\in J}$ is equivalent to the natural basis of the space $l_1(J)$ [resp., $c_0(J)$].

Our results show that if the basis $(v_j)_{j\in J}$ is merely unconditional, then it must contain large l_1- [resp., c_0-] subbases. Unlike in [6], where the above result was obtained via some renorming considerations, our arguments will be purely combinatorial. The l_1 part of Troyanski's result plays a crucial role in the author's recent paper [1]; the present work is, in a sense, a continuation of [1].

In general, our Banach space terminology and notation is that of [4] and [5].

Throughout, F will be a (nonseparable) Banach space with an uncountable unconditional basis $(v_j)_{j\in J}$. Recall (cf. [5], [6]) that this means that for every y in F there is a unique family of scalars $(t_j)_{j\in J}$ such that $y = \sum_{j\in J} t_j v_j$ (unconditional convergence or summability). Let $(v_j^*)_{j\in J} \subset F^*$ be the dual family, biorthogonal to $(v_j)_{j\in J}$. Then, for y in F, we define the *support* of y as

$$s(y) \doteq \{j\in J\colon\ v_j^*(y) \neq 0\};$$

clearly, $|s(y)| \leqslant \aleph_0$. ($|A|$ denotes the cardinal number of the set A.) The natural unit vector bases in the spaces $l_1(A)$ and $c_0(A)$ will be denoted by $(e_\alpha^1)_{\alpha\in A}$ and

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$(e^0_\alpha)_{\alpha\in A}$, respectively. Instead of saying that an unconditional basic family $(u_\alpha)_{\alpha\in A}$ is equivalent to the basis $(e^1_\alpha)_{\alpha\in A}$, we will often say that it is of l_1-*type*; c_0-*type* is understood similarly.

Conventions: Let $\mathfrak{m}$, $\mathfrak{n}$ be cardinal numbers. Then $\mathfrak{n} \leqslant_r \mathfrak{m}$ means $\mathfrak{n} = \mathfrak{m}$ when $\mathfrak{m}$ is regular (i.e., $\mathrm{cf}(\mathfrak{m}) = \mathfrak{m}$), and $\mathfrak{n} < \mathfrak{m}$ otherwise. Similarly, $\mathfrak{n} \leqslant_c \mathfrak{m}$ means $\mathfrak{n} = \mathfrak{m}$ when $\mathrm{cf}(\mathfrak{m}) > \aleph_0$, and $\mathfrak{n} < \mathfrak{m}$ otherwise. ($\mathrm{cf}(\mathfrak{m})$, the *cofinality* of $\mathfrak{m}$, is the smallest cardinal $\mathfrak{a}$ such that a set of cardinality $\mathfrak{m}$ can be written as the union of $\mathfrak{a}$ subsets, each having cardinality strictly less than $\mathfrak{m}$.)

In the proof of Theorem 1 we will need the following combinatorial result (see [2, p. 87]):

LEMMA 1. *Let $(S_\alpha)_{\alpha\in A}$ be an uncountable family of finite sets. Then, if $\mathfrak{n} \leqslant_r |A|$, there exists $B \subset A$ with $|B| = \mathfrak{n}$ and a finite set S such that*

$$S_\beta \cap S_{\beta'} = S \quad \textit{for all distinct } \beta,\ \beta' \textit{ in } B.$$

THEOREM 1. *Suppose F has a subspace isomorphic to $l_1(A)$, where $\mathfrak{m} = |A| > \aleph_0$. Then, if $\mathfrak{n} \leqslant_r \mathfrak{m}$, there exists $J' \subset J$ with $|J'| = \mathfrak{n}$ such that the subbasis $(v_j)_{j\in J'}$ is of l_1-type.*

Proof. By assumption, F contains an unconditional basic family $(w_\alpha)_{\alpha\in A}$ of l_1-type; thus $a\sum|t_\alpha| \leqslant \|\sum t_\alpha w_\alpha\| \leqslant b\sum|t_\alpha|$ for all (t_α) in $l_1(A)$ and some constants a, b. It is well known (and easily verified) that if for each α in A we choose a vector w'_α in F so that $\|w_\alpha - w'_\alpha\| \leqslant r$, where $0 \leqslant r < a$, then $(w'_\alpha)_{\alpha\in A}$ is also an unconditional basic family of l_1-type. Using this fact, and cutting off sufficiently distant "tails" in the expansions of w_α's with respect to the basis (v_j), we may assume that $|s(w_\alpha)| < \aleph_0$ for all α in A. It is clear that there must exist a $k\in N$ such that if

$$A' = \{\alpha\in A\colon\ |s(w_\alpha)| = k\},$$

then $|A'| = \mathfrak{m}$ if $\mathfrak{m}$ is regular, and $|A'| > \mathfrak{n}$ otherwise.

Now, by Lemma 1, we can find a finite subset L of J and a subset A'' of A' such that $|A''| = \mathfrak{m}$ if $\mathfrak{m}$ is regular, $|A''| > \mathfrak{n}$ otherwise, and

$$s(w_{\alpha_1}) \cap s(w_{\alpha_2}) = L \quad \text{for all distinct } \alpha_1,\ \alpha_2 \text{ in } A''.$$

Moreover, it is easy to see that we may also assume that the subbasis $(v_j)_{j\in K}$, where $K = \bigcup\{s(w_\alpha)\colon\ \alpha\in A''\}$, is seminormalized, i.e.,

$$0 < c = \inf\{\|v_j\|\colon\ j\in K\} \leqslant \sup\{\|v_j\|\colon\ j\in K\} < \infty. \tag{1}$$

Consider the operator $T\colon F\to F$ defined by $T(y) = \sum_{j\in L} v^*_j(y)v_j$. It is finite-dimensional, hence compact; therefore, if $S = \mathrm{id}_F - T$, then $\ker S$ is finite-dimensional, and S maps closed subspaces onto closed subspaces (cf. e.g. [3, Lemma 2]). It follows that there exists $A''' \subset A''$ with $A'' - A'''$ finite such that the restriction of S to the closed linear span of $(w_\alpha)_{\alpha\in A'''}$ is an isomorphic embedding. In particular, if $w'_\alpha = S(w_\alpha)$ $(= w_\alpha|(J-L))$, then $(w'_\alpha)_{\alpha\in A'''}$ is an unconditional basic family of l_1-type, and $|A'''| = |A''|$. Moreover, the vectors w'_α, $\alpha\in A'''$, have pairwise disjoint supports and, denoting $m = k - |L|$, we have

$$|s(w'_\alpha)| = m \quad \text{for all } \alpha \text{ in } A'''. \tag{2}$$

Since $(w'_\alpha)_{\alpha\in A'''}$ is of l_1-type, there exists a y^* in F^* such that $y^*(w'_\alpha) = 1$ for all α in A'''. For each α in A''' choose a $\varrho(\alpha)$ in $s(w'_\alpha)$ so that

$$|y^*(v_{\varrho(\alpha)})| = \max\{y^*(v_j)|\colon\ j\in s(w'_\alpha)\}.$$

Then, using (1) and (2), we have

$$\begin{aligned}1 = |y^*(w'_\alpha)| &\leqslant \sum_j |v^*_j(w'_\alpha)|\,|y^*(v_j)| \leqslant |y^*(v_{\varrho(\alpha)})| \sum_j |v^*_j(w'_\alpha)|\\ &= |y^*(v_{\varrho(\alpha)})| \sum_j \frac{1}{\|v_j\|}\,\|v^*_j(w'_\alpha)v_j\| \leqslant (mB/c)|y^*(v_{\varrho(\alpha)})|\,\|w'_\alpha\|\\ &\leqslant (mBM/c)|y^*(v_{\varrho(\alpha)})|,\end{aligned}$$

where B is the unconditional basis constant of $(v_j)_{j\in J}$, and $M = \sup\{\|w'_\alpha\|\colon \alpha\in A'''\} < \infty$. Thus $\inf\{|y^*(v_{\varrho(\alpha)})|\colon\ \alpha\in A'''\} > 0$; therefore, since by (1) $(v_{\varrho(\alpha)})_{\alpha\in A'''}$ is a bounded unconditional basic family in F, it must be of l_1-type. This concludes the proof: $J' = \{\varrho(\alpha)\colon\ \alpha\in A'''\}$ is as required. ∎

Remark. An inspection of the proof shows that, replacing $l_1(A)$ with $l_p(A)$, we have an analogue of the above result for unconditional bases in p-Banach spaces, $0 < p < 1$.

LEMMA 2. *Let $(y_\beta)_{\beta\in B}$ be a family in F consisting of nonzero vectors with pairwise disjoint supports. Let $\varrho\colon B\to J$ be any choice function such that $\varrho(\beta)\in s(y_\beta)$ for every β in B. Then there is an increasing sequence (B_n) of subsets of B with union B such that, for every $n\in N$, the families $(y_\beta)_{\beta\in B_n}$ and $(v_{\varrho(\beta)})_{\beta\in B_n}$ are seminormalized, and*

$$(y_\beta)_{\beta\in B_n} \succ (v_{\varrho(\beta)})_{\beta\in B_n},$$

i.e., $\sum_{\beta\in B_n} t_\beta v_{\varrho(\beta)}$ converges whenever $\sum_{\beta\in B_n} t_\beta y_\beta$ converges.

Proof. It is enough to set

$$B_n = \{\beta\in B\colon\ n^{-1} \leqslant \|y_\beta\| \leqslant n,\ n^{-1} \leqslant \|v_{\varrho(\beta)}\| \leqslant n,\ |v^*_{\varrho(\beta)}(y_\beta)| \geqslant n^{-1}\}.$$

Suppose a series $\sum_{\beta\in B_n} t_\beta y_\beta$ converges unconditionally to some y in F. Then the (unconditionally converging) expansion of y with respect to the basis (v_j) is $\sum\{t_\beta v^*_j(y_\beta)v_j\colon\ \beta\in B_n,\ j\in s(y_\beta)\}$; in consequence, the "subseries" $\sum_{\beta\in B_n} t_\beta v^*_{\varrho(\beta)}(y_\beta)\times$

$\times v_{\varrho(\beta)}$ converges unconditionally. Finally, since $|v^*_{\varrho(\beta)}(y_\beta)| \geqslant n^{-1}$ for all β in β_n, also the series $\sum_{\beta\in B_n} t_\beta v_{\varrho(\beta)}$ converges unconditionally. ∎

THEOREM 2. *Let* $T\colon c_0(A)\to F$ *be a continuous linear operator whose range has the density character* $\mathfrak{m} > \aleph_0$. *Then, if* $\mathfrak{n} \leqslant {}_c\mathfrak{m}$, *there is a subset* A' *of* A *and a subset* J' *of* J *such that* $|A'| = |J'| = \mathfrak{n}$, $T|c_0(A')$ *is an isomorphic embedding and the subbasis* $(v_j)_{j\in J'}$ *is of* c_0*-type.*

Proof. By [1, Lemma 4], the set $\{\alpha\in A\colon T(e^0_\alpha)\neq 0\}$ is of cardinality $\mathfrak{m}$; hence, by [1, Lemma 3] and Lemma 2 above, it contains a subset A' with $|A'| = \mathfrak{n}$ such that for some injective function $\varrho\colon A'\to J$ both the families $(T(e^0_\alpha))_{\alpha\in A'}$ and $(v_{\varrho(\alpha)})_{\alpha\in A'}$ are seminormalized, $(T(e^0_\alpha))_{\alpha\in A'} \succ (v_{\varrho(\alpha)})_{\alpha\in A'}$, and the vectors $T(e^0_\alpha)$, $\alpha\in A'$, have pairwise disjoint supports. Thus $(T(e^0_\alpha))_{\alpha\in A'}$ is an unconditional basic family, and since $(e^0_\alpha)_{\alpha\in A'} \succ (T(e^0_\alpha))_{\alpha\in A'}$ (by the continuity of T) and $(v_{\varrho(\alpha)})_{\alpha\in A'} \succ (e^0_\alpha)_{\alpha\in A'}$ (because the former family is seminormalized), the assertions of the theorem, with $J' = \{\varrho(\alpha)\colon \alpha\in A'\}$, follow easily. ∎

Remark. As the identity operator from $l_1(A)$ into $c_0(A)$ shows, the l_1-version of the above result is false.

EXAMPLES. In the two examples below, we show that the distinction between the cases $\mathrm{cf}(\mathfrak{m}) = \mathfrak{m}$ and $\mathrm{cf}(\mathfrak{m}) < \mathfrak{m}$ in Theorem 1, as well as the cases $\mathrm{cf}(\mathfrak{m}) > \aleph_0$ and $\mathrm{cf}(\mathfrak{m}) = \aleph_0$ in Theorem 2, is essential.

Let $(J_n)_{n\in N}$ be a sequence of pairwise disjoint infinite sets whose cardinal numbers $\mathfrak{m}_n = |J_n|$ form a strictly increasing sequence, let J be the union of these sets, and let $\mathfrak{m} = |J|$. Then $\mathrm{cf}(\mathfrak{m}) = \aleph_0 < \mathfrak{m}$.

For each n, let $\mathscr{P}_n$ be the class of all n-subsets of J_n, and let $(K_\alpha)_{\alpha\in A_n}$ be a family of pairwise disjoint n-subsets of J_n with union J_n. Thus $|J_n| = |A_n| = \mathfrak{m}_n$. If K is a subset of J_n, let e_K be its characteristic function; $e_j = e_{\{j\}}$. The usual l_1- and c_0-norms are denoted by $\|\cdot\|_1$ and $\|\cdot\|_\infty$, respectively.

1) For each $n\in N$ let $F_n = (l_1(J_n), |||\cdot|||_n)$, where

$$|||y|||_n = \max(n^{-1}\|y\|_1, \|y\|_\infty).$$

Then

$$n^{-1}\|y\|_1 \leqslant |||y|||_n \leqslant \|y\|_1 \quad \text{for all } y\in l_1(J_n). \tag{3}$$

Since $|||e_j|||_n = 1 = \|e_j\|_1$ for all $j\in J_n$ and $|||e_K|||_n = 1 = n^{-1}\|e_K\|_1$ for all $K\in\mathscr{P}_n$, it follows that

(∗) the estimates (3) cannot be improved on any subspace $l_1(K)$ of $l_1(J_n)$, where $K\subset J_n$ and $|K|\geqslant n$.

For $\alpha\in A_n$, let $u_\alpha = e_{K_\alpha}$. Then, for every (t_α) in $l_1(A_n)$,

$$\Big|\Big|\Big|\sum_{\alpha\in A_n} t_\alpha u_\alpha\Big|\Big|\Big|_n = \max\Big(n^{-1}\sum_{\alpha\in A_n} n|t_\alpha|, \sup_{\alpha\in A_n}|t_\alpha|\Big) = \|(t_\alpha)\|_1,$$

and so $(u_\alpha)_{\alpha\in A_n}$ is isometrically equivalent to $(e^1_\alpha)_{\alpha\in A_n}$.

Now, consider the l_1-sum of the spaces F_n,

$$F = \Big(\sum_{n=1}^{\infty} F_n\Big)_{l_1};$$

F can be thought of as a space of functions defined on J, and each F_n can be identified with the subspace of functions vanishing off J_n. Then $(e_j)_{j\in J}$ is a 1-unconditional basis of F. For each n, it contains a subbasis, viz., $(v_j)_{j\in J_n}$, of cardinality $\mathfrak{m}_n$ that is of l_1-type. Moreover, F has a subspace, viz., $[u_\alpha\colon \alpha\in A]$, $A = \bigcup_n A_n$, isometric to $l_1(A)$, where $|A| = \mathfrak{m}$. Nevertheless, as is easily seen using (∗), the basis $(e_j)_{j\in J}$ does not have any l_1-subbasis of cardinality $\mathfrak{m}$.

2) For each $n\in N$ let $F_n = (c_0(J_n), |||\cdot|||_n)$, where

$$|||y|||_n = \sup_{K\in\mathscr{P}_n}\sum_{j\in K}|y(j)|.$$

Then

$$\|y\|_\infty \leqslant |||y|||_n \leqslant n\|y\|_\infty \quad \text{for all } y\in c_0(J_n). \tag{4}$$

Since $|||e_j|||_n = 1 = \|e_j\|_\infty$ for all $j\in J_n$ and $|||e_K|||_n = n = n\|e_K\|_\infty$ for all $K\in\mathscr{P}_n$, it follows that

(∗∗) the estimates (4) cannot be improved on any subspace $c_0(K)$ of $c_0(J_n)$, where $K\subset J_n$ and $|K|\geqslant n$.

For $\alpha\in A_n$, let $u_\alpha = n^{-1}e_{K_\alpha}$. Then we verify easily that $(u_\alpha)_{\alpha\in A_n}$ is isometrically equivalent to $(e^0_\alpha)_{\alpha\in A_n}$. Let

$$F = \Big(\sum_{n=1}^{\infty} F_n\Big)_{c_0}.$$

Then $(e_j)_{j\in J}$ is a 1-unconditional basis of F and, for each n, it has a c_0-subbasis of cardinality $\mathfrak{m}_n$. Moreover, F has a subspace isometric to $c_0(A)$, $|A| = \mathfrak{m}$. Nevertheless, (∗∗) implies that the basis $(e_j)_{j\in J}$ does not have any c_0-subbasis of cardinality $\mathfrak{m}$.

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$\mathscr{L}_\pi$-Spaces and cone summing operators

by

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Abstract. Let E be a real Banach lattice, X a real Banach space, and $T\colon E \to X$ a linear operator. Suppose $1 \leqslant p < \infty$ and that there is a constant $K > 0$ such that for all $n \in N$ and any $u_1, \ldots, u_n$ in E

$$\Big(\sum_{i=1}^{n} \|Tu\|^p\Big)^{1/p} \leqslant K \sup\Big\{\Big(\sum_{i=1}^{n} \langle \varphi, |u_i| \rangle^p\Big)^{1/p}\colon\ \varphi \in \operatorname{ball} E^*_+\Big\}.$$

We show that T has a (sub)factorization through a class of Banach lattices closely related to the $L_p(L_1)$-spaces. We recover as special cases some classical results on p-absolutely summing operators.

1. Introduction.

1.1. Definition. Let E be a Banach lattice, X a Banach space and $1 \leqslant p < \infty$. A linear operator $T\colon E \to X$ is *cone p-summing* if there is a constant $K > 0$ such that for each positive integer n and any vectors $u_1, \ldots, u_n$ in E,

$$\Big(\sum_{j=1}^{n} \|Tu_j\|^p\Big)^{1/p} \leqslant K \sup\Big\{\Big(\sum_{j=1}^{n} \langle \varphi, |u_j| \rangle^p\Big)^{1/p}\colon\ \varphi \in \operatorname{ball} E^*_+\Big\}.$$

We denote by $\hat{\pi}_p(T)$ the least K for which this inequality holds for all n and all choices of n vectors in E; and $\hat{\Pi}_p(E, X)$ is the set of cone p-summing operators $E \to X$.

1.2. Remarks. When $p = 1$ these operators have been studied by Schaefer [7].

Let $1 \leqslant p < \infty$, let E be a Banach lattice and X a Banach space. Let $\Pi_p(E, X)$ denote the p-absolutely summing operators $E \to X$ in the sense of Pietsch [6] and $C_p(E, X)$ the p-concave operators $E \to X$ in the sense of Lindenstrauss and Tzafriri [4]. Then we have the relations:

(i) $\Pi_p(E, X) \subseteq \hat{\Pi}_p(E, X) \subseteq C_p(E, X)$.
(ii) $\Pi_p(E, X) = \hat{\Pi}_p(E, X) = C_p(E, X)$ whenever E is a $C(K)$-space.
(iii) $\hat{\Pi}_1(E, X) = C_1(E, X)$ for all E and all X.

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