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CR-SUBMANIFOLDS OF LOCALLY CONFORMAL KAEHLER MANIFOLDS AND RIEMANNIAN SUBMERSIONS

BY

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We consider a Riemannian submersion $\pi : M \to N$, where M is a CR-submanifold of a locally conformal Kaehler manifold L with the Lee form ω which is strongly non-Kaehler and N is an almost Hermitian manifold. First, we study some geometric structures of N and the relation between the holomorphic sectional curvatures of L and N. Next, we consider the leaves M of the foliation given by $\omega = 0$ and give a necessary and sufficient condition for M to be a Sasakian manifold.

1. Introduction. Let L be an almost Hermitian manifold with almost complex structure J. Let M be a real submanifold of L and TM its tangent bundle. We set $T^{\mathrm{h}}M = TM \cap J(TM)$. Then we have

(a) $JT_p^{\mathrm{h}}M = T_p^{\mathrm{h}}M$ for each $p \in M$.

Let M be a CR-submanifold of an almost Hermitian manifold L such that the differentiable distribution $T^{\mathrm{h}}M : p \to T_p^{\mathrm{h}}M \subset T_pM$ on M satisfies the following conditions:

(b) $JT_p^{\mathrm{v}}M \subset T_pM^{\perp}$ for each $p \in M$, where $T^{\mathrm{v}}M$ is the complementary orthogonal distribution of $T^{\mathrm{h}}M$ in TM;

(c) J interchanges $T^{\mathrm{v}}M$ and $TM^{\perp}$;

(d) there is a Riemannian submersion $\pi : M \to N$ of M onto an almost Hermitian manifold N such that (i) $T^{\mathrm{v}}M$ is the kernel of π_* and (ii) $\pi_* : T_p^{\mathrm{h}}M \to T_{\pi(p)}N$ is a complex isometry for every $p \in M$.

This set up is similar to the set up of symplectic geometry. Indeed, one has the following analogue (due to S. Kobayashi) of the symplectic reduction theorem of Marsden–Weinstein.

THEOREM 1 ([7]). *Let L be a Kaehler manifold. Under the assumptions stated above, N is a Kaehler manifold. If H^L and H^N denote the holomorphic sectional curvatures of L and N, then, for any horizontal unit vector*

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$X \in T^{\mathrm{h}}M$, we have

$$H^L(X) = H^N(\pi_* X) - 4|\sigma(X,X)|^2,$$

where σ denotes the second fundamental form of M in L.

In the above theorem, L is a Kaehler manifold. In this paper, we consider the case where L is a locally conformal Kaehler manifold which is strongly non-Kaehler. Then $T^{\mathrm{v}}M$ is integrable [3]. Let B^{h}, B^{v} and $B^{\perp}$ be the horizontal part, the vertical part and the normal part of the Lee vector field B respectively. First, we show the following theorem:

THEOREM 2. *Under the assumptions* (a)–(d), *assume further that L is a locally conformal Kaehler manifold. Then the Lee vector field $B \in T^{\mathrm{h}}M \oplus TM^{\perp}$ and for any horizontal unit vector $X \in T^{\mathrm{h}}M$, we have*

$$H^L(X) = H^N(\pi_* X) - 3|A_X JX|^2 - |\sigma(X,X)|^2,$$

where σ is the second fundamental form of M in L and A is the integrability tensor with respect to π. Moreover, if we assume in addition that the horizontal component B^{h} of the Lee vector field B is basic and $\dim N \geq 4$ *then N is also a locally conformal Kaehler manifold. In particular, if L is a generalized Hopf manifold and if the Lee vector field B is basic and horizontal then N is also a generalized Hopf manifold.*

Next, we consider the case where the Lee vector field $B \in TM^{\perp}$.

THEOREM 3. *Under the assumptions* (a)–(d), *if L is a locally conformal Kaehler manifold and $B \in TM^{\perp}$, then N is a Kaehler manifold.*

THEOREM 4. *Under the assumptions* (a)–(d), *if L is a P_0K-manifold and M is a totally umbilical submanifold whose mean curvature vector is parallel and $B \in TM^{\perp}$, then N is a locally symmetric Kaehler manifold and the holomorphic sectional curvature H^N of N is $H^N(\widetilde{X}) > 0$, where $\widetilde{X}$ is any unit tangent vector.*

Next, let L be a locally conformal Kaehler manifold which is strongly non-Kaehler, ω the Lee form and $\mathcal{M}$ the distribution defined by $\omega = 0$. Since $d\omega = 0$, $\mathcal{M}$ is integrable. Let M be a maximal connected integral submanifold of $\mathcal{M}$, that is, M is an orientable hypersurface of L. Then M is a CR-submanifold satisfying (a)–(c) such that $TM^{\perp} = \{B\}$ and $T^{\mathrm{v}}M = \{JB\}$. In the case where L is P_0K-manifold, we get the following theorem.

THEOREM 5. *Let L be a complete P_0K-manifold and M a maximal connected integral submanifold of $\mathcal{M}$. Let N be an almost Hermitian manifold and $\pi : M \to N$ be a Riemannian submersion satisfying the condition* (d). *Then N is isometric to the complex projective space $P_m(\mathbb{C})$.*

It is known that every orientable hypersurface of an almost Hermitian manifold has an almost contact metric structure (ϕ, V, η, g) (see [2], [17]). We show the following theorem:

THEOREM 6. *Let L be a locally conformal Kaehler manifold and M a maximal connected integral submanifold of $\mathcal{M}$. Then (M, ϕ, V, η, g) is a Sasakian manifold if and only if*

$$k = \big(\tfrac{1}{2}\sqrt{\omega(B)} - 1\big)g + \alpha\eta \otimes \eta,$$

where k is the second fundamental form of M and α is a function.

Remark 1. (I) In [17], I. Vaisman proved that if L is a locally conformal Kaehler manifold with parallel Lee form, then a maximal connected integral submanifold M of $\mathcal{M}$ is a totally geodesic submanifold of L and M is a Sasakian manifold. In Theorem 6, we obtain a necessary and sufficient condition for M to be a Sasakian manifold without the assumption that the Lee form is parallel.

(II) It is known that if M is an orientable hypersurface of a Kaehler manifold L, then the induced almost contact metric structure (ϕ, V, η, g) is Sasakian if and only if $k = -g + \alpha\eta \otimes \eta$, where k is the second fundamental form of M and α is a function [14]. When L is a locally conformal Kaehler manifold, from Theorem 6 we obtain a similar result.

2. Preliminaries. Let L be an almost Hermitian manifold with metric g, complex structure J and fundamental 2-form Ω. The manifold L will be called a *locally conformal Kaehler manifold* if every $x \in L$ has an open neighborhood U with a differentiable function $\gamma : U \to \mathbb{R}$ such that $g'_U = e^{-\gamma} g|_U$ is a Kaehler metric on U. The locally conformal Kaehler manifold L is characterized by

$$d\Omega = \omega \wedge \Omega, \qquad d\omega = 0, \tag{1}$$

where ω is a globally defined 1-form on L. We call ω the *Lee form*. Since for $\dim L = 2$ we have $d\Omega = 0$, we may suppose $\dim L \geq 4$. Next we define the *Lee vector field* B by

$$g(X, B) = \omega(X). \tag{2}$$

The *Weyl connection* ${}^{\mathrm{W}}\nabla$ is the linear connection defined by

$${}^{\mathrm{W}}\nabla_X Y := \nabla_X Y - \tfrac{1}{2}\omega(X)Y - \tfrac{1}{2}\omega(Y)X + \tfrac{1}{2}g(X, Y)B, \tag{3}$$

where ∇ is the Levi-Civita connection of g. It is shown in [15] that an almost Hermitian manifold L is a locally conformal Kaehler if and only if there is a closed 1-form ω on L such that

$${}^{\mathrm{W}}\nabla_X J = 0. \tag{4}$$

The equation (4) is equivalent to

$$(5)\quad \nabla_X JY - \tfrac{1}{2}\omega(JY)X + \tfrac{1}{2}g(X, JY)B = J\nabla_X Y - \tfrac{1}{2}\omega(Y)JX + \tfrac{1}{2}g(X,Y)JB,$$

where X and Y are vector fields on L.

The *Riemannian curvature tensor field* R^L of L is given by

$$(6)\qquad R^L(X,Y) = \nabla_X\nabla_Y - \nabla_Y\nabla_X - \nabla_{[X,Y]}.$$

We set

$$(7)\qquad R^L(W,Z,X,Y) = g(R^L(X,Y)Z, W).$$

Let ${}^{\mathrm{W}}R$ be the curvature tensor field of the Weyl connection ${}^{\mathrm{W}}\nabla$. Then

$$\begin{aligned}(8)\quad & {}^{\mathrm{W}}R(X,Y)Z \\ &= R^L(X,Y)Z - \tfrac{1}{2}\big\{\big[(\nabla_X\omega)Z + \tfrac{1}{2}\omega(X)\omega(Z)\big]Y \\ &\quad - \big[(\nabla_Y\omega)Z + \tfrac{1}{2}\omega(Y)\omega(Z)\big]X - g(Y,Z)\big(\nabla_X B + \tfrac{1}{2}\omega(X)B\big) \\ &\quad + g(X,Z)\big(\nabla_Y B + \tfrac{1}{2}\omega(Y)B\big)\big\} - \tfrac{1}{4}|\omega|^2(g(Y,Z)X - g(X,Z)Y),\end{aligned}$$

where X, Y and Z are any vector fields on L [18].

A locally conformal Kaehler manifold (L, J, g) is said to be a *generalized Hopf manifold* if the Lee form is *parallel*, that is, $\nabla\omega = 0$ ($\omega \neq 0$). A generalized Hopf manifold is called a P_0K-*manifold* if the Weyl curvature tensor is zero, that is, ${}^{\mathrm{W}}R(X,Y) = 0$. In this paper, we consider the case where L is a locally conformal Kaehler manifold which is strongly non-Kaehler in the sense that $d\Omega \neq 0$ (and so $\omega \neq 0$) at every point of L.

The *Hopf manifolds* are defined as $H^n_\lambda = (\mathbb{C}^n - \{0\})/\Delta_\lambda$, $n > 1$, where $\mathbb{C}$ is the complex plane, $\lambda \in \mathbb{C}$, $|\lambda| \neq 0, 1$ and Δ_λ is the group generated by the transformation $z \mapsto \lambda z$, $z \in \mathbb{C}^n - \{0\}$ (see [15]). On the manifold H^n_λ, we consider the Hermitian metric

$$ds^2 = \frac{1}{\sum_{k=1}^n z^k \bar{z}^k}\sum_{j=1}^n dz^j \otimes d\bar{z}^j,$$

where z^j ($j = 1, \ldots, n$) are complex Cartesian coordinates on $\mathbb{C}^n$. The Hopf manifold H^n_λ is an example of a P_0K-manifold which is strongly non-Kaehler.

Let M be a submanifold of a Riemannian manifold L. We denote by the same g the Riemannian metric tensor field induced on M from that of L. Let ∇^M denote covariant differentiation of M. Then the Gauss formula for M is written as

$$(9)\qquad \nabla_X Y = \nabla^M_X Y + \sigma(X,Y)$$

for any vector fields X, Y tangent to M, where σ denotes the second fundamental form of M in L. Let M be an n-dimensional submanifold of L.

The *mean curvature vector* ϱ of M is defined by $\varrho = \frac{1}{n}\,\mathrm{trace}(\sigma)$. A submanifold M is called *totally umbilical* if the second fundamental form σ satisfies $\sigma(X,Y) = g(X,Y)\varrho$. A submanifold M is called *totally geodesic* if the second fundamental form vanishes identically, that is, $\sigma = 0$.

Let R^M be the Riemannian curvature tensor field of M. Then we have the equation of Gauss

$$R^L(W,Z,X,Y) = R^M(W,Z,X,Y) + g(\sigma(X,Z),\sigma(Y,W)) - g(\sigma(Y,Z),\sigma(X,W)). \tag{10}$$

Let N be an almost Hermitian manifold with almost complex structure J' and $\pi : M \to N$ a Riemannian submersion such that $TM \cap J(TM)$ is the horizontal part of TM and, at each point $p \in M$, π_* is a complex isometry of $T_p^{\mathrm{h}}M = T_pM \cap J(T_pM)$ onto $T_{\pi(p)}N$. Let X denote a tangent vector at $p \in M$. Then X decomposes as $\mathcal{V}X + \mathcal{H}X$, where $\mathcal{V}X$ is tangent to the fiber through p and $\mathcal{H}X$ is perpendicular to it. We define tensors T and A associated with the submersion by

$$T_XY := \mathcal{V}\nabla^M_{\mathcal{V}X}\mathcal{H}Y + \mathcal{H}\nabla^M_{\mathcal{V}X}\mathcal{V}Y, \tag{11}$$

$$A_XY := \mathcal{V}\nabla^M_{\mathcal{H}X}\mathcal{H}Y + \mathcal{H}\nabla^M_{\mathcal{H}X}\mathcal{V}Y, \tag{12}$$

for any vector fields X, Y on M. Then T and A have the following properties [11].

(i) T_X and A_X are skew symmetric linear operators on the tangent space of M, and interchange the horizontal and vertical parts.

(ii) $T_X = T_{\mathcal{V}X}$ while $A_X = A_{\mathcal{H}X}$.

(iii) For V, W vertical, T_VW is symmetric, that is, $T_VW = T_WV$. For X, Y horizontal, A_XY is skew symmetric, that is, $A_XY = -A_YX$.

A vector field X on M is said to be *basic* if X is horizontal and π-related to a vector field $\widetilde{X}$ on N. Every vector field $\widetilde{X}$ on N has a unique horizontal lift X to M, and X is basic. We denote it by $X = \mathrm{h.l.}(\widetilde{X})$.

LEMMA 1 ([11]). *Let X and Y be any basic vector fields on M. Then*

(i) $g(X,Y) = \overline{g}(\widetilde{X},\widetilde{Y}) \circ \pi$;

(ii) $\mathcal{H}[X,Y]$ *is the basic vector field corresponding to* $[\widetilde{X},\widetilde{Y}]$;

(iii) $\mathcal{H}\nabla^M_XY$ *is the basic vector field corresponding to* $\nabla^N_{\widetilde{X}}\widetilde{Y}$, *where* $\overline{g}$ *is the metric of* N *and* ∇^N *is the covariant differentiation on* N.

Let R^N denote the curvature tensor field of N. The horizontal lift of the curvature tensor R^N of N will also be denoted by R^N. We recall the following curvature identity which will be needed in the sequel:

$$R^M(W,Z,X,Y) = R^N(\widetilde{W},\widetilde{Z},\widetilde{X},\widetilde{Y}) - g(A_YZ,A_XW) + g(A_XZ,A_YW) + 2g(A_XY,A_ZW), \tag{13}$$

where X, Y, Z, W are any basic vector fields on M. As before, this result is proven in [11].

Let X and Y be any basic vector fields on M. We define the operator $\overline{\nabla}^N$ by

$$\overline{\nabla}^N_X Y := \mathcal{H}\nabla^M_X Y. \tag{14}$$

Then, by Lemma 1(iii), $\overline{\nabla}^N_X Y$ is a basic vector field and

$$\pi_*(\overline{\nabla}^N_X Y) = \nabla^N_{\tilde{X}} \widetilde{Y}. \tag{15}$$

Next, we give the definition of a Sasakian manifold. A Riemannian manifold (M, g) is said to be a *Sasakian manifold* if there exist a tensor field ϕ of type $(1,1)$, a unit vector field V and a 1-form η such that

$$\begin{aligned} &\phi V = 0, \quad \eta(\phi X) = 0, \quad \phi^2 X = -X + \eta(X)V, \\ &g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \\ &(\nabla^M_X \phi)Y = g(X,Y)V - \eta(Y)X, \end{aligned} \tag{16}$$

for any vector fields X, Y on M [2].

3. Proof of Theorem 2. We put $B = B^{\mathrm{h}} + B^{\mathrm{v}} + B^{\perp}$, where $B^{\mathrm{h}}, B^{\mathrm{v}}$ and $B^{\perp}$ are the horizontal part, the vertical part and the normal part of the Lee vector field B respectively.

From (9) and (12), for any horizontal vector fields X and Y, we have

$$\nabla_X Y = \mathcal{H}\nabla^M_X Y + A_X Y + \sigma(X,Y). \tag{17}$$

Since M is a CR-submanifold of L, using (5) and (17), we obtain

$$\begin{aligned} \mathcal{H}\nabla^M_X JY - \tfrac{1}{2}\omega(JY)X + \tfrac{1}{2}g(X,JY)B^{\mathrm{h}} = J\mathcal{H}\nabla^M_X Y - \tfrac{1}{2}\omega(Y)JX& \\ + \tfrac{1}{2}g(X,Y)JB^{\mathrm{h}} \in T^{\mathrm{h}}M&, \end{aligned} \tag{18}$$

$$A_X JY + \tfrac{1}{2}g(X,JY)B^{\mathrm{v}} = J\sigma(X,Y) + \tfrac{1}{2}g(X,Y)JB^{\perp} \in T^{\mathrm{v}}M, \tag{19}$$

$$\sigma(X,JY) + \tfrac{1}{2}g(X,JY)B^{\perp} = JA_X Y + \tfrac{1}{2}g(X,Y)JB^{\mathrm{v}} \in TM^{\perp}, \tag{20}$$

where X and Y are any horizontal vector fields on M.

From (19) and (20), for any horizontal vector fields X and Y, we obtain

$$\sigma(JX,JY) = \sigma(X,Y) + g(JX,Y)JB^{\mathrm{v}}, \qquad A_{JX}JY = A_X Y - g(X,Y)B^{\mathrm{v}},$$

because $A_X Y$ is skew symmetric. In the last equation, we set $X = Y$; then we have $A_{JX}JX = A_X X - g(X,X)B^{\mathrm{v}}$. Since $A_X X = 0$, we obtain $B^{\mathrm{v}} = 0$.

Since $B^{\mathrm{v}} = 0$, for any horizontal vector fields X and Y, we obtain

$$\sigma(JX,JY) = \sigma(X,Y), \qquad A_{JX}JY = A_X Y. \tag{21}$$

Next, we compare the holomorphic sectional curvatures of L and N. We set $Z = JW$ and $Y = JX$ in (10) and (13) to obtain

$$
\begin{aligned}
(22)\quad & R^L(W, JW, X, JX) \\
&= R^N(\widetilde{W}, J'\widetilde{W}, \widetilde{X}, J'\widetilde{X}) \\
&\quad - g(A_{JX}JW, A_X W) - g(A_X JW, A_W JX) \\
&\quad - 2g(A_X JX, A_W JW) + g(\sigma(X, JW), \sigma(JX, W)) \\
&\quad - g(\sigma(JX, JW), \sigma(X, W)),
\end{aligned}
$$

where X and W are any basic vector fields on M.

Setting $X = W$ in the above equation, using (21), by $\sigma(X, JX) = 0$, we obtain

$$
(23)\quad R^L(X, JX, X, JX) = R^N(\widetilde{X}, J'\widetilde{X}, \widetilde{X}, J'\widetilde{X}) - 3|A_X JX|^2 - |\sigma(X, X)|^2.
$$

Thus, for any horizontal unit vector X on M, we obtain

$$
(24)\qquad H^L(X) = H^N(\pi_* X) - 3|A_X JX|^2 - |\sigma(X, X)|^2.
$$

Now, we assume that the horizontal component B^{h} of the Lee vector field B is basic and $\dim N \geq 4$. We put $\widetilde{B} := \pi_*(B^{\mathrm{h}})$. Let ω' be the 1-form on M induced by the Lee form ω on L. For any vector field $\widetilde{X}$ on N, we set $\widetilde{\omega}(\widetilde{X}) := \overline{g}(\widetilde{X}, \widetilde{B})$. Then $(\pi^*\widetilde{\omega})(X) = \omega'(X)$, where X is any basic vector field. Since π^* commutes with d and π is a Riemannian submersion, $\widetilde{\omega}$ is closed.

From the definition of $\widetilde{\omega}$, we obtain

$$
(25)\qquad \overline{g}(\widetilde{X}, \widetilde{B}) \circ \pi = \widetilde{\omega}(\widetilde{X}) \circ \pi = \omega'(X) = \omega(X) = g(X, B),
$$

where $\widetilde{X}$ is any vector field on N and $X = \mathrm{h.l.}(\widetilde{X})$. We define the Weyl connection ${}^{\mathrm{W}}\nabla^N$ of N by

$$
(26)\qquad {}^{\mathrm{W}}\nabla^N_{\widetilde{X}}\widetilde{Y} = \nabla^N_{\widetilde{X}}\widetilde{Y} - \tfrac{1}{2}\widetilde{\omega}(\widetilde{X})\widetilde{Y} - \tfrac{1}{2}\widetilde{\omega}(\widetilde{Y})\widetilde{X} + \tfrac{1}{2}\overline{g}(\widetilde{X}, \widetilde{Y})\widetilde{B}.
$$

From Lemma 1, (18), (25) and (26), for any vector fields $\widetilde{X}$, $\widetilde{Y}$ and $\widetilde{Z}$, we obtain

$$
\begin{aligned}
(27)\quad & \overline{g}(({}^{\mathrm{W}}\nabla^N_{\widetilde{X}} J')\widetilde{Y}, \widetilde{Z}) \circ \pi \\
&= \overline{g}({}^{\mathrm{W}}\nabla^N_{\widetilde{X}} J'\widetilde{Y}, \widetilde{Z}) \circ \pi - \overline{g}(J'({}^{\mathrm{W}}\nabla^N_{\widetilde{X}}\widetilde{Y}), \widetilde{Z}) \circ \pi \\
&= g\big(\mathcal{H}\nabla^M_X JY - \tfrac{1}{2}\omega(JY)X + \tfrac{1}{2}g(X, JY)B \\
&\qquad - J\mathcal{H}\nabla^M_X Y + \tfrac{1}{2}\omega(Y)JX - \tfrac{1}{2}g(X, Y)JB, Z\big) = 0,
\end{aligned}
$$

where X, Y and Z are the horizontal lifts of $\widetilde{X}$, $\widetilde{Y}$ and $\widetilde{Z}$ respectively. Therefore ${}^{\mathrm{W}}\nabla^N_{\widetilde{X}} J' = 0$, that is, N is a locally conformal Kaehler manifold.

Let L be a generalized Hopf manifold and let the Lee vector field B be basic and horizontal. Since the Lee form ω of L is parallel, for any vector field X tangent to M, we have $\nabla_X B = 0$. Hence, by $\nabla_X B = \nabla^M_X B + \sigma(X, B)$, we have $\nabla^M_X B = 0$. From Lemma 1 and (25), we obtain

$$\begin{aligned}\overline{g}(\nabla^N_{\widetilde{X}}\widetilde{B}, \widetilde{Y}) \circ \pi &= (\widetilde{X}\overline{g}(\widetilde{B}, \widetilde{Y}) - \overline{g}(\widetilde{B}, \nabla^N_{\widetilde{X}}\widetilde{Y})) \circ \pi \\ &= Xg(B, Y) - g(B, \nabla^M_X Y) = g(\nabla^M_X B, Y) = 0,\end{aligned}$$

where $\widetilde{X}$, $\widetilde{Y}$ are any vector fields tangent to N, and X, Y are their horizontal lifts. Hence we obtain $\nabla^N_{\widetilde{X}}\widetilde{B} = 0$, that is, N is a generalized Hopf manifold.

R e m a r k 2. In this theorem, let L be a locally conformal Kaehler manifold and M a totally umbilical CR-submanifold of L and the Lee vector field $B \in T^{\mathrm{h}}M$. It is known that if B is tangent to M, then a totally umbilical proper CR-submanifold M of L is totally geodesic [6]. For $X, Y \in T^{\mathrm{h}}M$, we have $A_X Y = \frac{1}{2}\mathcal{V}[X, Y]$ (see [11]). Therefore, using (19), we see that the horizontal distribution $T^{\mathrm{h}}M$ is integrable and the integral submanifolds are totally geodesic.

4. Proof of Theorem 3. Since $B \in TM^{\perp}$, for any vector field X tangent to M, we have $\omega(X) = 0$. Since M is a CR-submanifold of L, (5) implies

$$\nabla_X JY + \tfrac{1}{2}g(X, JY)B = J\nabla_X Y + \tfrac{1}{2}g(X, Y)JB, \tag{28}$$

where X and Y are horizontal vector fields. Using (17) and (28), we obtain

$$\mathcal{H}\nabla^M_X JY = J\mathcal{H}\nabla^M_X Y \in T^{\mathrm{h}}M, \tag{29}$$

$$A_X JY = J\sigma(X, Y) + \tfrac{1}{2}g(X, Y)JB \in T^{\mathrm{v}}M, \tag{30}$$

$$\sigma(X, JY) + \tfrac{1}{2}g(X, JY)B = JA_X Y \in TM^{\perp}, \tag{31}$$

where X and Y are any horizontal vector fields on M.

Since π_* is a complex isometry, we have $\pi_* \circ J = J' \circ \pi_*$. Therefore, if X is a basic vector field, JX is also a basic vector field. Using Lemma 1, (15) and (29), we have

$$\nabla^N_{\widetilde{X}} J'\widetilde{Y} = J'\nabla^N_{\widetilde{X}}\widetilde{Y}.$$

Hence N is a Kaehler manifold.

5. Proof of Theorem 4. Since L is a P_0K-manifold, we have ${}^{\mathrm{W}}R = 0$ and $\nabla\omega = 0$. We set $c := |\omega|/2$. Since $\nabla\omega = 0$, we have $\nabla B = 0$ and $c = \text{constant}$ (see [17]). From (8), we have

$$\begin{aligned}R^L(X, Y)Z = \tfrac{1}{4}\{&[\omega(X)Y - \omega(Y)X]\omega(Z) \\ &+ [g(X, Z)\omega(Y) - g(Y, Z)\omega(X)]B\} \\ &+ c^2(g(Y, Z)X - g(X, Z)Y).\end{aligned} \tag{32}$$

Using $\nabla\omega = 0$ and $\nabla B = 0$, we obtain $\nabla R^L = 0$ (see [6]). Since $B \in TM^{\perp}$, using (10) and (32), for any vector fields X, Y, Z and W tangent to M, we have

$$R^M(W,Z,X,Y) = c^2(g(Y,Z)g(X,W) - g(X,Z)g(Y,W)) + g(\sigma(Y,Z),\sigma(X,W)) - g(\sigma(X,Z),\sigma(Y,W)). \tag{33}$$

Since M is a totally umbilical submanifold of L and the mean curvature vector is parallel, the second fundamental form is parallel. Thus M is a locally symmetric space. Using (33) and $\sigma(X,Y) = g(X,Y)\varrho$, for $X,Y,Z \in T^{\mathrm{h}}M$ and $V \in T^{\mathrm{v}}M$, we obtain $R^M(X,Y,Z,V) = 0$. Moreover, since $\sigma(X,Y) = g(X,Y)\varrho$ and $B \in TM^{\perp}$, the fibers of π are totally geodesic [6]. Hence the reflections $\varphi_{\pi^{-1}(x)}$ with respect to the fibers are isometries [4]. Therefore N is a locally symmetric space [4], [9]. From Theorem 3, N is a Kaehler manifold. Using (32), for any horizontal unit vector X, we get $H^L(X) = c^2$. Thus, from (24), we have $H^N(\widetilde{X}) > 0$, where $\widetilde{X}$ is any unit tangent vector.

6. Proof of Theorem 5. Since L is a P_0K-manifold, the maximal integral submanifold M of $\mathcal{M}$ is a totally geodesic submanifold of L (see [17]). From (33), we have

$$R^M(W,Z,X,Y) = c^2(g(Y,Z)g(X,W) - g(X,Z)g(Y,W)), \tag{34}$$

where X, Y, Z and W are any vector fields tangent to M and c $(= |\omega|/2)$ is constant. Using (13) and (34), we obtain

$$R^N(\widetilde{X},\widetilde{Y},\widetilde{X},\widetilde{Y}) = c^2(g(Y,Y)g(X,X) - g(X,Y)g(X,Y)) + 3g(A_XY, A_XY), \tag{35}$$

where $\widetilde{X}$, $\widetilde{Y}$ are vector fields on N and X, Y are their respective horizontal lifts. For each plane p in the tangent space T_xN, the sectional curvature $K^N(p)$ of N is

$$K^N(p) = c^2 + 3|A_XY|^2, \tag{36}$$

where $\widetilde{X}$, $\widetilde{Y}$ is an orthonormal basis for p. Let $\{\widetilde{X_i}, J'\widetilde{X_i}\}$ $(i = 1,\dots,m)$ be an orthonormal basis for T_xN, $\dim(N) = 2m$. We denote the Ricci tensor of N by Ric^N. Then

$$\mathrm{Ric}^N(\widetilde{X},\widetilde{X}) = \sum_{i=1}^{m} R^N(\widetilde{X_i},\widetilde{X},\widetilde{X_i},\widetilde{X}) + \sum_{i=1}^{m} R^N(J'\widetilde{X_i},\widetilde{X},J'\widetilde{X_i},\widetilde{X}).$$

From (30) and (31), we get $A_{X_i}X_j = 0$, $A_{JX_i}X_j = 0$ $(i \neq j)$, $A_{JX_i}X_i = -\frac{1}{2}JB$ and $A_{JX_i}JX_j = 0$, $(i,j = 1,\dots,m)$. Now, we compute the scalar curvature $s^N(x)$ of N:

$$s^N(x) = \sum_{j=1}^{m} \mathrm{Ric}^N(\widetilde{X_j},\widetilde{X_j}) + \sum_{j=1}^{m} \mathrm{Ric}^N(J'\widetilde{X_j},J'\widetilde{X_j}) = c^2(4m^2 + 6m).$$

Since L is complete and M is a totally geodesic submanifold of L, M is complete. Since M is complete and $\pi : M \to N$ is a Riemannian submersion, N is complete [11]. From Theorem 3, N is a Kaehler manifold.

It is known that a complete Kaehler manifold with constant scalar curvature and with positive sectional curvature is isometric to the complex projective space $P_m(\mathbb{C})$ (see [1]). Therefore N is isometric to $P_m(\mathbb{C})$.

7. Proof of Theorem 6. For the Lee vector field B, we set

$$C := B/\sqrt{g(B,B)}. \tag{37}$$

We define a vector field V, a 1-form η and a tensor field ϕ of type $(1,1)$ on M by

$$V = JC, \quad \eta(X) = g(X,V), \quad JX = \phi X - \eta(X)C. \tag{38}$$

Since L is a Hermitian manifold, (M,ϕ,V,η,g) admits an almost contact metric structure [2], [17].

Let $\mathcal{H}X$ and $\mathcal{V}X$ be the $T^{\mathrm{h}}M$ part and $T^{\mathrm{v}}M$ part of $X \in TM$ respectively. We set $\sigma(X,Y) = -k(X,Y)C$. From (5), for any vector field X in $T^{\mathrm{h}}M$, we obtain

$$\nabla_V JX = J\nabla_V X. \tag{39}$$

Using $\nabla_V X = \nabla^M_V X - k(V,X)C$, by (39), we have the following equations:

$$\mathcal{H}\nabla^M_V JX = J\mathcal{H}\nabla^M_V X \in T^{\mathrm{h}}M, \tag{40}$$

$$\mathcal{V}\nabla^M_V JX = -k(V,X)V \in T^{\mathrm{v}}M, \tag{41}$$

$$-k(V,JX)C = J\mathcal{V}\nabla^M_V X \in TM^{\perp}, \tag{42}$$

where X is any vector field in $T^{\mathrm{h}}M$. From (38) and (40), for any vector fields X and Y in $T^{\mathrm{h}}M$, we obtain

$$\begin{aligned} g((\nabla^M_V\phi)X,Y) &= g(\nabla^M_V\phi X - \phi\nabla^M_V X, Y) \\ &= g(\mathcal{H}\nabla^M_V JX - J\mathcal{H}\nabla^M_V X, Y) = 0. \end{aligned} \tag{43}$$

From the $T^{\mathrm{h}}M$ part of (5) and (38), for any vector fields X and Y in $T^{\mathrm{h}}M$, we obtain

$$\mathcal{H}\nabla^M_X\phi Y = \phi\mathcal{H}\nabla^M_X Y. \tag{44}$$

Now, for any vector fields X and Y tangent to M, we assume $k(X,Y) = \big(\frac{1}{2}\sqrt{\omega(B)} - 1\big)g(X,Y) + \alpha\eta(X)\eta(Y)$. Let V and W be any vector fields in $T^{\mathrm{v}}M$ and X be any vector field in $T^{\mathrm{h}}M$. From (42), we obtain $\mathcal{V}\nabla^M_V X = 0$, because $k(X,V) = 0$. Using (5), we obtain $g(J\mathcal{H}\nabla^M_V W, X) = g(\mathcal{H}\nabla_V JW, X) = -g(\sigma(V,X), JW) = 0$. Hence, we get $\mathcal{H}\nabla^M_V W = 0$.

We shall prove that (M,ϕ,V,η,g) admits a Sasakian structure. Let X, Y and Z be any vector fields tangent to M. Using (44) and the above result, we have

$$
\begin{aligned}
&g((\nabla^M_X\phi)Y,Z)\\
&\quad= g((\nabla^M_X\phi)Y,\mathcal{H}Z)+g((\nabla^M_X\phi)Y,\mathcal{V}Z)\\
&\quad= g(\nabla^M_X\phi Y,\mathcal{H}Z)-g(\phi\nabla^M_X Y,\mathcal{H}Z)+g(\nabla^M_X\phi Y,\mathcal{V}Z)\\
&\qquad -g(\phi\nabla^M_X Y,\mathcal{V}Z)\\
&\quad= g(\nabla^M_{\mathcal{H}X}\phi\mathcal{H}Y,\mathcal{H}Z)+g(\nabla^M_{\mathcal{H}X}\phi\mathcal{V}Y,\mathcal{H}Z)+g(\nabla^M_{\mathcal{V}X}\phi\mathcal{H}Y,\mathcal{H}Z)\\
&\qquad +g(\nabla^M_{\mathcal{V}X}\phi\mathcal{V}Y,\mathcal{H}Z)-g(\phi\nabla^M_{\mathcal{H}X}\mathcal{H}Y,\mathcal{H}Z)-g(\phi\nabla^M_{\mathcal{H}X}\mathcal{V}Y,\mathcal{H}Z)\\
&\qquad -g(\phi\nabla^M_{\mathcal{V}X}\mathcal{H}Y,\mathcal{H}Z)-g(\phi\nabla^M_{\mathcal{V}X}\mathcal{V}Y,\mathcal{H}Z)+g(\nabla^M_{\mathcal{H}X}\phi\mathcal{H}Y,\mathcal{V}Z)\\
&\qquad +g(\nabla^M_{\mathcal{H}X}\phi\mathcal{V}Y,\mathcal{V}Z)+g(\nabla^M_{\mathcal{V}X}\phi\mathcal{H}Y,\mathcal{V}Z)+g(\nabla^M_{\mathcal{V}X}\phi\mathcal{V}Y,\mathcal{V}Z)\\
&\qquad -g(\phi\nabla^M_{\mathcal{H}X}\mathcal{H}Y,\mathcal{V}Z)-g(\phi\nabla^M_{\mathcal{H}X}\mathcal{V}Y,\mathcal{V}Z)-g(\phi\nabla^M_{\mathcal{V}X}\mathcal{H}Y,\mathcal{V}Z)\\
&\qquad -g(\phi\nabla^M_{\mathcal{V}X}\mathcal{V}Y,\mathcal{V}Z)\\
&\quad= g(\nabla^M_{\mathcal{V}X}\phi\mathcal{H}Y,\mathcal{H}Z)-g(\phi\nabla^M_{\mathcal{V}X}\mathcal{H}Y,\mathcal{H}Z)+g(\nabla^M_{\mathcal{H}X}\phi\mathcal{H}Y,\mathcal{V}Z)\\
&\qquad -g(\phi\nabla^M_{\mathcal{H}X}\mathcal{V}Y,\mathcal{H}Z)\\
&\quad= g((\nabla^M_{\mathcal{V}X}\phi)\mathcal{H}Y,\mathcal{H}Z)+g(V,\mathcal{V}Z)g(\nabla^M_{\mathcal{H}X}\phi\mathcal{H}Y,V)\\
&\qquad -g(V,\mathcal{V}Y)g(\nabla^M_{\mathcal{H}X}\phi\mathcal{H}Z,V).
\end{aligned}
$$

Using the $T^{\mathrm{v}}M$ part of (5) and the assumption, we obtain

$$
\begin{aligned}
g(\nabla^M_{\mathcal{H}X}\phi\mathcal{H}Y,V) &= g(\mathcal{V}\nabla^M_{\mathcal{H}X}J\mathcal{H}Y,V)\\
&= -k(\mathcal{H}X,\mathcal{H}Y)+\tfrac{1}{2}g(\mathcal{H}X,\mathcal{H}Y)\sqrt{\omega(B)}\\
&= g(\mathcal{H}X,\mathcal{H}Y).
\end{aligned}
\tag{45}
$$

Thus, by (43) and (45),

$$
g((\nabla^M_X\phi)Y,Z)=g(V,\mathcal{V}Z)g(\mathcal{H}X,\mathcal{H}Y)-g(V,\mathcal{V}Y)g(\mathcal{H}X,\mathcal{H}Z). \tag{46}
$$

On the other hand,

$$
\begin{aligned}
g(g(X,Y)V-\eta(Y)X,Z) &= g(\mathcal{H}X,\mathcal{H}Y)g(V,\mathcal{V}Z)+g(\mathcal{V}X,\mathcal{V}Y)g(V,\mathcal{V}Z)\\
&\quad -g(\mathcal{H}X,\mathcal{H}Z)g(V,\mathcal{V}Y)-g(\mathcal{V}X,\mathcal{V}Z)g(V,\mathcal{V}Y)\\
&= g(V,\mathcal{V}Z)g(\mathcal{H}X,\mathcal{H}Y)-g(V,\mathcal{V}Y)g(\mathcal{H}X,\mathcal{H}Z).
\end{aligned}
$$

Therefore

$$
(\nabla^M_X\phi)Y=g(X,Y)V-\eta(Y)X. \tag{47}
$$

Hence (M,ϕ,V,η,g) is a Sasakian manifold.

Conversely, assume that (M,ϕ,V,η,g) is a Sasakian manifold. Let X and Y be any vector fields tangent to M. From (9) and (38), we obtain

$$\begin{aligned}\nabla_X JY - J\nabla_X Y &= \nabla_X(\phi Y - \eta(Y)C) - J(\nabla^M_X Y + \sigma(X,Y))\\ &= \nabla_X\phi Y - \nabla_X(\eta(Y)C) - \phi\nabla^M_X Y + \eta(\nabla^M_X Y)C - J\sigma(X,Y)\\ &= (\nabla^M_X\phi)Y - k(X,\phi Y)C - X\eta(Y)C\\ &\quad - \eta(Y)\nabla_X C + \eta(\nabla^M_X Y)C + k(X,Y)V.\end{aligned}$$

On the other hand, by (5),

$$\begin{aligned}\nabla_X JY - J\nabla_X Y &= \tfrac{1}{2}\omega(JY)X - \tfrac{1}{2}g(X,JY)B + \tfrac{1}{2}g(X,Y)JB\\ &= -\tfrac{1}{2}\sqrt{\omega(B)}\eta(Y)X - \tfrac{1}{2}g(X,\phi Y)B + \tfrac{1}{2}\sqrt{\omega(B)}g(X,Y)V.\end{aligned}$$

From these equations and (47), we have

$$\begin{aligned}&g(X,Y)V - \eta(Y)X - k(X,\phi Y)C - X\eta(Y)C\\ &\qquad - \eta(Y)\nabla_X C + \eta(\nabla^M_X Y)C + k(X,Y)V\\ &\quad = -\tfrac{1}{2}\sqrt{\omega(B)}\eta(Y)X - \tfrac{1}{2}g(X,\phi Y)B + \tfrac{1}{2}\sqrt{\omega(B)}g(X,Y)V.\end{aligned}$$

The V component of this equation is

$$\begin{aligned}&g(X,Y) - \eta(Y)\eta(X) - \eta(Y)g(\nabla_X C,V) + k(X,Y)\\ &\qquad = -\tfrac{1}{2}\sqrt{\omega(B)}\eta(Y)\eta(X) + \tfrac{1}{2}\sqrt{\omega(B)}g(X,Y).\end{aligned}$$

Thus

$$\begin{aligned}k(X,Y) &= \big(\tfrac{1}{2}\sqrt{\omega(B)} - 1\big)g(X,Y)\\ &\quad - \big(\tfrac{1}{2}\sqrt{\omega(B)} - 1\big)\eta(X)\eta(Y) + \eta(Y)g(\nabla_X C,V).\end{aligned}$$

Since $k(X,Y)$ is symmetric, we have $\eta(Y)g(\nabla_X C,V) = \eta(X)g(\nabla_Y C,V)$. This equation shows that $g(\nabla_X C,V) = \beta\eta(X)$, where β is a function. We set $\alpha = -\frac{1}{2}\sqrt{\omega(B)} + 1 + \beta$; then we have

$$k(X,Y) = \big(\tfrac{1}{2}\sqrt{\omega(B)} - 1\big)g(X,Y) + \alpha\eta(X)\eta(Y).$$

8. Examples. (I) Let (M,ϕ,V,η,g) be a Sasakian manifold and S^1 the circle with length element $\omega = dt$. Then $S^1\times M$ is a generalized Hopf manifold with metric $\omega^2 + g$ and Lee form ω (see [17]).

Let $\mathbb{C}^{n+m}$ be the complex vector space of all $(n+m)$-tuples of complex numbers $z = (z_1,\dots,z_{n+m})$ and a_{kj} be positive integers and α_{kj} be real numbers, $k = 1,\dots,m$, $j = 1,\dots,n+m$. Let

$$f_k(z_1,\dots,z_{n+m}) = \sum_{j=1}^{n+m}\alpha_{kj}z_j^{a_{kj}},\qquad k = 1,\dots,m,$$

be a collection of complex polynomials. Let $F = \bigcap_{k=1}^m f_k^{-1}(0)$. Let $d_k = \mathrm{LCM}(a_{k1},a_{k2},\dots,a_{k,n+m})$, $q_{kj} = d_k/a_{kj}$. Suppose that

(i) F is a complete intersection of the $f_k^{-1}(0)$.
(ii) F has an isolated singularity at the origin.
(iii) q_{kj} is independent of k (let $q_j = q_{kj}$).

Let $B^{2n-1} = F \cap S^{2(n+m)-1} \subset \mathbb{C}^{n+m}$. Then B^{2n-1} is called a *generalized Brieskorn manifold* [12]. It is a $(2n-1)$-dimensional submanifold in $S^{2(n+m)-1}$. Let $(S^{2(n+m)-1}, \phi, V, \eta, g)$ be the unit sphere with the standard Sasakian structure and imbedded in $\mathbb{C}^{n+m}$. Denoting by $x_1, y_1, \dots, x_{n+m}$, y_{n+m} the real coordinates of $\mathbb{C}^{n+m}$ such that $z_j = x_j + \sqrt{-1}\, y_j$ ($j = 1, \dots, n+m$), we define a real vector field $\widetilde{V}$ on $\mathbb{C}^{n+m}$ by

$$\widetilde{V} = \sum_{j=1}^{n+m} A_j (x_j \partial/\partial y_j - y_j \partial/\partial x_j),$$

where $A_j = \gamma q_j$ for a positive constant γ ($j = 1, \dots, n+m$). We set

$$\mu = \widetilde{V} - V, \quad \widetilde{\eta} = (1 + \eta(\mu))^{-1} \eta, \quad \widetilde{\phi}(X) = \phi(X - \widetilde{\eta}(X)\widetilde{V}),$$
$$\widetilde{g}(X, Y) = (1 + \eta(\mu))^{-1} g(X - \widetilde{\eta}(X)\widetilde{V}, Y - \widetilde{\eta}(Y)\widetilde{V}) + \widetilde{\eta}(X)\widetilde{\eta}(Y),$$

where X and Y are vector fields on $S^{2(n+m)-1}$. Then, by the theorem of Takahashi [13], $(S^{2(n+m)-1}, \widetilde{\phi}, \widetilde{V}, \widetilde{\eta}, \widetilde{g})$ is also a Sasakian manifold. Let $\iota : B^{2n-1} \to S^{2(n+m)-1}$ be the inclusion mapping. We define four tensor fields $(\widehat{\phi}, \widehat{V}, \widehat{\eta}, \widehat{g})$ on B^{2n-1} by

$$\widehat{\phi} = \widetilde{\phi}_{|B^{2n-1}}, \quad \widehat{V} = \widetilde{V}_{|B^{2n-1}}, \quad \widehat{\eta} = \iota^* \widetilde{\eta}, \quad \widehat{g} = \iota^* \widetilde{g}.$$

Using calculations similar to those of [13], we can prove that every generalized Brieskorn manifold $(B^{2n-1}, \widehat{\phi}, \widehat{V}, \widehat{\eta}, \widehat{g})$ admits many Sasakian structures. Therefore, $S^1 \times B^{2n-1}$ is a generalized Hopf submanifold of the generalized Hopf manifold $S^1 \times S^{2(n+m)-1}$.

(II) Let $E^{2n-1}(-3)$ be the Sasakian space form with constant ϕ-sectional curvature -3 with standard Sasakian structure in a Euclidean space. Let $S^1(r_i)$ be a circle of radius r_i, $i = 1, \dots, p$. A pythagorean product $E^{2(n-p)-1}(-3) \times S^1(r_1) \times \dots \times S^1(r_p)$ is a pseudo-umbilical generic submanifold of $E^{2n-1}(-3)$ ($p \geq 2$) (see [20]). Let S^1 be the circle with length element ω. Then ω is the Lee form of the generalized Hopf manifold $S^1 \times E^{2n-1}(-3)$. Hence $S^1 \times E^{2(n-p)-1}(-3) \times S^1(r_1) \times \dots \times S^1(r_p)$ is a CR-submanifold of $S^1 \times E^{2n-1}(-3)$ satisfying the conditions (a)–(c) and $S^1 \times E^{2(n-p)-1}(-3)$ is tangent to the Lee vector field of $S^1 \times E^{2n-1}(-3)$. The projection

$$\pi : S^1 \times E^{2(n-p)-1}(-3) \times S^1(r_1) \times \dots \times S^1(r_p) \to S^1 \times E^{2(n-p)-1}(-3)$$

is a Riemannian submersion satisfying (d). $S^1 \times E^{2(n-p)-1}(-3)$ is also a generalized Hopf manifold.

(III) The Hopf manifold $H^n_{e^2}$ is isometric to $S^1(1/\pi) \times S^{2n-1}$ (see [17]). S^{2n-1} is a real hypersurface of $H^n_{e^2}$ and the Lee vector field of $H^n_{e^2}$ is normal to S^{2n-1}. S^{2n-1} is a CR-submanifold of $H^n_{e^2}$ satisfying the conditions (a)–(c). $\pi : S^{2n-1} \to P_{n-1}(\mathbb{C})$ is a Riemannian submersion satisfying (d). From O'Neill [11], for orthonormal horizontal vectors X, Y, $A_X Y = -g(X, JY)JC$, where J is an almost complex structure on $H^n_{e^2}$ and C is the unit normal vector to S^{2n-1}. The holomorphic sectional curvature H of $P_{n-1}(\mathbb{C})$ is $H(\widetilde{X}) = 1 + 3|A_X JX|^2 = 4$, where $\widetilde{X}$ is any unit vector tangent to $P_{n-1}(\mathbb{C})$ and $X = \mathrm{h.l.}(\widetilde{X})$.

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