STUDIA MATHEMATICA, T. XCII. (1989)

Weighted rearrangements and higher integrability results

by

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Abstract. In this paper we study relations between reverse integral inequalities and higher integrability in some cases. We use reduction to the one-dimensional case via decreasing rearrangement of functions.

1. Introduction. Let Ω be a bounded domain in $\boldsymbol{R}^n$ and f a nonnegative measurable function on Ω. We say that f satisfies a *reverse Hölder inequality* if for some constants $p > q$

$$\Big(\fint_Q f^p\,dx\Big)^{1/p} \leqslant K\Big(\fint_Q f^q\,dx\Big)^{1/q} \tag{1.1}$$

for any cube $Q \subset \Omega$, where $|Q|$ is the Lebesgue measure of Q and $\fint_Q f\,dx = |Q|^{-1}\int_Q f\,dx$.

It is well known that from (1.1) we can deduce higher integrability for f. The most famous result of this kind is Gehring's theorem:

THEOREM 1.1 (Gehring [10]). *Let $q > 1$, $f \in L^q_{\mathrm{loc}}(\Omega)$ and suppose that*

$$\Big(\fint_Q |f|^q\,dx\Big)^{1/q} \leqslant K\fint_Q |f|\,dx$$

for every cube $Q \subset \Omega$. Then there exists $p = p(n, q, K)$ such that $p > q$ and $f \in L^p_{\mathrm{loc}}(\Omega)$. Moreover,

$$\Big(\fint_Q |f|^p\,dx\Big)^{1/p} \leqslant C\Big(\fint_Q |f|^q\,dx\Big)^{1/q},$$

where $C = C(n, q, K)$.

It is interesting to consider weighted versions of (1.1). In particular, in [19] a weighted Gehring theorem if the weight w is doubling is proved.

In [16], [9], [4] another reverse integral inequality is considered, intimately connected with the Fefferman–Stein sharp function [8]. For nonnegative $f \in L^1_{\mathrm{loc}}(\Omega)$ it is assumed that

$$\fint_Q \Big|f - \fint_Q f\,dx\Big|\,dx \leqslant K\fint_Q f \tag{1.2}$$

for $0 < K < 2$. From (1.2) the higher integrability for f is deduced, by different methods.

The aim of the present paper is to consider weighted versions of (1.2) and deduce from them some higher integrability results.

The method used is, in principle, the reduction to the one-dimensional case using the decreasing rearrangement. In fact we note that if f satisfies a reverse integral inequality, its decreasing rearrangement f^* satisfies essentially the same inequality. The key results in our proofs are a weighted version [19] of Herz's important theorem [15], a weighted version of the Fefferman–Stein inequality and the Besicovitch–De Giorgi covering lemma [6].

In Section 2 we list some properties of decreasing rearrangements with respect to a weight w and state the useful weighted Herz theorem.

In Section 3 we prove the higher integrability theorem 3.2, which is proved in the unweighted case in [9] (see also [14] and [16]).

In Section 4 we prove a weighted version of the Fefferman–Stein inequality with the majorization constant depending linearly (and not exponentially) on the constant $\hat{p}$. We use this result to generalize a higher integrability result of Iwaniec [16]. We obtain this result starting from a condition like (1.2):

$$w(\sigma Q)^{-1}\int_{\sigma Q}\Big|f-w(\sigma Q)^{-1}\int_{\sigma Q} fw\,dx\Big|\,w\,dx \leqslant Kw(Q)^{-1}\int_{Q} fw\,dx, \tag{1.3}$$

where $w(Q)=\int_Q w(x)\,dx$, $0<\sigma\leqslant 1$, and σQ is the cube with the same center as Q and σ times its size. For conditions of this kind see [12], [16] and [4].

Finally, we note that reverse integral inequalities occur in many different fields of analysis, e.g. in the theory of embedding for weighted Sobolev spaces and in partial differential equations [7], [20].

2. Some notation and useful results. Let $Q\subset \boldsymbol{R}^n$ be a cube which is a translate of $[0,s]^n$, $0<s<\infty$. We fix a cube Q_0 and consider subcubes Q of Q_0. We denote by $|Q|$ the Lebesgue measure of Q. Let $w\in L^1(Q_0)$ be a nonnegative weight function and set for any $E\subseteq Q_0$

$$w(E)=\int_E w(x)\,dx.$$

For any measurable nonnegative function f defined on Q_0, we set

$$\lambda_f(y)=w(\{x\in Q_0\colon f(x)>y\})$$

and we define the *nonincreasing rearrangement* of f with respect to the measure $w\,dx$ by

$$f^*(t)=\inf\{y>0\colon \lambda_f(y)\leqslant t\},\qquad t\in\,]0,w(Q_0)[. \tag{2.1}$$

The following results hold [5], [19], [20]:

$$\int_0^{w(Q_0)} f^*(s)^p\,ds=\int_{Q_0} f^p w\,dx,\qquad p\geqslant 1, \tag{2.2}$$

$$\int_0^t f^*(s)\,ds=\sup_{\substack{E\subset Q_0\\ w(E)\leqslant t}}\int_E fw\,dx,\qquad t\in\,]0,w(Q_0)[, \tag{2.3}$$

$$f^*(t)=\sup_{\substack{E\subset Q_0\\ w(E)=t}}\inf_E f,\qquad t\in\,]0,w(Q_0)[. \tag{2.4}$$

We suppose that w satisfies the following condition:

$$w(Q)\leqslant A\,(|Q|/|E|)^p\,w(E) \tag{2.5}$$

for every cube $Q\subseteq Q_0$ and $E\subset Q$, where A, $p\in\,]1,+\infty[$ are independent of Q and E. The property (2.5) follows easily from the A^∞ condition of Muckenhoupt [18]. It is also easy to deduce from (2.5) the *doubling property* for w:

$$w(2Q)\leqslant d\,w(Q),\qquad Q\subset \tfrac{1}{2}Q_0,$$

with $d>0$ independent of Q, where δQ denotes the cube with the same center as Q but δ times its size.

We also write

$$f^{**}(t)=t^{-1}\int_0^t f^*(s)\,ds, \tag{2.6}$$

$$Mf_w(x)=\sup w(Q)^{-1}\int_Q f(y)\,w\,dy, \tag{2.7}$$

$$f_w^{\#}(x)=\sup w(Q)^{-1}\int_Q\Big|f(y)-w(Q)^{-1}\int_Q fw\,dy\Big|\,w\,dy, \tag{2.8}$$

where the suprema are taken over all cubes containing x.

We need the following result concerning the relation between $f^{**}(t)$ and $(Mf_w)^*(t)$. The result is a weighted version of an important theorem of Herz [15].

THEOREM 2.1 [19]. *Let $f\in L^1(Q_0,w\,dx)$, $f\geqslant 0$. Then for any $t\in\,]0,w(Q_0)[$*

$$C_1(Mf_w)^*(t)\leqslant f^{**}(t)\leqslant C_2(Mf_w)^*(t),$$

where C_1 and C_2 depend only on w and n.

Now we state the following Besicovitch–De Giorgi covering lemma.

LEMMA 2.2 [6]. *Let A be a bounded domain in $\boldsymbol{R}^n$ and for each $x\in A$ let Q_x be a cube centered at x. Then there exists a sequence $(Q_j)_{j\in\boldsymbol{N}}$ contained in $(Q_x)_{x\in A}$ such that*

$$A\subset\bigcup_j Q_j,\qquad \sum_j \chi_{Q_j}(x)\leqslant\delta,\qquad x\in\boldsymbol{R}^n, \tag{2.9}$$

where δ depends only on the dimension n.

3. A first weighted higher integrability result. The following covering lemma, a weighted version of a covering lemma of Bennett–De Vore–Sharpley [2], will be used.

LEMMA 3.1. *Let $B \leqslant 2^{(n+1)p} A^2$ and let G be a relatively open subset of Q_0 such that $w(G) < B^{-1} w(Q_0)$. Then there exists a sequence of cubes $(Q_j)_{j\in N}$ with pairwise disjoint interiors such that*

$$\text{(3.1)}\quad \begin{aligned} &(1) && w(Q_j) < A2^p\, w(G^c \cap Q_j),\\ &(2) && \sum_j w(Q_j) \leqslant B w(G),\\ &(3) && G \subset \bigcup_j Q_j \subset Q_0. \end{aligned}$$

Proof. It is sufficient to use Lemma 3.1 of [1] and then apply the property (2.5) of w.

Set $\lambda = 3A^2\, 2^{(n+1)p}(A2^p+1)(1+\delta A 3^{np})$ and

$$K < 1/\lambda, \tag{3.2}$$

where A, p are from (2.5) and δ is from (2.9). Let $f \geqslant 0$ belong to $L^1(Q_0, w\,dx)$ and suppose that

$$w(Q)^{-1}\int_Q \Big|f(x) - w(Q)^{-1}\int_Q fw\,dx\Big|\, w\,dx \leqslant K w(Q)^{-1}\int_Q fw\,dx \tag{3.3}$$

for every $Q \subseteq Q_0$. Then we have

THEOREM 3.2. *If* (3.2) *and* (3.3) *hold then f belongs to $L^p(Q_0, w\,dx)$ for every $p \in [1, 1/(K\lambda)[$.*

Remark. The constant λ depends only on n and w.

Proof. Taking the supremum over all cubes containing x in (3.3) we obtain $f_w^{\#}(x) \leqslant K Mf_w(x)$ and then, from the known properties of decreasing rearrangements [5],

$$(f_w^{\#})^*(t) \leqslant K(Mf_w)^*(t). \tag{3.4}$$

We now need the following estimate: for any $t \in]0, (3B)^{-1} w(Q_0)[$

$$f^{**}(t) - f^*(t) \leqslant 3B(A2^p+1)(f_w^{\#})^*(t). \tag{3.5}$$

To see this, fix $0 < t < (3B)^{-1} w(Q_0)$ and set

$$E = \{x \in Q_0 \colon f(x) > f^*(t)\}, \qquad F = \{x \in Q_0 \colon f_w^{\#}(x) > (f_w^{\#})^*(t)\}.$$

We have $w(E \cup F) \leqslant 2t$. Fix a relatively open subset G of Q_0 such that $w(G) \leqslant 3t$, $E \cup F \subset G \subset Q_0$ and in particular $w(G) < B^{-1} w(Q_0)$. Let $(Q_j)_j$ be a covering of G satisfying (3.1). By (2.3) we have

$$\begin{aligned} t\big(f^{**}(t) - f^*(t)\big) &= \int_E \big(f(x) - f^*(t)\big) w\,dx = \sum_j \int_{E\cap Q_j} \big(f(x) - f^*(t)\big) w\,dx \\ &\leqslant \sum_j \int_{Q_j} \Big|f(x) - w(Q_j)^{-1}\int_{Q_j} f(x) w\,dx\Big|\, w\,dx \\ &\quad + \sum_j w(E\cap Q_j)\Big(w(Q_j)^{-1}\int_{Q_j} fw\,dx - f^*(t)\Big). \end{aligned} \tag{3.6}$$

Summing only over those j for which

$$f^*(t) \leqslant w(Q_j)^{-1}\int_{Q_j} fw\,dx$$

we obtain

$$\begin{aligned} &\sum_j{}' w(E\cap Q_j)\Big(w(Q_j)^{-1}\int_{Q_j} fw\,dx - f^*(t)\Big) \\ &\qquad \leqslant \sum_j{}' w(G\cap Q_j)\Big(w(Q_j)^{-1}\int_{Q_j} fw\,dx - f^*(t)\Big), \end{aligned}$$

and then by (1) of (3.1),

$$\begin{aligned} &\sum_j{}' w(E\cap Q_j)\Big(w(Q_j)^{-1}\int_{Q_j} fw\,dx - f^*(t)\Big) \\ &\qquad \leqslant A2^p \sum_j{}' \int_{G^c\cap Q_j} \Big(w(Q_j)^{-1}\int_{Q_j} fw\,dx - f^*(t)\Big) w\,dx \\ &\qquad \leqslant A2^p \sum_j \int_{Q_j} \Big|f(x) - w(Q_j)^{-1}\int_{Q_j} fw\,dx\Big|\, w\,dx. \end{aligned} \tag{3.7}$$

From (3.6) and (3.7) we deduce

$$t\big(f^{**}(t) - f^*(t)\big) \leqslant (A2^p+1)\sum_j \int_{Q_j} \Big|f(x) - w(Q_j)^{-1}\int_{Q_j} fw\,dx\Big|\, w\,dx. \tag{3.8}$$

But for every j, $F^c \cap Q_j \neq \emptyset$; let x_j belong to $F^c \cap Q_j$. We have $(f_w^{\#})^*(t) \geqslant f_w^{\#}(x_j)$ and then

$$\begin{aligned} &\sum_j \int_{Q_j} \Big|f(x) - w(Q_j)^{-1}\int_{Q_j} fw\,dx\Big|\, w\,dx \\ &\qquad = \sum_j w(Q_j)\, w(Q_j)^{-1}\int_{Q_j} \Big|f(x) - w(Q_j)^{-1}\int_{Q_j} fw\,dx\Big|\, w\,dx \\ &\qquad \leqslant \sum_j w(Q_j) f_w^{\#}(x_j) \leqslant \sum_j w(Q_j)(f_w^{\#})^*(t). \end{aligned}$$

Then by (2) of (3.1) and (3.8)

$$t\big(f^{**}(t) - f^*(t)\big) \leqslant (A2^p+1)\, B (f_w^{\#})^*(t)\, w(G) \leqslant 3B(A2^p+1)\, t\, (f_w^{\#})^*(t),$$

which is (3.5).

From (3.4) and (3.5) we obtain

$$f^{**}(t) - f^*(t) \leqslant 3KB(A2^p+1)(Mf_w)^*(t). \tag{3.9}$$

We note at this point that in Theorem 2.1 we can take

$$C_1 = 1/(1+\delta A 3^{np}).$$

Then from (3.9) we deduce

$$f^{**}(t)-f^{*}(t) \leqslant 3KB(A2^p+1)(1+\delta A3^{np}) f^{**}(t),$$

and by (3.2)

$$f^{**}(t) \leqslant \frac{1}{1-3KB(A2^p+1)(1+\delta A3^{np})} f^{*}(t). \tag{3.10}$$

We now use the following lemma of Muckenhoupt [19].

LEMMA 3.3. *Let $h(t)$ be positive and nonincreasing on $]0, a[$ and assume that there exists $D > 1$ such that*

$$t^{-1}\int_0^t h(s)\,ds \leqslant Dh(t), \quad t \in]0, a/2[.$$

Then for $r \in [1, D/(D-1)[$ we have

$$a^{-1}\int_0^a h(s)^r\,ds \leqslant \frac{2^r D}{D-r(D-1)}\Big(a^{-1}\int_0^a h(s)\,ds\Big)^r.$$

Applying Lemma 3.3 starting from (3.10) we conclude the proof of the theorem, since the average of a nonincreasing function is still nonincreasing.

An unweighted version of (3.5) is in [2] and [9], while Theorem 3.1 without weight can be found in [9], [14], [4], [16].

4. A weighted Fefferman–Stein inequality and another higher integrability result. Using the covering lemma 2.2 we prove the following

THEOREM 4.1. *Let $f \geqslant 0$, $f \in L^1_{\mathrm{loc}}(\mathbf{R}^n, w\,dx)$. For any $t \in]0, +\infty[$, we have*

$$(Mf_w)^*(t) \leqslant C(f_w^\#)^*(2t)+(Mf_w)^*(2t),$$

where $C > 0$ is a constant depending only on w and n.

Proof. Let G be a relatively open subset of Q_0 such that $w(G) < 5t$ and $O \subset G$, where

$$O = \{x \in Q_0\colon f_w^\#(x) > (f_w^\#)^*(2t)\} \cup \{x \in Q_0\colon Mf_w(x) > (Mf_w)^*(2t)\}.$$

We can find (see [1], Lemma 3.2) a covering $(Q_j)_j$ of G such that the Q_j are nonoverlapping and

$$\sum_j w(Q_j) \leqslant Cw(G), \tag{1}$$

$$[Q \cap Q_j \neq \emptyset,\ |Q_j| \leqslant |Q|] \Rightarrow |Q| \leqslant 2|Q \cap G^c|. \tag{2}$$

Let Q be a cube of the cover, $\gamma > (Mf_w)^*(2t)$ and set $E = \{x \in Q\colon Mf_w(x) > \gamma\}$.

For each $x \in E$ we find $Q(x)$ such that $x \in Q(x)$ and

$$w(Q(x))^{-1}\int_{Q(x)} f(y)\,w\,dy > \gamma.$$

Obviously, $Q(x) \subset G$ and then, by (2), $|Q(x)| < |Q|$, whence $Q(x) \subset 3Q$. Then $3Q \cap G^c \neq \emptyset$ and

$$w(3Q)^{-1}\int_{3Q} f(x)\,w\,dx \leqslant (Mf_w)^*(2t).$$

Consequently,

$$(\gamma-(Mf_w)^*(2t))\,w(Q(x)) \leqslant \int_{Q(x)} \Big(f(y)-w(3Q)^{-1}\int_{3Q} fw\,dx\Big)w\,dy.$$

Using Lemma 2.2 we can extract from $(Q(x))_{x\in E}$ a sequence $(Q(x_j))_j$ such that

$$w(E) \leqslant \delta \sum_j w(Q(x_j)), \tag{4.1}$$

where δ depends only on n. But $\bigcup_j Q(x_j) \subset 3Q$, and then

$$w(E)(\gamma-(Mf_w)^*(2t)) \leqslant \delta\int_{3Q}\Big|f(y)-w(3Q)^{-1}\int_{3Q} fw\,dx\Big|\,w\,dy$$
$$\leqslant \delta w(3Q)(f_w^\#)^*(2t) \leqslant A\delta 3^{np}\,w(Q)(f_w^\#)^*(2t).$$

Let

$$\gamma = \varepsilon^{-1}A\delta 3^{np}(f_w^\#)^*(2t)+(Mf_w)^*(2t)$$

with $\varepsilon < 1/(5C)$, where C is from (1). Then we have

$$w(E) = w(\{x \in Q\colon Mf_w(x) > \gamma\}) \leqslant \varepsilon w(Q)$$

and by (1)

$$w(\{x\colon Mf_w(x) > \gamma\}) \leqslant \frac{1}{5C}Cw(G) < t.$$

The proof is complete.

From Theorem 4.1 we deduce

COROLLARY 4.2. *For nonnegative $f \in L^1_{\mathrm{loc}}(\mathbf{R}^n, w\,dx)$ we have*

$$(Mf_w)^*(t) \leqslant C\int_t^{+\infty}(f_w^\#)^*(s)\,ds/s+\lim_{r\to\infty}(Mf_w)^*(r) \tag{4.2}$$

and, if $\lim_{r\to\infty}(Mf_w)^*(r) = 0$,

$$\Big(\int_{\mathbf{R}^n}|Mf_w(x)|^p w(x)\,dx\Big)^{1/p} \leqslant Cp\Big(\int_{\mathbf{R}^n}|f_w^\#(x)|^p w(x)\,dx\Big)^{1/p}, \tag{4.3}$$

where C is a constant independent of p.

We note that (4.3) is a weighted Fefferman–Stein inequality in which the constant on the right depends linearly and not exponentially on p. For similar results see [22], [4], [1].

With slight modifications we obtain the following local version of (4.3).

COROLLARY 4.3. *For nonnegative $f \in L^1(Q_0, w\,dx)$ we have*

$$(4.4)\qquad (Mf_w)^*(t) \leqslant C\Big(\int_t^{|Q_0|} (f_w^\#)^*(s)\,ds/s + w(Q_0)^{-1}\int_{Q_0} f(x)\,w\,dx\Big).$$

Starting from Corollary 4.3 we can prove the following higher integrability result which generalizes some Gurov–Reshetnyak [14] and Iwaniec [1] results, as in the paper [4] of Bojarski.

For Ω a bounded domain in $\boldsymbol{R}^n$ we have

THEOREM 4.4. *Let $f \in L^1_{\mathrm{loc}}(\Omega, w\,dx)$ and nonnegative, $0 < \sigma \leqslant 1$, $0 < K < 2$, and*

$$w(\sigma Q)^{-1}\int_{\sigma Q}\Big|f - w(\sigma Q)^{-1}\int_{\sigma Q} f(x)\,w\,dx\Big|\,w\,dx \leqslant K w(Q)^{-1}\int_Q f(x)\,w\,dx$$

for each cube $Q \subseteq \Omega$. Then there exists a constant C, depending only on w and n, such that $f \in L^r_{\mathrm{loc}}(\Omega, w\,dx)$ for $r < C/K$. Moreover,

$$\Big(w(\tfrac{1}{2}Q)^{-1}\int_{(1/2)Q} f^r\,dy\Big)^{1/r} \leqslant b\,w(Q)^{-1}\int_Q f\,dy$$

with $b \geqslant 0$ depending only on σ, n, r.

Proof. The proof is similar to that of Lemma 2 of [16] with the only essential modification of using Corollary 4.3 instead of Lemma 4 from [16].

Finally, we note that in Theorems 3.2 and 4.4 the order of the optimal integrability exponent is exact.

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Received September 19, 1986 (2213)