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KRENGEL–LIN DECOMPOSITION FOR NONCOMPACT GROUPS

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1. Introduction. Let G be a locally compact σ-compact topological Hausdorff group with a right Haar measure λ. The class of all such G with equivalent right and left uniform structures is denoted by **SIN**. A comprehensive review of properties of such groups can be found in [HR] or in a more recent paper [HT]. We note that if G is in addition second countable then $G \in \mathbf{SIN}$ if and only if there exists a (two-sided) invariant metric on G.

We denote by $P(G)$ the convex convolution semigroup of all Borel (Radon) probability measures on G. For a fixed $\mu \in P(G)$, and $f \in L^p(\lambda)$ or $f \in C_0(G)$, we define

$$P_\mu f(x) = \int f(xg)\, d\mu(g).$$

To simplify, and in accordance with [DL2], we write $T(t)$ for P_{δ_t}. Clearly, P_μ is a positive linear contraction on each $L^p(\lambda)$ where $1 \le p \le \infty$ as well as on $C_0(G)$. We notice that P_μ considered as a linear contraction on $L^\infty(\lambda)$ may be treated as adjoint to $P_{\check{\mu}}$, which obviously acts on $L^1(\lambda)$. Moreover, it is doubly stochastic on $L^1(\lambda)$. $P_{\check{\mu}}(\nu) = \nu \star \check{\mu}$ is an extension of $P_{\check{\mu}}$ to $M(G)$, the (AL) Banach lattice of all bounded (Radon) measures on G. Clearly $P_\mu^*(\nu) = \nu \star \mu$, where P_μ is now an operator on $C_0(G)$.

The smallest closed subgroup which contains the topological support $S(\mu)$ of μ is denoted by $G(\mu)$. If $G(\mu) = G$ then we say that μ is *adapted*. We introduce another subgroup which is strongly responsible for asymptotic properties of the iterates P_μ^n. Namely, we denote by $h(\mu)$ the smallest closed subgroup $H \subseteq G$ such that

$$gH = Hg \quad \text{and} \quad \mu(gH) = 1 \quad \text{for each } g \in S(\mu).$$

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In [DL2], $h(\mu)$ is identified as the closed subgroup generated by

$$\bigcup_{n=1}^{\infty}[S(\check{\mu}^{\star n}\star\mu^{\star n})\cup S(\mu^{\star n}\star\check{\mu}^{\star n})].$$

Our notation is taken mainly from [DL2]. In particular, $\check{\mu}$ stands for the symmetric reflection of μ and $\star$ denotes as usual the convolution operation.

In this paper we shall study the following:

Problem. Characterize $\mu\in P(G)$ such that for all $f\in L^2(\lambda)$ we have

$$(\star)\qquad \lim_{n\to\infty}\|P_\mu^n f\|_2=0.$$

On the other hand, if for some $f\in L^2(\lambda)$ the convergence $(\star)$ does not hold, then identify all such $f\in L^2(\lambda)$.

In 1984 Y. Derriennic and M. Lin [DL1] proved that if G is Abelian then $(\star)$ holds for all $f\in L^2(\lambda)$ if and only if $h(\mu)$ is noncompact. We introduce the class $\mathcal{G}_{\mathrm{DL}}$ of all locally compact, σ-compact groups with the property that $(\star)$ holds if and only if $h(\mu)$ is noncompact. It was shown in [B2] that all countable groups belong to $\mathcal{G}_{\mathrm{DL}}$. Subsequently in [B3] it was proved that all Polish, locally compact groups with invariant metrics are in $\mathcal{G}_{\mathrm{DL}}$. Proposition 1 of [B4] gives examples of Lie groups without invariant metrics which still belong to $\mathcal{G}_{\mathrm{DL}}$. Although a full characterization of the class $\mathcal{G}_{\mathrm{DL}}$ is not provided here, we show that $\mathbf{SIN}\subset\mathcal{G}_{\mathrm{DL}}$.

This fact will be used in the third section which includes the main result of the paper. We extend the Krengel–Lin decomposition to the class **SIN**. Namely, we show that if μ is adapted then $L^2(\lambda)=E_0\oplus E_1$ where $E_0=\{f\in L^2(\lambda):\lim_{n\to\infty}\|P_\mu^n f\|_2=0\}$ and $E_1=L^2(G,\Sigma_{\mathrm{d}}(P_\mu),\lambda)$, where according to [F], $\Sigma_{\mathrm{d}}(P_\mu)$ stands for the *deterministic σ-field* of the Markov operator P_μ. We recall that it is defined as

$$\Sigma_{\mathrm{d}}(P_\mu)=\{A\subseteq G: A \text{ is measurable and } \forall_{n\in\mathbb{N}}\ \exists B_n\ P_\mu^n\mathbf{1}_A=\mathbf{1}_{B_n}\}.$$

It is proved in [KL] that the *tail σ-field*

$$\Sigma_{\mathrm{t}}(P_\mu)=\{A\subseteq G: A \text{ is measurable and } \forall_{n\in\mathbb{N}}\ \exists B_n\ P_\mu^n\mathbf{1}_{B_n}=\mathbf{1}_A\}$$

coincides with $\Sigma_{\mathrm{d}}(P_{\check{\mu}})$. We will prove that if E_1 is nontrivial then $\Sigma_{\mathrm{d}}(P_\mu)$ is atomic and consists of classes of the group $h(\mu)$. As a result we get $\Sigma_{\mathrm{d}}(P_\mu)=\Sigma_{\mathrm{d}}(P_{\check{\mu}})$ and consequently, the deterministic part $G_1(\mu)$ defined as

$$\operatorname{ess\,sup}\{A: A\in\Sigma_{\mathrm{d}}(P_\mu)\cap\Sigma_{\mathrm{t}}(P_\mu) \text{ with } \lambda(A)<\infty\}$$

is the whole group G.

Convention. All groups considered in this paper are locally compact, Hausdorff, and σ-compact. Measures are Borel and Radon.

2. Concentrated probabilities. We start with some auxiliary results and provide the necessary definitions. Most of them are taken from [B3]. We say that $\mu \in P(G)$ is *concentrated* if there exist a compact set $K \subseteq G$ and a sequence $g_n \in G$ such that

$$\mu^{\star n}(g_n K) \equiv 1 \quad \text{for all natural } n.$$

A measure $\mu \in P(G)$ is said to be *scattered* if $(\star)$ holds.

The following lemma, which actually is a version of the theorem on convergence of alternating sequences, has been proved in [B3]. In its proof we used Lemma 1.2 and part a) of the proof of Theorem 3.1, both from [C]. It may be easily checked that even though the separability assumption is essential for most of the proofs in [C], the results we quote are valid for general topological groups.

LEMMA 1. *Let $\mu \in P(G)$. Then either μ is scattered, or there exists a probability measure $\varrho \in P(G)$ such that $\check{\mu}^{\star n} \star \mu^{\star n} \Rightarrow \varrho$ in the weak measure topology. Moreover, we have the obvious identity*

$$\check{\mu} \star \varrho \star \mu = \varrho. \tag{1}$$

The above convergence has also been studied in [E] and from the general point of view in [AB].

As in [B3], for a unimodular group G and $\mu \in P(G)$ we define

$$T_\mu f(g) = \iint f(ygz^{-1})\, d\mu(y)\, d\mu(z).$$

We recall that any group $G \in \mathbf{SIN}$ is unimodular (see [HR], p. 278).

LEMMA 2. *Let $\mu \in P(G)$ and $G \in \mathbf{SIN}$. Then the following conditions are equivalent*:

(α) *there exists a measure $\varrho \in P(G)$ such that $\check{\mu} \star \varrho \star \mu = \varrho$,*
(β) *$T_\mu(f_*) = f_*$ for some nonnegative and nonzero $f_* \in L^1(\lambda) \cap L^\infty(\lambda)$.*

Proof. Only $(\alpha)\Rightarrow(\beta)$ needs to be proved. For $0 < \varepsilon < 1/2$ let f be a continuous function with compact support K such that $0 \le f \le 1$ and $\int f\, d\varrho > 1 - \varepsilon$. Then $\int_K T_\mu^n f\, d\varrho > 1 - 2\varepsilon$, so $T_\mu^n f(x_n) > 1 - 2\varepsilon$ for some $x_n \in K$. Since the family $\{T_\mu^n f : n \text{ natural}\}$ is equicontinuous ([HR], (4.14)(g)), there exists a compact neighbourhood W of e such that $T_\mu^n f(g) > 1 - 2\varepsilon$ whenever $g \in W x_n$. Consequently,

$$\int\limits_{WK} T_\mu^n f\, d\lambda \ge (1-2\varepsilon)\lambda(Wx_n) = (1-2\varepsilon)\lambda(W).$$

This implies that the Cesàro L^2 limit of the sequence $T_\mu^n f$ (we denote it by f_*) does not vanish, and is T_μ-invariant. Clearly $f_* \in L^1(\lambda) \cap L^\infty(\lambda)$ since T_μ is doubly stochastic. ∎

The following theorem extends some results of [B3] to **SIN** groups. We will apply this result in the next section. The proof is omitted as it easily follows from Theorem 2 of [B3] and Lemma 2.

THEOREM 1. *Let $\mu \in P(G)$ be adapted and $G \in$ **SIN**. Then the following conditions are equivalent*:

(i) *there is a compact set K and g_n, $\widetilde{g}_n \in G$ so that $\mu^{\star n}(g_n K) = \check{\mu}^{\star n}(\widetilde{g}_n K) \equiv 1$ for all n*,
(ii) *μ is nonscattered*,
(iii) *there exists $f \in L^2(\lambda)$ such that* $\lim_{n\to\infty} \|P_\mu^n f\|_2 > 0$,
(iv) *there exists $\varrho \in P(G)$ such that $\check{\mu} \star \varrho \star \mu = \varrho$*,
(v) *$h(\mu)$ is compact.*

We notice that for noncompact G and adapted $\mu \in P(G)$, if $h(\mu)$ is compact then it has positive Haar measure. This follows from the identity $G/h(\mu) = \mathbb{Z}$, which may be easily inferred from [DL2], Proposition (1.6). Using Baire category methods it may be shown that the interior of $h(\mu)$ is nonempty.

3. Krengel–Lin decomposition. In this section we extend the Krengel–Lin decomposition from compact groups to the class **SIN**. We begin with the following

LEMMA 3. *Let $G \in$ **SIN**. Then for any $\mu \in P(G)$ and $f \in L^2(\lambda)$ the set*

$$G_{f,\mu} = \{t \in G : \lim_{n\to\infty} \|T(t)P_\mu^n f - P_\mu^n f\|_2 = 0\}$$

is a closed subgroup containing $h(\mu)$. As a result, if μ is nonscattered and $\varrho = \lim_{n\to\infty} \check{\mu}^{\star n} \star \mu^{\star n}$, then for any $f \in L^2(\lambda)$ we have

$$\lim_{n\to\infty} \|P_\varrho P_{\check{\mu}}^n f - P_{\check{\mu}}^n f\|_2 = 0. \tag{2}$$

Proof. Without loss of generality we may assume that the considered functions f are taken from $C_0(G) \cap L^2(\lambda)$. Clearly $G_{f,\mu}$ is a subgroup. Now let $t_\alpha \to t_0$, where $t_\alpha \in G_{f,\mu}$. We find that independently of n,

$$\begin{aligned}\|T(t_\alpha)P_\mu^n f &- T(t_0)P_\mu^n f\|_2^2 \\ &= \int \Big| \int (f(xt_\alpha y) - f(xt_0 y))\, d\mu^{\star n}(y)\Big|^2 d\lambda(x) \\ &\le \int\int |f(xt_\alpha y) - f(xt_0 y)|^2\, d\mu^{\star n}(y)\, d\lambda(x) \\ &= \int\int |f(xy^{-1}t_0^{-1}t_\alpha y) - f(x)|^2\, d\lambda(x)\, d\mu^{\star n}(y) \le (\varepsilon/2)^2\end{aligned}$$

for $t_0^{-1}t_\alpha$ close to e, the neutral element of G. Therefore for sufficiently "large" α and n we have

$$\|T(t_0)P_\mu^n f - P_\mu^n f\|_2 \le \|T(t_0)P_\mu^n f - T(t_\alpha)P_\mu^n f\|_2 + \|T(t_\alpha)P_\mu^n f - P_\mu^n f\|_2 \le \varepsilon.$$

This implies that $t_0 \in G_{f,\mu}$.

From the above arguments it is easy to conclude that all sets

$$\{t \in G : \lim_{j\to\infty} \|T(t)P_\mu^{n_j} f - P_\mu^{n_j} f\|_2 = 0\}, \tag{3}$$

where $n_j \to \infty$ are arbitrary, are closed.

Now, let

$$G_{L^2,\mu} = \bigcap_{f\in L^2(\lambda)} G_{f,\mu}.$$

Clearly it is a closed subgroup of G. We prove $h(\mu) \subseteq G_{L^2,\mu}$. It follows from the convergence

$$\lim_{n\to\infty} \int \|T(t)P_\mu^n f - P_\mu^n f\|_2 \, d\eta(t) = 0,$$

where

$$\eta = \sum_{k=1}^{\infty} \frac{1}{2^k} \nu^{\star k} \quad \text{and} \quad \nu = \sum_{k=2}^{\infty} \frac{1}{2^k} (\mu^{\star k} \star \check{\mu}^{\star k} + \check{\mu}^{\star k} \star \mu^{\star k})$$

(see [DL2]), that for any sequence $m_j \to \infty$ there exists a subsequence $n_j \to \infty$ such that

$$\lim_{j\to\infty} \|T(t)P_\mu^{n_j} f - P_\mu^{n_j} f\|_2 = 0, \tag{4}$$

where t runs over a set of full η measure. By (3) the convergence (4) holds for all $t \in S(\eta) = h(\mu)$ and the inclusion $h(\mu) \subseteq G_{L^2,\mu}$ is proved.

The second part of the lemma is an easy consequence of the first one. For nonscattered $\mu \in P(G)$ we have $S(\varrho) \subseteq S(\eta) = h(\check{\mu})$. This implies that for all $f \in L^2(\lambda)$,

$$\|P_\varrho P_{\check{\mu}}^n f - P_{\check{\mu}}^n f\|_2 \le \int \|T(t)P_{\check{\mu}}^n f - P_{\check{\mu}}^n f\|_2 \, d\varrho(t) \to 0. \ \blacksquare$$

PROPOSITION 1. *Let $G \in$ **SIN** and $\mu \in P(G)$ be adapted. If μ is nonscattered then $S(\varrho) = h(\mu)$, where $\varrho = \lim_{n\to\infty} \check{\mu}^{\star n} \star \mu^{\star n}$. If in addition the group G is noncompact, then*

$$\varrho = \frac{\lambda|_{h(\mu)}}{\lambda(h(\mu))} \tag{5}$$

is the normalized Haar measure on $h(\mu)$ and $\tau = \lim_{n\to\infty} \mu^{\star n} \star \check{\mu}^{\star n} = \varrho$.

Proof. Firstly we notice that $e \in S(\varrho)$. For this, let W be a compact neighbourhood of e. By Theorem 1 the group $h(\mu)$ is compact, so it may be covered by a finite union $\bigcup_{j=1}^{p} x_j W$. Hence

$$S(\mu^{\star n}) \subseteq g^n h(\mu) \subseteq \bigcup_{j=1}^{p} g^n x_j W,$$

where $g \in S(\mu)$ is arbitrary. Therefore for any n there exists x_{j_n} such that $\mu^{\star n}(g^n x_{j_n} W) > 1/p$. Consequently,

$$\check{\mu}^{\star n} \star \mu^{\star n}(W^{-1}W) = \check{\mu}^{\star n} \star \mu^{\star n}(W^{-1}x_{j_n}^{-1} g^{-n} g^n x_{j_n} W) \geq \frac{1}{p^2}.$$

Passing with n to infinity we get $\varrho(W^{-1}W) \geq 1/p^2$. Since W is arbitrary, it follows that $e \in S(\varrho)$. In the same way we obtain $e \in S(\tau)$.

Now we apply Theorem 1 (condition (iv)). For all natural n we have

$$\mu^{\star n} \star \check{\mu}^{\star n} \star \varrho \star \mu^{\star n} \star \check{\mu}^{\star n} = \mu^{\star n} \star \varrho \star \check{\mu}^{\star n}.$$

The left side tends to $\tau \star \varrho \star \tau$. By (2) the right side is convergent to τ. As a result,

$$\tau \star \varrho \star \tau = \tau \tag{6}$$

and consequently $\tau \star \varrho \star \tau \star \varrho = \tau \star \varrho$. It is well known (see [H]) that then $\tau \star \varrho$ must be the normalized Haar measure of the compact subgroup $S(\tau \star \varrho)$.

Now we show that $S(\tau)$ is a group. It is obvious that this closed set is symmetric and contains the neutral element of the group. It remains to prove that it is a semigroup. This follows from

$$S(\tau)S(\tau) = S(\tau)eS(\tau) \subseteq S(\tau)S(\varrho)S(\tau) = S(\tau \star \varrho \star \tau) = S(\tau).$$

Notice that $S(\varrho) \subseteq S(\tau)S(\varrho) = S(\tau \star \varrho)$. Hence $\varrho \star \tau \star \varrho = \tau \star \varrho$. Interchanging ϱ with τ in (6) we have $\varrho \star \tau \star \varrho = \varrho$, so $\varrho = \tau \star \varrho$. Since $S(\tau) \subseteq S(\tau \star \varrho)$, it follows that $\tau = \tau \star \varrho \star \tau = \varrho$. The inclusion

$$\bigcup_{n=1}^{\infty} (S(\check{\mu}^{\star n} \star \mu^{\star n}) \cup S(\mu^{\star n} \star \check{\mu}^{\star n})) \subseteq \bigcup_{n=1}^{\infty} (S(\check{\mu}^{\star n} \star \varrho \star \mu^{\star n}) \cup S(\mu^{\star n} \star \tau \star \check{\mu}^{\star n})) = S(\varrho)$$

is obvious and we get $h(\mu) \subseteq S(\varrho)$. The opposite inclusion $S(\varrho) \subseteq h(\mu)$ is always valid so $S(\varrho) = h(\mu)$.

It is noticed in (5) that $\lambda(h(\mu)) > 0$ for noncompact G. Therefore the measure ϱ may be identified as

$$\varrho = \frac{\lambda|_{h(\mu)}}{\lambda(h(\mu))}$$

and the proof is complete. ■

It is well known that on compact groups left and right uniform structures are equivalent. So, the following theorem extends the Krengel–Lin decomposition which is discussed in [KL] only for compact groups.

THEOREM 2. *Let μ be an adapted probability measure on a noncompact group $G \in \mathbf{SIN}$. If μ is nonscattered then*:

(a) $\Sigma_{\mathrm{d}}(P_\mu) = \sigma(\{g^n h(\mu) : n \in \mathbb{Z}$ *and* $g \in S(\mu)$ *arbitrary*$\})$,

(b) $\lim_{n\to\infty} \|P_\mu^n(f - \mathbb{E}_{\mathrm{d}} f)\|_2 = 0$ *for all* $f \in L^2(\lambda)$, *where* $\mathbb{E}_{\mathrm{d}}$ *stands for the conditional expectation operator with respect to* $\Sigma_{\mathrm{d}}(P_\mu)$.

Proof. For natural n and j we have $P_\mu^n \mathbf{1}_{g^j h(\mu)} = \mathbf{1}_{g^{j-n} h(\mu)}$, so

$$\sigma(\{g^n h(\mu) : n \in \mathbb{Z},\ g \in S(\mu)\}) \subseteq \Sigma_{\mathrm{d}}(P_\mu).$$

It follows from [F] that

$$L^2(G, \Sigma_{\mathrm{d}}(P_\mu), \lambda) = \{f \in L^2(\lambda) : P_{\check{\mu}^{\star n} \star \mu^{\star n}} f = f \text{ for any natural } n\}.$$

This gives $P_\varrho f = f$ for $f \in L^2(G, \Sigma_{\mathrm{d}}(P_\mu), \lambda)$ where ϱ is defined in Proposition 1. Using that proposition we have

$$f(x) = \int f(xy)\, d\varrho(y) = \int {}_x f(y)\, d\varrho(y) = \frac{1}{\lambda(h(\mu))} \int\limits_{h(\mu)} {}_x f(y)\, d\lambda(y).$$

If $\widetilde{x} \in x h(\mu)$ then

$$f(\widetilde{x}) = \frac{1}{\lambda(h(\mu))} \int\limits_{h(\mu)} {}_x f(x^{-1}\widetilde{x} y)\, d\lambda(y) = \frac{1}{\lambda(h(\mu))} \int\limits_{h(\mu)} {}_x f(y)\, d\lambda(y) = f(x).$$

This means that f is constant on cosets of $h(\mu)$, and (a) is proved. Here we notice that $\Sigma_{\mathrm{d}}(P_\mu) = \Sigma_{\mathrm{d}}(P_{\check{\mu}}) = \Sigma_{\mathrm{t}}(P_\mu)$. Since $\lambda(h(\mu))$ is finite, the deterministic part is the whole group.

To prove (b) we must show that $\lim_{n\to\infty} \|P_\mu^n f\|_2 = 0$ for any $f \in L^2(\lambda)$ satisfying

$$\int\limits_{gh(\mu)} f\, d\lambda = 0 \quad \text{for all } g \in G.$$

We notice that the above condition is equivalent to $P_\varrho f = 0$. Now,

$$\lim_{n\to\infty} \|P_\mu^n f\|_2^2 = \lim_{n\to\infty} \int P_\mu^{*n} P_\mu^n f \cdot f\, d\lambda = \int P_\varrho f \cdot f\, d\lambda = 0. \ \blacksquare$$

COROLLARY 1. *For any adapted probability measure μ on a noncompact group $G \in \mathbf{SIN}$ there exists a decomposition*

$$L^2(\lambda) = E_0 \oplus L^2(G, \Sigma_{\mathrm{d}}(P_\mu), \lambda),$$

where $\lim_{n\to\infty} \|P_\mu^n f\|_2 = 0$ *for all* $f \in E_0$, *and if nontrivial*, *then* $(L^2(G, \Sigma_{\mathrm{d}}(P_\mu), \lambda), P_\mu)$ *is isomorphic to the bilateral shift* $(\ell^2(\mathbb{Z}), \sigma)$.

In the above decomposition it may happen that $E_0 = L^2(\lambda)$ (μ is scattered) or that E_0 is trivial ($G = \mathbb{Z}$, and $\mu = \delta_1$).

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