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The nonexistence of expansive homeomorphisms of chainable continua

by

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Abstract. A homeomorphism $f : X \to X$ of a compactum X with metric d is expansive if there is $c > 0$ such that if $x, y \in X$ and $x \neq y$, then there is an integer $n \in \mathbb{Z}$ such that $d(f^n(x), f^n(y)) > c$. In this paper, we prove that if a homeomorphism $f : X \to X$ of a continuum X can be lifted to an onto map $h : P \to P$ of the pseudo-arc P, then f is not expansive. As a corollary, we prove that there are no expansive homeomorphisms on chainable continua. This is an affirmative answer to one of Williams' conjectures.

1. Introduction. All spaces considered in this paper are assumed to be separable metric spaces. *Maps* are continuous functions. By a *compactum* we mean a compact metric space. A *continuum* is a connected, nondegenerate compactum. A homeomorphism $f : X \to X$ of a compactum X with metric d is called *expansive* [15] if there is $c > 0$ such that for any $x, y \in X$ and $x \neq y$, there is an integer $n \in \mathbb{Z}$ such that

$$d(f^n(x), f^n(y)) > c.$$

A homeomorphism $f : X \to X$ of a compactum X is *continuum-wise expansive* [7] if there is $c > 0$ such that if A is a nondegenerate subcontinuum of X, then there is an integer $n \in \mathbb{Z}$ such that $\operatorname{diam} f^n(A) > c$, where $\operatorname{diam} B = \sup\{d(x, y) \mid x, y \in B\}$ for a set B. Such a positive number c is called an *expansive constant* for f. Note that each expansive homeomorphism is continuum-wise expansive, but the converse assertion is not true. There are many important continuum-wise expansive homeomorphisms which are not expansive (e.g., see [7]). Expansiveness and continuum-wise expansiveness do not depend on the choice of the metric d of X. These

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notions are important in topological dynamics, ergodic theory and continuum theory.

For closed subsets A, B of a compactum X, we define $d(A,B) = \inf\{d(a,b) \mid a \in A, b \in B\}$ and $d_{\mathrm{H}}(A,B) = \inf\{\varepsilon > 0 \mid B \subset U_\varepsilon(A),\ A \subset U_\varepsilon(B)\}$, where $U_\varepsilon(A)$ denotes the ε-neighborhood of A in X. For a continuum X, let $C(X)$ be the set of all nonempty subcontinua of X. Then $C(X)$ is a continuum with *the Hausdorff metric* d_{H} (eg., see [10] or [14]). The space $C(X)$ is called the *hyperspace* of X. An *order arc* in $C(X)$ is an arc α in $C(X)$ such that if $A, B \in \alpha$, then $A \subset B$ or $B \subset A$. It is well known that if X is a continuum, $A, B \in C(X)$ and A is a proper subset of B, then there is an order arc from A to B in $C(X)$ (see [10] or [14]). For a map $f : X \to Y$, we define a map $C(f) : C(X) \to C(Y)$ by $C(f)(A) = f(A)$ for $A \in C(X)$.

A *chain* $C = [C_1, \ldots, C_m]$ of X is a finite collection of open subsets of X with the following property: $\mathrm{Cl}(C_i) \cap \mathrm{Cl}(C_j) \neq \emptyset$ iff $|i - j| \leq 1$. Moreover, if for each $i = 1, \ldots, m$, $\mathrm{diam}(C_i) < \varepsilon$, i.e., $\mathrm{mesh}(C) < \varepsilon$, then we say that the chain C is an ε-*chain*. For a chain $C = [C_1, \ldots, C_m]$ and two points $p, q \in X$, if $p \in C_1$ and $q \in C_m$, we say that $C = [C_1, \ldots, C_m]$ is a *chain from p to q*. A continuum X is *chainable* if for any $\varepsilon > 0$, there is an ε-chain covering of X. A continuum X is called a *tree-like* continuum if for any $\varepsilon > 0$ there is an onto map $g : X \to T$ such that $\mathrm{diam}\, g^{-1}(y) < \varepsilon$ for each $y \in T$, where T is a tree.

Let $f : X \to X$ be an onto map of a compactum X. If there exists an onto map $h : Y \to Y$ of a compactum Y and an onto map $\psi : Y \to X$ such that $\psi h = f \psi$, then we say that f can be *lifted to* an onto map $h : Y \to Y$.

The typical nonseparating plane continua are chainable continua. Concerning expansive homeomorphisms, the following conjectures by Williams remain open:

Conjecture 1.1. *No nonseparating plane continuum admits an expansive homeomorphism.*

Conjecture 1.2. *No chainable continuum admits an expansive homeomorphism.*

In [5–7], we proved that if X is a tree-like continuum admitting a continuum-wise expansive homeomorphism, it must contain an indecomposable subcontinuum. Also, Knaster's chainable continua and the pseudo-arc admit continuum-wise expansive homeomorphisms. In [9], we proved that Knaster's chainable continua admit no expansive homeomorphisms.

The aim of this paper is to give the complete solution of (1.2). In fact, we prove that if $f : X \to X$ is a homeomorphism of a continuum X and f can be lifted to an onto map $h : P \to P$ of the pseudo-arc P, then f is not expansive. To prove this result, we use a method similar to [9]. In [13, Theorem 4.1], W. Lewis proved that every onto map between chainable continua can be lifted to a homeomorphism of the pseudo-arc P. As a corollary, we obtain the following main theorem of this paper: Chainable continua admit no expansive homeomorphisms.

2. There are no expansive homeomorphisms on chainable continua. A continuum X is *decomposable* if there are two proper subcontinua A and B of X such that $A \cup B = X$. A continuum X is *indecomposable* if it is not decomposable. A continuum X is *hereditarily indecomposable* if each subcontinuum of X is indecomposable. For a continuum X and a point $p \in X$, let

$$\kappa(p) = \{x \in X \mid \text{there is a proper subcontinuum } A \text{ of } X \\ \text{such that } p, x \in A\}.$$

The set $\kappa(p)$ is called the *composant* of X containing p. Note that $\kappa(p)$ is dense in X. It is well known that if X is an indecomposable continuum, then X admits an uncountable collection of mutually disjoint composants. *The pseudo-arc* is characterized [2] as a (nondegenerate) hereditarily indecomposable chainable continuum. The pseudo-arc has many remarkable properties in topology and chaotic dynamics (e.g., see [1–3, 10–13]). For example, the pseudo-arc is homogeneous [1], each onto map of the pseudo-arc is a near homeomorphism [13], and the pseudo-arc admits chaotic homeomorphisms in the sense of Devaney (see [11]).

First, we shall prove the following theorem.

Theorem 2.1. *If $f : X \to X$ is a homeomorphism of a continuum X and f can be lifted to an onto map $h : P \to P$ of the pseudo-arc, then f is not expansive.*

To prove the above theorem, we need the following results. By [4], we know the following.

Lemma 2.2. *Every chainable continuum has the fixed point property.*

By the proofs of [1, Theorem 12 and 13], we obtain the following (see also [13, Lemma 3]).

Lemma 2.3. *Let P be the pseudo-arc and let $C = [C_1, \ldots, C_m]$ be a chain covering of P. Suppose that P_n $(n = 1, 2)$ are nondegenerate subcontinua of P and p_n, q_n are two points of P_n $(n = 1, 2)$ respectively, such that $p_1, p_2 \in C_1$ and $q_1, q_2 \in C_m$, i.e., the chain C is a chain from p_1 (resp. p_2) to q_1 (resp. q_2), and moreover, p_n and q_n $(n = 1, 2)$ belong to different composants of P_n for each $n = 1, 2$. Then there is a homeomorphism $k : P_1 \to P_2$ such that $k(p_1) = p_2, k(q_1) = q_2$, and $k(C_j) \subset \mathrm{st}(C_j; C)^*$ for each $C_j \in C$* (see Figure 1), *where $\mathrm{st}(C_j; C)^* = \bigcup\{C_i \in C \mid C_j \cap C_i \neq \emptyset\}$.*

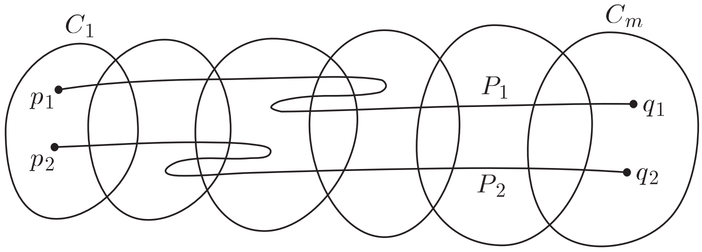

Fig. 1

Also, we need the following useful theorem [13, Theorem 4.1] which was proved by W. Lewis.

THEOREM 2.4. *If $f : X \to X$ is an onto map of a chainable continuum X to itself, then there exists a homeomorphism $h : P \to P$ of the pseudo-arc P and an onto map $\psi : P \to X$ such that $f\psi = \psi h$. In particular, every homeomorphism $f : X \to X$ of a chainable continuum X can be lifted to a homeomorphism of the pseudo-arc P.*

The next lemma follows from [7, (2.4)].

LEMMA 2.5. *Let $f : X \to X$ be a continuum-wise expansive homeomorphism. Then there is $\delta > 0$ such that for each $\gamma > 0$ there is a natural number $N > 0$ satisfying the following condition: if $A \in C(X)$ and* $\operatorname{diam} A \geq \gamma$, *then* $\operatorname{diam} f^n(A) \geq \delta$ *for all $n \geq N$ or* $\operatorname{diam} f^{-n}(A) \geq \delta$ *for all $n \geq N$.*

Proof of Theorem 2.1. Suppose, on the contrary, that $f : X \to X$ is an expansive homeomorphism. Let $c > 0$ be an expansive constant for f and let $c/2 > \varepsilon > 0$. First, we prove that f has the following property $(*)$:

$(*)$ For any $\tau > 0$ there are two points x, y of X and a natural number $n(\tau)$ such that $d(x, y) \leq \tau$, $d(f^{n(\tau)}(x), f^{n(\tau)}(y)) \leq \tau$, and $\varepsilon \leq \sup\{d(f^j(x), f^j(y)) \mid 0 \leq j \leq n(\tau)\} \leq 2\varepsilon$.

By the assumption, there is an onto map $h : P \to P$ of the pseudo-arc P and an onto map $\psi : P \to X$ such that $\psi h = f\psi$. Since P is chainable, by Theorem 2.4, we may assume that h is a homeomorphism. By Lemma 2.2, there is a fixed point p of h, i.e., $h(p) = p$. Consider the following set:

$$C_p = \{A \in C(P) \mid p \in A\} \subset C(P).$$

Since P is hereditarily indecomposable, C_p is the unique order arc from $\{p\}$ to P in $C(P)$ (see [10] or [14]). Note that $C(h)|C_p : C_p \to C_p$ is a homeomorphism of the arc C_p. Since $C(\psi)|C_p : C_p \to C(\psi)(C_p)$ is a monotone map from an arc C_p, $\mathcal{A} = C(\psi)(C_p)$ is an arc from $\psi(p)$ to X in $C(X)$. Note that $C(f)(\mathcal{A}) = \mathcal{A}$. Also, we can choose the subcontinuum $P_0 \in C_p$ (i.e., $p \in P_0$) such that $\psi(P_0) = \psi(p)$ and if A is any subcontinuum of P such that A contains P_0 as a proper subset, then $\psi(A)$ is nondegenerate.

Note that $P_0 \neq P$ and $h(P_0) = P_0$. Also, since f is a continuum-wise expansive homeomorphism, P_0 is an isolated point of the set of fixed points of the homeomorphism $C(h)|C_p : C_p \to C_p$. Hence we can choose $P_1 \in C_p$ such that P_1 contains P_0 as a proper subset, $h(P_1) = P_1$ and the homeomorphism $C(h)|[P_0, P_1] : [P_0, P_1] \to [P_0, P_1]$ has the only two fixed points P_0 and P_1, where $[P_0, P_1]$ denotes the order arc from P_0 to P_1. We may assume that $C(h)|[P_0, P_1]$ is increasing. If necessary, we consider $C(h^{-1})$. Note that if $A \in (P_0, P_1) = [P_0, P_1] - \{P_0, P_1\}$, then $\lim_{n\to\infty} h^n(A) = P_1$, and $\lim_{n\to\infty} h^{-n}(A) = P_0$.

Let $\tau > 0$. Choose an open covering $\mathcal{U}$ of X with $\operatorname{mesh}(\mathcal{U}) \le \tau$. Choose a subcontinuum $A \in (P_0, P_1)$ such that $0 < \operatorname{diam} \psi(A) = \tau_1 \le \tau$. Since $\lim_{n\to\infty} \operatorname{diam} f^{-n}(\psi(A)) = 0$ and f is expansive, we can choose a natural number $N_1 > 0$ such that if $x, y \in \psi(A)$ and $d(x, y) \ge \tau_1/3$, then

$$\sup\{d(f^i(x), f^i(y)) \mid 0 \le i \le N_1\} > c.$$

Choose a point $a \in A$ such that $d(\psi(p), \psi(a)) > \tau_1/3$. Choose two subcontinua E, K of A such that $d(\psi(E), \psi(K)) > \tau_1/3$, E contains P_0 as a proper subset, i.e., $\psi(E)$ is nondegenerate, K contains the point a, and $\psi(K)$ is a nondegenerate subcontinuum of X. Then we shall show the following claim:

CLAIM. *The set* $\limsup_{n\to\infty} h^n(K)$ *contains a point* $q \in P_1$ *such that* q *is not contained in the composant of* P_1 *containing* p.

Suppose, on the contrary, that the claim is not true. Then there is $\varepsilon_1 > 0$ such that $d_{\mathrm{H}}(h^n(K), P_1) \ge \varepsilon_1$ for all $n \ge 0$. Take a subsequence $\{n(i) \mid i = 1, 2, \ldots\}$ of natural numbers such that $\lim_{i\to\infty} h^{n(i)}(K) = K_0$. By the assumption, K_0 is contained in the composant of P_1 containing p. Choose a subcontinuum $K' \supset K_0$ such that K' is contained in the composant of P_1 containing p. Since $\lim_{n\to\infty} \operatorname{diam} f^{-n}(\psi(K')) = 0$, we see that $\lim_{n\to\infty} \operatorname{diam} f^{-n}(\psi(K_0)) = 0$.

On the other hand, by Lemma 2.5, there is a natural number $N_2 > 0$ such that if $n \ge N_2$, then $\operatorname{diam} f^n(\psi(K)) \ge \delta$ for some $\delta > 0$, since K is contained in the composant of P_1 containing p and hence $\lim_{n\to\infty} \operatorname{diam} f^{-n}(\psi(K)) = 0$. Since $\lim_{i\to\infty}(n(i) - N_2) = \infty$, $\operatorname{diam} f^n(\psi(K)) \ge \delta$ for all $n \ge N_2$ and $\lim_{i\to\infty} f^{n(i)}(\psi(K)) = \lim_{i\to\infty} \psi(h^{n(i)}(K)) = \psi(K_0)$, we can prove that

$$\operatorname{diam} f^{-n}(\psi(K_0)) \ge \delta \qquad \text{for all } n \ge 0.$$

In fact, suppose, on the contrary, that there is $n_0 \ge 0$ such that

$$\operatorname{diam} f^{-n_0}(\psi(K_0)) < \delta.$$

Choose $\delta_1 > 0$ such that (I) if $T \in C(X)$ and $d_{\mathrm{H}}(T, \psi(K_0)) < \delta_1$, then $\operatorname{diam} f^{-n_0}(T) < \delta$. Since $\lim_{i\to\infty} f^{n(i)}(\psi(K)) = \psi(K_0)$, there is a sufficiently large natural number $n(i_0)$ such that (II) $n(i_0) - n_0 \ge N_2$ and

$d_{\mathrm{H}}(f^{n(i_0)}(\psi(K)), \psi(K_0)) < \delta_1$. By (I) and (II),

$$\delta > \operatorname{diam} f^{-n_0}(f^{n(i_0)}(\psi(K))) = \operatorname{diam} f^{n(i_0)-n_0}(\psi(K)).$$

However, $n(i_0) - n_0 \geq N_2$ implies that $\operatorname{diam} f^{n(i_0)-n_0}(\psi(K)) \geq \delta$. Therefore we see that $\operatorname{diam} f^{-n}(\psi(K_0)) \geq \delta$ for all $n \geq 0$. This is a contradiction. Hence the claim is true.

Since p and q are points belonging to different composants of P_1, we can choose a chain covering $C = [C_1, \ldots, C_m]$ of P_1 such that $\mathrm{st}(C) = \{\mathrm{st}(C_j; C)^* \mid C_j \in C\}$ is a refinement of $\psi^{-1}(\mathcal{U}) = \{\psi^{-1}(U) \mid U \in \mathcal{U}\}$, and C is a chain from p to q, i.e., $p \in C_1$ and $q \in C_m$ (see the proof of [1, Theorem 13]). Since $\lim_{n\to\infty} h^n(E) = P_1$, there is a natural number $N > N_1$ such that $h^N(E) \cap C_m \neq \emptyset$ and $h^N(K) \cap C_m \neq \emptyset$. Choose two points $u \in E$ and $v \in K$ such that $h^N(u), h^N(v) \in C_m$. Since A is indecomposable and each composant of A is dense in A, we can choose two points $q_1, q_2 \in A$ such that q_1 is sufficiently near to u, q_1 is not contained in the composant of A containing p, q_2 is sufficiently near to v, and q_2 is not contained in the composant of A containing p. We may assume that $d(\psi(q_1), \psi(q_2)) > \tau_1/3$ and $h^N(q_1), h^N(q_2) \in C_m$. By Lemma 2.3, there is a homeomorphism $k : h^N(A) \to h^N(A)$ such that $k(p) = p, k(h^N(q_1)) = h^N(q_2)$ and for each $x \in h^N(A)$, $k(x)$ and x are contained in an element $\mathrm{st}(C_j; C)^*$ of $\mathrm{st}(C)$. Choose a sufficiently small $\gamma > 0$. Then we can choose a γ-chain covering $D = [D_1, \ldots, D_s]$ of $h^N(A)$ from $h^N(p) = p$ to $h^N(q_1)$, because $h^N(p) = p$ is not contained in the composant of $h^N(A)$ containing $h^N(q_1)$ (see the proof of [1, Theorem 13]). Since $\gamma > 0$ is sufficiently small, we may assume that if $D_i \in D$ $(i = 1, \ldots, s)$, then $k(D_i)$ and D_i are contained in an element $\mathrm{st}(C_j; C)^*$ of $\mathrm{st}(C)$. Set

$$\begin{aligned} D(1) &= \psi h^{-N}(D) = [\psi h^{-N}(D_1), \ldots, \psi h^{-N}(D_s)] = [D(1)_1, \ldots, D(1)_s], \\ D(2) &= \psi h^{-N}(k(D)) = [\psi h^{-N}(k(D_1)), \ldots, \psi h^{-N}(k(D_s))] \\ &= [D(2)_1, \ldots, D(2)_s]. \end{aligned}$$

Then $D(n)$ is a covering of $\psi(A)$ from $\psi(p)$ to $\psi(q_n)$ for each $n = 1, 2$. We may assume that if $x, y \in D(n)_i \cup D(n)_{i+1}$ for $i = 1, \ldots, s-1$ and $n = 1, 2$ respectively, then $\sup\{d(f^j(x), f^j(y)) \mid 0 \leq j \leq N\} < \varepsilon/2$. Choose sequences $\psi(p) = a_1, \ldots, a_{s-1}, a_s = \psi(q_1)$ and $\psi(p) = b_1, \ldots, b_{s-1}$, $b_s = \psi(q_2)$ of points of A such that $a_i \in D(1)_i, b_i \in D(2)_i$ for each $i = 1, \ldots, s$. Note that $d(f^N(a_i), f^N(b_i)) < \tau$ for each i. Consider the finite sequence r_i $(i = 1, \ldots, s)$ of positive numbers defined by

$$r_i = \sup\{d(f^j(a_i), f^j(b_i)) \mid 0 \leq j \leq N\}.$$

Then $|r_i - r_{i+1}| < \varepsilon$ and $r_1 = 0 < \varepsilon$ and $r_s > c > 2\varepsilon$. We can choose i such that $\varepsilon \leq r_i \leq 2\varepsilon$. Then the two points $a_i = x$ and $b_i = y$ satisfy the conditions of $(*)$. Hence the property $(*)$ is satisfied.

Let $\{\varepsilon_i\}_{i=1}^{\infty}$ be a sequence of positive numbers such that $\lim_{i\to\infty}\varepsilon_i = 0$. By $(*)$, there are two points x_i, y_i of X and a natural number $n(i)$ such that $d(x_i, y_i) < \varepsilon_i, d(f^{n(i)}(x_i), f^{n(i)}(y_i)) < \varepsilon_i$ and

$$\varepsilon \le \sup\{d(f^j(x_i), f^j(y_i)) \mid 0 \le j \le n(i)\} \le 2\varepsilon.$$

Choose $0 < m(i) < n(i)$ such that $d(f^{m(i)}(x_i), f^{m(i)}(y_i)) \ge \varepsilon$. We may assume that $\{f^{m(i)}(x_i)\}$ and $\{f^{m(i)}(y_i)\}$ are convergent to x_0 and y_0, respectively. Note that

$$\lim_{i\to\infty}(n(i) - m(i)) = \infty = \lim_{i\to\infty} m(i).$$

Then $x_0 \ne y_0$ and $d(f^n(x_0), f^n(y_0)) \le 2\varepsilon < c$ for all $n \in \mathbb{Z}$. This is a contradiction.

By Theorems 2.1 and 2.4, we obtain the following main theorem of this paper.

THEOREM 2.6. *Chainable continua admit no expansive homeomorphisms. In other words, if X is any chainable continuum and $f : X \to X$ is any homeomorphism of X, then for any $\varepsilon > 0$ there exist two different points $x, y \in X$ such that $d(f^n(x), f^n(y)) < \varepsilon$ for all $n \in \mathbb{Z}$.*

A continuum X is *weakly chainable* ([3] or [12]) if it is a continuous image of a chainable continuum, in particular an image of the pseudo-arc P. Note that there exists a tree-like continuum which is not weakly chainable. In [13], W. Lewis posed the following problem:

PROBLEM 2.7. *If X is a weakly chainable, tree-like continuum and $f : X \to X$ is an onto map, does there exist an onto map $\psi : P \to X$ of the pseudo-arc P and a homeomorphism $h : P \to P$ such that $\psi h = f\psi$?*

Concerning tree-like continua, we have the following problem:

PROBLEM 2.8. *Does there exist a tree-like continuum X admitting an expansive homeomorphism?*

A positive answer to Problem 2.7 would show that weakly chainable, tree-like continua admit no expansive homeomorphisms.

Let $f : X \to X$ be a map of a compactum X. Consider the following inverse limit space:

$$(X, f) = \{(x_n)_{n=1}^{\infty} \mid x_n \in X \text{ and } f(x_{n+1}) = x_n\}.$$

Define the *shift map* $\widetilde{f} : (X, f) \to (X, f)$ of f by $\widetilde{f}(x_1, x_2, \ldots,) = (f(x_1), x_2, \ldots,)$.

In [6, (2.9)], we obtained the following.

PROPOSITION 2.9. *If $f : G \to G$ is an onto map of a finite graph G such that the shift map $\widetilde{f} : (G, f) \to (G, f)$ is expansive, then each point*

$\widetilde{x} \in (G, f)$ is contained in an arc in (G, f). In particular, (G, f) is not hereditarily indecomposable.

Concerning this proposition, the following problem arises naturally.

PROBLEM 2.10. *Does there exist a hereditarily indecomposable continuum admitting an expansive homeomorphism?*

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