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The $*$-holonomy group of the Stefan suspension of a diffeomorphism

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Abstract. The definition of a Stefan suspension of a diffeomorphism is given. If $\mathcal{G}_g$ is the Stefan suspension of the diffeomorphism g over a Stefan foliation $\mathcal{G}$, and $G_0 \in \mathcal{G}$ satisfies the condition $g|G_0 = \mathrm{id}_{G_0}$, then we compute the $*$-holonomy group for the leaf $F_0 \in \mathcal{G}_g$ determined by G_0. A representative element of the $*$-holonomy along the standard imbedding of S^1 into F_0 is characterized. A corollary for the case when G_0 contains only one point is derived.

0. Introduction. Our base is the notion of a Stefan foliation introduced in [4]. In the present paper, "$*$-holonomy" has the same meaning as holonomy defined in [2]. This new terminology is introduced in order to distinguish it from Ehresmann holonomy ([1], [5]).

Let N be a smooth manifold and let $\mathcal{G}$ be a Stefan foliation of N. Let $g : N \to N$ be a diffeomorphism which maps leaves into leaves. In Section 1 we define the Stefan suspension of g over $\mathcal{G}$.

Let $G_0 \in \mathcal{G}$ satisfy the condition $g|G_0 = \mathrm{id}_{G_0}$, let $\mathcal{F}$ be the Stefan suspension of g over $\mathcal{G}$ and let $F_0 \in \mathcal{F}$ be determined by G_0. Section 2 contains theorems on the $*$-holonomy group of F_0. Theorem (2.1) asserts that this group is isomorphic to the product of the $*$-holonomy group of G_0 and the group generated by the $*$-holonomy along the standard imbedding of S^1 into F_0. Theorem (2.2) says that, for an arbitrary transversal Σ containing $y_0 \in G_0$, there exists a representative element of the $*$-holonomy conjugate to $g|\Sigma$. As a corollary we obtain the following fact: if G_0 contains the single point y_0, then the $*$-holonomy group of F_0 is isomorphic to the group generated by the class of the diffeomorphism g.

We adopt the terminology and notation from [2]. The only exception is the symbol $*\text{-}\mathrm{Hol}_{x_0}(\mathcal{F}, \varphi)$ instead of $\mathrm{Hol}_{x_0}(\mathcal{F}, \varphi)$ used in [2].

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1. A Stefan suspension of a diffeomorphism. Let N be a smooth manifold of dimension n and $\mathcal{G}$ a Stefan foliation of N. Let $g : N \to N$ be a diffeomorphism which maps leaves of $\mathcal{G}$ into leaves of $\mathcal{G}$.

In $N \times \mathbb{R}$, define the equivalence relation $\sim$ in the following way: $(y,t) \sim (y',t')$ if and only if $t-t' = k \in \mathbb{Z}$ and $y' = g^k(y)$. In other words, consider on $N\times\mathbb{R}$ the diffeomorphism $\overline{g}(y,t) = (g^{-1}(y), t+1)$. Then $(y,t) \sim (y',t')$ if and only if $(y',t') = \overline{g}^k(y,t)$ for some $k \in \mathbb{Z}$. It is well known that $M := N\times\mathbb{R}/{\sim}$ is a manifold of dimension $n+1$ and the canonical projection $\pi : N\times\mathbb{R} \to M$ is a covering.

Consider in $N\times\mathbb{R}$ a foliation $\mathcal{F}_0 := \mathcal{G}\times\widetilde{\mathbb{R}}$ ([5]) where $\widetilde{\mathbb{R}}$ is the foliation of $\mathbb{R}$ consisting of a single leaf. Note that $\mathcal{F}_0$ is invariant under the diffeomorphism $\overline{g}$. It is easy to see that there exists a Stefan foliation $\mathcal{F}$ of M such that $\mathcal{F}_0 = \pi^*(\mathcal{F})$ ([3], [5]). Leaves of $\mathcal{F}$ are submanifolds of M of the form $\pi(G\times\mathbb{R})$ where $G \in \mathcal{G}$ and the foliation $\mathcal{F}$ is locally isomorphic to $\mathcal{F}_0$. The foliation $\mathcal{F}$ is called the *Stefan suspension* of g over $\mathcal{G}$.

A simple computation proves that the following facts hold:

(1.1) If ψ is a distinguished chart of $\mathcal{G}$ around y_0, then $\psi\circ g$ is a distinguished chart of this foliation around $g^{-1}(y_0)$ with the domain $g^{-1}(D_\psi)$.

(1.2) If ψ is a distinguished chart of $\mathcal{G}$ around y_0, and $t_0 \in \mathbb{R}$, then the mapping

$$\begin{aligned}\varphi : \pi(D_\psi \times (t_0 - 1/2, t_0 + 1/2)) \ni \pi(y,t) \\ \mapsto (t - t_0, \psi(y)) \in (-1/2, 1/2) \times U_\psi \times W_\psi\end{aligned}$$

$(y \in D_\psi, t \in (t_0 - 1/2, t_0 + 1/2))$ is a distinguished chart of $\mathcal{F}$ around $\pi(y_0, t_0) \in M$.

Introduce the following notation for the natural projections: $\mathrm{pr}_1 : U_\psi \times W_\psi \to U_\psi$, $\mathrm{pr}_2 : U_\psi \times W_\psi \to W_\psi$, $\mathrm{Pr}_1 : (-1/2,1/2) \times U_\psi \times W_\psi \to (-1/2,1/2) \times U_\psi = U_\varphi$ and $\mathrm{Pr}_2 : (-1/2,1/2) \times U_\psi \times W_\psi \to W_\psi = W_\varphi$.

2. The $*$-holonomy group of a Stefan suspension. Let $G_0 \in \mathcal{G}$ be a leaf for which

$$g|G_0 = \mathrm{id}_{G_0} . \tag{1}$$

Let $F_0 = \pi(G_0 \times \mathbb{R}) \in \mathcal{F}$. Note that $F_0 = G_0 \times S^1$ by (1). Denote by p_{G_0} and p_{S^1} the natural projections of F_0 onto G_0 and S^1, respectively. We have

$$\pi_1(F_0) \cong \pi_1(G_0) \times \pi_1(S^1) . \tag{2}$$

It is easy to check that each element of $\pi_1(G_0)$ commutes with each element of $\pi_1(S^1)$ in $\pi_1(F_0)$.

Let $y_0 \in G_0$ and $x_0 = \pi(y_0, 0)$. Fix a distinguished chart ψ of $\mathcal{G}$ around y_0 and let φ be the distinguished chart of $\mathcal{F}$ defined as in (1.2) with $t_0 = 0$.

At the point x_0 consider the loop

$$\gamma : [0,1] \ni s \mapsto \pi(y_0, s) \in F_0 .$$

Under the above assumptions, we prove the following

(2.1) THEOREM. $*\text{-Hol}_{x_0}(\mathcal{F}, \varphi) \cong *\text{-Hol}_{y_0}(\mathcal{G}, \psi) \times \langle [f_{\gamma;\varphi,\varphi}] \rangle$.

(Here, $\langle [f_{\gamma;\varphi,\varphi}] \rangle$ denotes the subgroup of $\mathcal{A}_\varphi/\equiv$ generated by $[f_{\gamma;\varphi,\varphi}]$.)

Proof. Define

$$\Phi : *\text{-Hol}_{y_0}(\mathcal{G}, \psi) \times \langle [f_{\gamma;\varphi,\varphi}] \rangle \to *\text{-Hol}_{x_0}(\mathcal{F}, \varphi)$$

by the formula

$$\Phi(h_{\mathcal{G},\psi}([\alpha]), [f_{\gamma;\varphi,\varphi}]^k) = h_{\mathcal{F},\varphi}([\overline{\alpha}] \cdot [\gamma]^k) \tag{3}$$

(with h being the holonomy homomorphism of the respective foliation), where $k \in \mathbb{Z}$, α is a loop in G_0 at y_0 and $\overline{\alpha} : [0,1] \ni s \mapsto \pi(\alpha(s), 0) \in F_0$.

By using chains of charts described in (1.2), it is easy to check that the definition of Φ is correct. Note that Φ takes its values in $*\text{-Hol}_{x_0}(\mathcal{F}, \varphi)$ by (3).

We show that Φ is a group homomorphism. Using the commutativity mentioned after (2), we have

$$\begin{aligned} h_{\mathcal{F},\varphi}([\overline{\alpha}]) \cdot [f_{\gamma;\varphi,\varphi}]^k &= h_{\mathcal{F},\varphi}([\overline{\alpha}]) \cdot h_{\mathcal{F},\varphi}([\gamma]^k) = h_{\mathcal{F},\varphi}([\overline{\alpha}] \cdot [\gamma]^k) \\ &= h_{\mathcal{F},\varphi}([\gamma]^k \cdot [\overline{\alpha}]) \ = [f_{\gamma;\varphi,\varphi}]^k \cdot h_{\mathcal{F},\varphi}([\overline{\alpha}]) . \end{aligned}$$

Therefore, by simple computations, we get

$$\begin{aligned} &\Phi((h_{\mathcal{G},\psi}([\alpha]), [f_{\gamma;\varphi,\varphi}]^k) \cdot (h_{\mathcal{G},\psi}([\alpha']), [f_{\gamma;\varphi,\varphi}]^{k'})) \\ &\qquad = \Phi(h_{\mathcal{G},\psi}([\alpha]), [f_{\gamma;\varphi,\varphi}]^k) \cdot \Phi(h_{\mathcal{G},\psi}([\alpha']), [f_{\gamma;\varphi,\varphi}]^{k'}) . \end{aligned}$$

Define

$$\Psi : *\text{-Hol}_{x_0}(\mathcal{F}, \varphi) \to *\text{-Hol}_{y_0}(\mathcal{G}, \psi) \times \langle [f_{\gamma;\varphi,\varphi}] \rangle$$

by the formula

$$\Psi(h_{\mathcal{F},\varphi}([\delta])) = (h_{\mathcal{G},\psi}([p_{G_0} \circ \delta]), [f_{\gamma;\varphi,\varphi}]^k) \tag{4}$$

where δ is a loop in F_0 at x_0 and k is an integer such that $[p_{S^1} \circ \delta] = [\beta]^k$ with $\beta : [0,1] \ni s \mapsto e^{2\pi i s} \in S^1$.

We show that the above definition is correct. Fix δ for a moment. For each $s \in [0,1]$, take an arbitrary distinguished chart $\psi^{(s)}$ ($\psi^{(0)} = \psi^{(1)} = \psi$) of $\mathcal{G}$ around $y(s) := p_{G_0} \circ \delta(s)$. Let $t : [0,1] \to \mathbb{R}$ be the unique lift of $p_{S^1} \circ \delta$ to the universal covering of S^1 with $t(0) = 0$. Note that

$$\delta(s) = \pi(y(s), t(s)) . \tag{5}$$

Define a distinguished chart $\varphi^{(s)}$ around $\delta(s)$ as in (1.2), using the chart $\psi^{(s)}$ and setting $t_0 = t(s)$. From the family $\{\varphi^{(s)} : s \in [0,1]\}$ choose a finite subfamily $\{\varphi_0, \varphi_1, \ldots, \varphi_r\}$ (with $\varphi_0 = \varphi^{(0)}, \varphi_r = \varphi^{(1)}$ and $\varphi_i = \varphi^{(s_i)}$ for $i = 1, \ldots, r-1$) such that the sequence

$$\widetilde{\mathcal{C}} = (\varphi_0, 0; \varphi_1, s_1; \ldots; \varphi_r, 1; \varphi_0, 1)$$

is a chain along δ. We prove that the sequence

$$\mathcal{C} = (\psi_0, 0; \psi_1, s_1; \ldots; \psi_r, 1)$$

is a chain along $p_{G_0} \circ \delta$, where $\psi_0 = \psi^{(0)} = \psi^{(1)} = \psi_r = \psi$ and $\psi_i = \psi^{(s_i)}$ for $i = 1, \ldots, r-1$. To this end, we prove

LEMMA A. *If* $\widetilde{s} \in \delta^{-1}(D_{\varphi_i})_{s_i}$ (*the connected component of* $\delta^{-1}(D_{\varphi_i})$ *containing* s_i), *then* $t(\widetilde{s}) \in (t(s_i) - 1/2, t(s_i) + 1/2)$.

Proof. It follows directly from the definitions of φ_i, t and from (5) that

$$t(\delta^{-1}(D_{\varphi_i})_{s_i}) \subset (t(s_i) - 1/2, t(s_i) + 1/2)\,.$$

In particular, $t(\widetilde{s}) \in (t(s_i) - 1/2, t(s_i) + 1/2)$. ∎

Lemma A implies

LEMMA B. $\delta^{-1}(D_{\varphi_i})_{s_i} \subset (p_{G_0} \circ \delta)^{-1}(D_{\psi_i})_{s_i}$.

Proof. If $\widetilde{s} \in \delta^{-1}(D_{\varphi_i})_{s_i}$, then $\pi(y(\widetilde{s}), t(\widetilde{s})) = \pi(y', t')$ for some $y' \in D_{\psi_i}, t' \in (t(s_i) - 1/2, t(s_i) + 1/2)$ and, using Lemma A, we obtain $y(\widetilde{s}) = (p_{G_0} \circ \delta)(\widetilde{s}) \in D_{\psi_i}$, which gives the assertion. ∎

Directly from Lemma B it follows that if $\delta^{-1}(D_{\varphi_i})_{s_i} \cap \delta^{-1}(D_{\varphi_{i+1}})_{s_{i+1}} \neq \emptyset$, then $(p_{G_0} \circ \delta)^{-1}(D_{\psi_i})_{s_i} \cap (p_{G_0} \circ \delta)^{-1}(D_{\psi_{i+1}})_{s_{i+1}} \neq \emptyset$. Thus $\mathcal{C}$ is a chain along $p_{G_0} \circ \delta$.

We now show that the $*$-holonomy diffeomorphism determined by the part

$$(\varphi_0, 0; \varphi_1, s_1; \ldots; \varphi_r, 1)$$

of $\widetilde{\mathcal{C}}$ is equal to $f_{\mathcal{C}}$. Indeed, let $\widetilde{s}_i \in \delta^{-1}(D_{\varphi_i})_{s_i} \cap \delta^{-1}(D_{\varphi_{i+1}})_{s_{i+1}}$, $i = 0, 1, \ldots, r-1$. Then, by Lemmas A and B, we have

$$f_{\varphi_i, \varphi_{i+1}; \delta(\tilde{s}_i)}(w) = f_{\psi_i, \psi_{i+1}; y(\tilde{s}_i)}(w)\,. \tag{6}$$

Suppose now that

$$h_{\mathcal{F},\varphi}([\delta]) = h_{\mathcal{F},\varphi}([\delta'])\,. \tag{7}$$

Take chains $\widetilde{\mathcal{C}}$ and $\widetilde{\mathcal{C}}'$ constructed as above along δ and δ', respectively. Then $f_{\widetilde{\mathcal{C}}} \equiv f_{\widetilde{\mathcal{C}}'}$. Along the curve $\eta = \delta * \delta'^{-1}$ we can construct a chain $\overline{\mathcal{C}}$ by composing links of $\widetilde{\mathcal{C}}$ and links of $\widetilde{\mathcal{C}}'$ in opposite order. We have

$$\begin{aligned}\overline{\mathcal{C}} = (&\varphi_0, 0; \varphi_1, (1/2)s_1; \ldots; \varphi_r, 1/2; \varphi_0, 1/2; \varphi_0, 1/2;\\ &\varphi'_{r'}, 1/2; \varphi'_{r'-1}, 1 - (1/2)s'_{r'-1}; \ldots; \varphi'_1, 1 - (1/2)s'_1; \varphi_0, 1)\,.\end{aligned}$$

By [2], the $*$-holonomy does not depend on the choice of the chain. We can cross out in $\overline{\mathcal{C}}$ two links of the form $(\varphi_0, 1/2)$. We get the chain

$$\overline{\overline{\mathcal{C}}} = (\varphi_0, 0; \varphi_1, (1/2)s_1; \ldots; \varphi_r, 1/2; \varphi'_{r'}, 1/2; \ldots; \varphi'_1, 1-(1/2)s'_1; \varphi_0, 1)\,.$$

Then

$$f_{\mathcal{C}'}^{-1} \circ f_{\mathcal{C}} = f_{\overline{\overline{\mathcal{C}}}} \equiv f_{\overline{\mathcal{C}}} = f_{\widetilde{\mathcal{C}}'}^{-1} \circ f_{\widetilde{\mathcal{C}}} \equiv \mathrm{id}_W$$

by (6). Thus $f_{\mathcal{C}} \equiv f_{\mathcal{C}'}$, so

$$h_{\mathcal{G},\psi}([p_{G_0} \circ \delta]) = h_{\mathcal{G},\psi}([p_{G_0} \circ \delta'])\,. \tag{8}$$

Therefore, the first coordinate of Ψ is correctly defined.

Note that from the properties of the isomorphism $\zeta : \pi_1(F_0, x_0) \to \pi_1(G_0, y_0) \times \pi_1(S^1, 1)$ it follows that

$$[\delta] = [\overline{p_{G_0} \circ \delta}] \cdot [\overline{p_{S^1} \circ \delta}] \tag{9}$$

for every loop δ in F_0 at x_0, where, for arbitrary curves $\alpha : [0,1] \to G_0$, $\varepsilon : [0,1] \to S^1$, we define $\overline{\alpha} : [0,1] \ni s \mapsto \pi(\alpha(s), 0) \in F_0$ and $\overline{\varepsilon} : [0,1] \ni s \mapsto (y_0, \varepsilon(s)) \in G_0 \times S^1 = F_0$.

Since $h_{\mathcal{F},\varphi}$ is a homomorphism, (7) implies

$$h_{\mathcal{F},\varphi}([\overline{p_{G_0} \circ \delta}]) \cdot h_{\mathcal{F},\varphi}([\overline{p_{S^1} \circ \delta}]) = h_{\mathcal{F},\varphi}([\overline{p_{G_0} \circ \delta'}]) \cdot h_{\mathcal{F},\varphi}([\overline{p_{S^1} \circ \delta'}])\,. \tag{10}$$

We have

$$h_{\mathcal{G},\psi}([p_{G_0} \circ \delta]) = h_{\mathcal{G},\psi}([p_{G_0} \circ \delta'])$$

by (8). It follows that $h_{\mathcal{F},\varphi}([\overline{p_{G_0} \circ \delta}]) = h_{\mathcal{F},\varphi}([\overline{p_{G_0} \circ \delta'}])$. Thus, multiplying (10) by the inverse of $h_{\mathcal{F},\varphi}([\overline{p_{G_0} \circ \delta}])$, we obtain

$$h_{\mathcal{F},\varphi}([\overline{p_{S^1} \circ \delta}]) = h_{\mathcal{F},\varphi}([\overline{p_{S^1} \circ \delta'}]),$$

which means that

$$[f_{\gamma;\varphi,\varphi}]^k = [f_{\gamma;\varphi,\varphi}]^{k'}$$

where k, k' are integers such that $[p_{S^1} \circ \delta] = [\beta]^k$, $[p_{S^1} \circ \delta'] = [\beta]^{k'}$. Consequently, the second coordinate of Ψ is correctly defined.

It is easy to check that Ψ is the inverse of Φ. ■

Let Σ be an arbitrary transversal of $\mathcal{G}$ containing y_0 ([5]). Then $\Sigma' = g(\Sigma)$ is a transversal of $\mathcal{G}$ containing y_0. We have

(2.2) Theorem. *There exist a distinguished chart φ of $\mathcal{F}$ around x_0 and a chain $\widetilde{\mathcal{C}} \in C^{\gamma}_{\varphi,\varphi}$ such that the diagram*

$$\begin{array}{ccc} G & \xrightarrow{f_{\tilde{\mathcal{C}}}} & G' \\ {\scriptstyle\sigma}\big\downarrow & & {\scriptstyle\tau}\big\downarrow \\ \Omega & \xrightarrow{g|\Sigma} & \Omega' \end{array} \tag{11}$$

commutes. Here G, G' *are open neighbourhoods of* 0 *in* W_φ, Ω, Ω' *are open neighbourhoods of* y_0 *in* Σ, Σ', *respectively, and the vertical mappings are diffeomorphisms compatible with the induced foliations.*

Proof. Let $x_0 = \pi(y_0, 0)$. Take a distinguished chart ψ of $\mathcal{G}$ around y_0 such that $\psi^{-1}(\{0\} \times W_\psi) \subset \Sigma$ ([2]). Then $\psi' = \psi \circ g$ is a distinguished chart of $\mathcal{G}$ around y_0 by (1.1). Set

$$\widetilde{\mathcal{C}} = (\varphi, 0; \varphi', 1/2; \varphi, 1)$$

where φ and φ' are defined by

$$\varphi : \pi(D_\psi \times (-1/2, 1/2)) \ni \pi(y,t) \mapsto (t, \psi(y)) \in (-1/2, 1/2) \times U_\psi \times W_\psi\,,$$
$$\varphi' : \pi(D_{\psi'} \times (0, 1)) \ni \pi(y,t) \mapsto (t - 1/2, \psi'(y)) \in (-1/2, 1/2) \times U_\psi \times W_\psi\,.$$

We show that $\widetilde{\mathcal{C}}$ is a chain along γ. Obviously, φ is a chart around $\gamma(0) = \gamma(1)$ and φ' is a chart around $\gamma(1/2)$. Thus all three terms of $\widetilde{\mathcal{C}}$ are links. Since

$$\gamma^{-1}(D_\varphi) = [0, 1/2) \cup (1/2, 1] \quad \text{and} \quad \gamma^{-1}(D_{\varphi'}) = (0, 1)\,,$$

we have

$$\gamma^{-1}(D_\varphi)_0 \cap \gamma^{-1}(D_{\varphi'})_{1/2} = (0, 1/2) \neq \emptyset\,,$$
$$\gamma^{-1}(D_{\varphi'})_{1/2} \cap \gamma^{-1}(D_\varphi)_1 = (1/2, 1) \neq \emptyset\,.$$

In order to define a $*$-holonomy diffeomorphism, take the points $\gamma(1/4)$ and $\gamma(3/4)$. By the definition of φ and φ' we have

$$\begin{aligned} f_{\widetilde{\mathcal{C}}}(w) &= \mathrm{Pr}_2\, \varphi\varphi'^{-1}(\mathrm{Pr}_1\, \varphi'\gamma(3/4), \mathrm{Pr}_2\, \varphi'\varphi^{-1}(\mathrm{Pr}_1\, \varphi\gamma(1/4), w)) \\ &= \mathrm{pr}_2\, \psi g \psi^{-1}(0, w)\,. \end{aligned} \tag{12}$$

It is easy to check that the mappings $\sigma : W_\psi \ni w \mapsto \psi^{-1}(0, w) \in \Sigma$ and $\mathrm{pr}_2\, \psi|\Sigma'$ are regular at 0 and y_0, respectively, by the transversality of Σ and Σ'. Consequently, there exist open neighbourhoods G, G' of 0 in W_φ and Ω, Ω' of y_0 in Σ and Σ', respectively, such that σ is a diffeomorphism of G onto Ω and $\mathrm{pr}_2\, \psi|\Sigma'$ is a diffeomorphism of Ω' onto G'. Set $\tau = (\mathrm{pr}_2\, \psi|\Omega')^{-1}$. The diffeomorphisms σ and τ are compatible with the induced foliations since ψ is a distinguished chart.

By (12), we have the commutativity of diagram (11). ∎

Consider the case when $G_0 = \{y_0\}$. Let $\mathcal{A}$ be the set of all diffeomorphisms $k : U \to V$ (U, V are open neighbourhoods of y_0 in N) such that $k(y_0) = y_0$ and k is compatible with the foliations $\mathcal{G}|U$ and $\mathcal{G}|V$. In $\mathcal{A}$ we introduce the relation $\equiv$ quite analogously to that in $\mathcal{A}_{\varphi,\varphi}$ ([2]). Then the set $\mathcal{A}/\equiv$ with multiplication determined by superposition of diffeomorphisms is a group. Moreover, note that $g \in \mathcal{A}$. From Theorems (2.1) and (2.2) we immediately get

(2.4) COROLLARY. *If* $G_0 = \{y_0\}$ *and* $g(y_0) = y_0$, *then* $*\text{-Hol}_{x_0}(\mathcal{F}, \varphi)$ *is isomorphic to the subgroup of* $\mathcal{A}/\equiv$ *generated by the equivalence class of the diffeomorphism* g. ■

References

[1] C. Ehresmann, *Structures feuilletées*, in: Proc. 5th Canad. Math. Congress, Montréal 1961, 109–172.

[2] A. Piątkowski, *A stability theorem for foliations with singularities*, Dissertationes Math. 267 (1988).

[3] —, *On the ∗-holonomy of the inverse image of a Stefan foliation*, Acta Univ. Lodz. Folia Math., to appear.

[4] P. Stefan, *Accessible sets, orbits and foliations with singularities*, Proc. London Math. Soc. 29 (1974), 699–713.

[5] P. Ver Eecke, *Le groupoïde fondamental d'un feuilletage de Stefan*, Publ. Sem. Mat. García de Galdeano, Ser. II, Sec. 3, No. 6, Universidad de Zaragoza, 1986.

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