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Hyperspaces of two-dimensional continua

by

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Abstract. Let X be a compact metric space and let $\mathcal{C}(X)$ denote the space of subcontinua of X with the Hausdorff metric. It is proved that every two-dimensional continuum X contains, for every $n \geq 1$, a one-dimensional subcontinuum T_n with $\dim \mathcal{C}(T_n) \geq n$. This implies that X contains a compact one-dimensional subset T with $\dim \mathcal{C}(T) = \infty$.

1. Introduction. Let X be a compact metrizable space. 2^X denotes the space of closed subsets of X endowed with the Hausdorff metric, and $\mathcal{C}(X)$ is the subset of 2^X which consists of the subcontinua of X. Both 2^X and $\mathcal{C}(X)$ are compact.

In [5] the authors proved that if $\dim X = 2$ then $\dim \mathcal{C}(X) = \infty$. In this note we improve this result by showing that actually the 1-dimensional subcontinua of X are responsible for the infinite dimensionality of $\mathcal{C}(X)$, more precisely: for every positive integer n, X contains a one-dimensional subcontinuum T_n with $\dim \mathcal{C}(T_n) \geq n$, and as a result, X contains a one-dimensional compact subset T with $\dim \mathcal{C}(T) = \infty$. The following problem is still left open:

QUESTION 1.1. *Let X be a 2-dimensional continuum. Does X contain a 1-dimensional subcontinuum T with* $\dim \mathcal{C}(T) = \infty$?

In two extreme cases the answer is affirmative. It is proved in [6] that if T is a 1-dimensional hereditarily indecomposable continuum then $\dim \mathcal{C}(T)$ is either 2 or ∞. Thus, if X is a 2-dimensional hereditarily indecomposable continuum then the 1-dimensional continuum $T_3 \subset X$ that we construct with $\dim \mathcal{C}(T_3) \geq 3$, actually satisfies $\dim \mathcal{C}(T_3) = \infty$ (see [3] for more information on hyperspaces of finite-dimensional hereditarily indecomposable continua). Note that this implies that every 3-dimensional continuum X contains a

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1-dimensional subcontinuum T with $\dim \mathcal{C}(T) = \infty$ since by [1], X contains a 2-dimensional hereditarily indecomposable continuum.

The hereditarily indecomposable continua are characterized by the property that their subcontinua do not intersect in a non-trivial manner (i.e. $A \cap B \neq \emptyset$ implies $A \subset B$ or $B \subset A$). If on the other hand a 2-dimensional continuum X is rich with mutually intersecting 1-dimensional subcontinua (e.g. if X is a Peano continuum or if X is the product of two 1-dimensional continua) then again Question 1.1 has a positive answer for X.

We shall need the following result from [5] and include a short proof for it.

THEOREM 1.2. *Let X be an n-dimensional compact metric space*, $n < \infty$. *There exists an n-dimensional hereditarily indecomposable continuum Y and a light map f of Y into X.*

Proof. We have $\dim X \times I = n + 1$, $I = [0, 1]$. By [1] there exists an n-dimensional hereditarily indecomposable continuum $Y \subset X \times I$. Let $P : X \times I \to X$ be the projection, and set $f = P|_Y$. Then f is light since a component of a fiber of f is a subcontinuum of both Y and I and hence must be a singleton. ■

Recall that a map $W : \mathcal{C}(X) \to \mathbb{R}^+$ is called a *Whitney map* if $W(\{x\}) = 0$ for all $x \in X$ and if $A \subset B, A \neq B$ in $\mathcal{C}(X)$ implies $W(A) < W(B)$. Whitney maps always exist (see [6]).

Let $\psi : X \to Q$ be a map of compacta. Set $Q_0 = \{z : z \in Q,\ \dim \psi^{-1}(z) \le 0\}$ and $Q_1 = Q \setminus Q_0 = \{z \in Q : \dim \psi^{-1}(z) \ge 1\}$. We shall need the following result.

THEOREM 1.3. *Let X be an n-dimensional compact space*, $n \ge 2$. *There exist a* 1*-dimensional compactum Q and a map $\psi : X \to Q$ such that* $\dim \psi^{-1}(Q_1) = n - 1$.

Proof. Let Q be a dendrite with a dense set of nonseparating points. It is proved in Theorem 2.2 of [7] that for every compact space X and every 0-dimensional σ-compact subset F of X, almost all maps $\psi \in C(X, Q)$ (i.e. all except a set of first category in the function space) satisfy $F \subset \{x \in X : \psi^{-1}(\psi(x)) = \{x\}\}$, and thus $\psi^{-1}(Q_1) \subset X \setminus F$.

If $\dim X = n$ there exists a σ-compact 0-dimensional subset F of X such that $\dim(X \setminus F) \le n - 1$ ([7], Proposition 3.1). It follows that for almost all $\psi \in C(X, Q)$, $\dim \psi^{-1}(Q_1) = n - 1$ (note that $\dim \psi \ge n - 1$ and hence $\dim \psi^{-1}(Q_1) \ge n - 1$).

Another, more elementary proof of Theorem 1.3 can be obtained by applying the results of [2]. There Lelek constructs, for each $n \ge 2$, a map $f : I^n \to Q$, where Q is a dendrite with $\dim f^{-1}(Q_1) = n-1$. (Lelek does not use the same terminology but it is easy to verify that f indeed satisfies this.)

Now, if $\dim X = n$, let $\varphi : X \to I^n$ be light; then for $\psi = f \circ \varphi : X \to Q$ we have $\dim \psi^{-1}(Q_1) = n - 1$. ∎

The general scheme of our note resembles that of [5] but it includes some additional ingredients and is more complicated.

2. Proofs

THEOREM 2.1. *Let X be a 2-dimensional continuum and let n be a positive integer. Then X contains a 1-dimensional continuum T_n with $\dim \mathcal{C}(T_n) > n$.*

COROLLARY 2.2. *Let X be a 2-dimensional continuum. Then X contains a 1-dimensional compact subset T such that $\dim \mathcal{C}(T) = \infty$.*

Proof. For each $n \geq 1$ let X_n be a 2-dimensional continuum with $\operatorname{diam} X_n \leq 1/n$ and $X_1 \supset X_2 \supset X_3 \supset \ldots$ Let $T_0 = \bigcap_{n=1}^{\infty} X_n$ (T_0 is a singleton) and by Theorem 2.1 let $T_n \subset X_n$ be a 1-dimensional continuum with $\dim \mathcal{C}(T_n) > n$. Take $T = \bigcup_{n=0}^{\infty} T_n$. ∎

LEMMA 2.3. *Let $f : Y \to X$ be a light map of compacta. For every $\varepsilon > 0$ there exist positive reals $\alpha(\varepsilon)$ and $\delta(\varepsilon)$ such that for every subset B of X with $\operatorname{diam} B \leq \delta(\varepsilon)$, $f^{-1}(B)$ is decomposable as $f^{-1}(B) = \bigcup_{s=1}^{t} W^s$ with $\operatorname{diam} W^s < \varepsilon$ and $\operatorname{dist}(W^s, W^r) \geq \alpha(\varepsilon)$ for $s \neq r$. (By $\operatorname{dist}(W^s, W^r)$ we mean $\inf\{d(x^s, x^r) : x^s \in W^s, x^r \in W^r\}$, where d is a metric).*

Proof. Let $\varepsilon > 0$. For $x \in X$, $\dim f^{-1}(x) = 0$. Hence $f^{-1}(x)$ can be covered by a finite family $\mathcal{U}_x$ of open subsets of Y with $\operatorname{mesh} \mathcal{U}_x < \varepsilon$ and $\alpha(x) = \min\{\operatorname{dist}(A, B) : A, B \in \mathcal{U}_x,\ A \neq B\} > 0$. Let V_x denote the union of the elements of $\mathcal{U}_x$. V_x is a neighborhood of $f^{-1}(x)$ in Y. Let W_x be an open neighborhood of x in X such that $f^{-1}(W_x) \subset V_x$. By compactness X is covered by some $W_{x_1}, \ldots, W_{x_n}$. Let $\delta(\varepsilon)$ be the Lebesgue number of this cover; i.e. each subset B of X with $\operatorname{diam} B \leq \delta$ is contained in some W_{x_i}, and the lemma holds with $\alpha(\varepsilon) = \min\{\alpha(x_i) : 1 \leq i \leq n\}$. ∎

LEMMA 2.4. *Let $\mathcal{K} \subset \mathcal{C}(Y)$ be a decomposition of Y which contains no singletons and which is closed in $\mathcal{C}(Y)$. Let $h : Y \to \mathcal{K}$ denote the corresponding (open) quotient map. Let f be a light map of Y into some continuum X, and let $g : Y \to \mathcal{C}(X)$ be defined by $g(y) = f(h(y))$. Then for every positive integer n and every positive real ε there exists a positive real $\alpha = \alpha(\varepsilon, n)$ such that for every closed subset $Y_0 \subset Y$ with $\dim g(Y_0) \leq n$ there exist closed subsets $Z_1, \ldots, Z_m$ of Y_0 with $\operatorname{diam} Z_i < \varepsilon$, $1 \leq i \leq m$ such that $\bigcup_{i=1}^{m} Z_i$ intersects every element of $\mathcal{K}$ which is contained in Y_0 and for $1 \leq i < j \leq m$ either $g(Z_i) \cap g(Z_j) = \emptyset$ or $\operatorname{dist}(Z_i, Z_j) \geq \varepsilon$.*

Proof. h and g are continuous since $\mathcal{K}$ is closed in $\mathcal{C}(Y)$. As $\mathcal{K}$ contains no singletons it follows that $\inf\{\operatorname{diam} K : K \in \mathcal{K}\} > 0$; and since f is light we see that $\inf\{\operatorname{diam} g(y) : y \in Y\} = \inf\{\operatorname{diam} f(K) : K \in \mathcal{K}\} = \lambda > 0$.

As all n-dimensional spaces are embeddable in the same Euclidean space there exists an integer $N = N(n)$ such that for every n-dimensional compact space H every open cover of H has an open refinement $\{V_1, \ldots, V_r\}$ so that each V_i intersects at most N of the other V_j. Let $\varepsilon > 0$ and n be given. Let $\delta_1 = \delta_1(\varepsilon)$ and $\alpha(\varepsilon)$ be as in Lemma 2.3.

Let $0 < \delta = \min\{\delta_1/2, \lambda/(6N)\}$ (note that δ depends on n and ε) and let $\alpha_1(\varepsilon, n) > 0$ be small enough such that $d(y_1, y_2) \le \alpha_1(\varepsilon, n)$ in Y implies that $d(f(y_1), f(y_2)) < \delta$ (in X). Finally, let $\alpha(\varepsilon, n) = \min\{\alpha(\varepsilon), \alpha_1(\varepsilon, n)\}$.

Note that

(i) If $B_1, \ldots, B_N$ are N subsets of X with $\operatorname{diam} B_i < 3\delta$ then $\{B_i\}_{i=1}^N$ do not cover $g(y)$ for all $y \in Y$. Moreover, for every $y \in Y$ there exists a point $x \in g(y)$ such that $\operatorname{dist}(x, B_i) \ge 3\delta$ for all $1 \le i \le N$. (Since $g(y)$ is a continuum of diameter $\ge \lambda$ and $\delta \le \lambda/(6N)$.)

Let $Y_0 \subset Y$ be closed with $\dim g(Y_0) \le n$. Let $\{\mathcal{V}_1, \ldots, \mathcal{V}_r\}$ be a closed cover of $g(Y_0)$ with mesh $< \delta$ (mesh with respect to the Hausdorff metric in $\mathcal{C}(X)$) such that each $\mathcal{V}_i$ intersects at most N of the other $\mathcal{V}_j$. Then

(ii) For every $1 \le i \le r$, for every $A \in \mathcal{V}_i$, and every $x \in A$, $B(x, \delta)$ (= closed δ-ball in X with center at x) intersects every $B \in \mathcal{V}_i$ (since otherwise the Hausdorff distance between A and B would be more than δ).

Now we construct inductively closed subsets $W_1, \ldots, W_r$ of Y_0 as follows: pick some $A_1 \in \mathcal{V}_1$ and $x_1 \in A_1$, and set $W_1 = f^{-1}(B(x_1, \delta)) \cap g^{-1}(\mathcal{V}_1) \cap Y_0$. Assume that $W_1, \ldots, W_{j-1}$ were constructed as $W_i = f^{-1}(B(x_i, \delta)) \cap g^{-1}(\mathcal{V}_i) \cap Y_0$ where $x_i \in A_i \in \mathcal{V}_i$, $1 \le i \le j-1$. Let $A_j \in \mathcal{V}_j$. At most N of $\mathcal{V}_i$, $1 \le i \le j-1$, intersect $\mathcal{V}_j$. Assume these are $\mathcal{V}_{i_1}, \ldots, \mathcal{V}_{i_N}$. By (i) there exists a point $x_j \in A_j$ such that $\operatorname{dist}(x_j, B(x_{i_l}, 3\delta)) \ge 3\delta$ for all $1 \le l \le N$. Hence

(iii) $\operatorname{dist}(B(x_j, \delta), B(x_{i_l}, \delta)) > \delta$ for all $1 \le l \le N$

and we take $W_j = f^{-1}(B(x_j, \delta)) \cap g^{-1}(\mathcal{V}_j) \cap Y_0$. It follows from (ii) that W_i, $1 \le i \le r$, intersects every element of $\mathcal{K}$ which is contained in $Y_0 \cap g^{-1}(\mathcal{V}_i)$ and so $\bigcup_{i=1}^r W_i$ intersects every element of $\mathcal{K}$ which is contained in Y_0.

From (iii) and the definition of $\alpha_1(\varepsilon, n)$ we obtain

(iv) for $1 \le i < j \le r$, if $g(W_i) = \mathcal{V}_i$ intersects $g(W_j) = \mathcal{V}_j$ then $\operatorname{dist}(B(x_i, \delta), B(x_j, \delta)) > \delta$ and hence $\operatorname{dist}(W_i, W_j) \ge \alpha_1(\varepsilon, n)$ (in Y) since $W_i \subset f^{-1}(B(x_i, \delta))$.

As $\delta \le \delta_1/2$ and $W_i \subset f^{-1}(B(x_i, \delta))$ we may apply Lemma 2.3 to decompose W_i as $W_i = \bigcup_{s=1}^{t_i} W_i^s$ with $\operatorname{diam} W_i^s < \varepsilon$ and $\operatorname{dist}(W_i^{s_1}, W_i^{s_2}) \ge$

$\alpha(\varepsilon)$. For $1 \le i < j \le r$, if $g(W_i^s) \cap g(W_j^t) \neq \emptyset$ then by (iv), $\operatorname{dist}(W_i^s, W_j^t) \ge \alpha_1(\varepsilon, n) \ge \alpha(\varepsilon, n)$ and we take $Z_1, \ldots, Z_m$ to be an enumeration of $\{W_i^s\}$, $1 \le i \le r$, $1 \le s \le t_i$. ■

Proof of Theorem 2.1. Let X be a 2-dimensional continuum. Apply Theorems 1.2 and 1.3 to find a 2-dimensional hereditarily indecomposable continuum Y with a light map $f : Y \to X$, and a 1-dimensional continuum Q with a map $\psi : X \to Q$ such that $\dim \psi^{-1}(Q_1) = 1$. Let $\psi \circ f = \varphi \circ p$ denote the monotone light decomposition of the map $\psi \circ f : Y \to Q$ with $p : Y \to V = p(Y)$ monotone.

$$\begin{tikzcd}
& & S \arrow[dl] \arrow[dr] & & \\
& V \arrow[dl, "\varphi"'] & & W^{-1}(t_0) \arrow[dr] & \\
Q & & Y \arrow[uu, "s"] \arrow[ul, "p"] \arrow[ur, "q"'] \arrow[rr, "h"] \arrow[dl, "f"] \arrow[dr, "g"'] & & W^{-1}(t) \arrow[dl, "f^*"] \\
& X \arrow[ul, "\psi"] & & \mathcal{C}(X) &
\end{tikzcd}$$

(The arrows not marked by letters in this diagram represent maps which exist, but are not referred to in the sequel.)

Let F_1 and F_2 be closed disjoint subsets of Y such that

(i) every closed separator between F_1 and F_2 must have a component of diameter $\ge r = r(F_1, F_2) > 0$.

Let $W : \mathcal{C}(Y) \to \mathbb{R}^+$ be a Whitney map, and let $t > 0$ be small enough such that

(ii) $\operatorname{mesh} W^{-1}(t) < r$.

$\mathcal{K} = W^{-1}(t)$ is a closed decomposition of Y which contains no singletons. Let $h : Y \to W^{-1}(t)$ denote the quotient map. Let n be a positive integer and set $\varepsilon = (1/3)\operatorname{dist}(F_1, F_2) > 0$. Let $\alpha(\varepsilon, n) > 0$ be the real obtained in Lemma 2.4. (Note that $g = f^* \circ h$, where $f^* : \mathcal{C}(Y) \to \mathcal{C}(X)$ is defined by $f^*(A) = f(A)$, i.e. $g(y) = f(h(y))$.)

Let $0 < t_0 < t$ be such that

(iii) $\operatorname{mesh} W^{-1}(t_0) < \min\{\alpha(\varepsilon, n), \varepsilon\}$.

Let $q : Y \to W^{-1}(t_0)$ be the quotient map. Then q is an open monotone map with no trivial fibers. Let $s = p \wedge q$ denote the product of the maps p and q, i.e. the fiber of s at $y \in Y$ is the intersection of the fibers of p and q at y (see [4]). Note that as Y is hereditarily indecomposable and

p and q are monotone, these fibers of p and q at y actually contain one another. Thus, each fiber of s is either a fiber of p or of q. Let S denote the range of s and let $\mathcal{S}$ denote the decomposition of Y induced by s. Set $Y_q = \{A : A \in \mathcal{S} \cap W^{-1}(t_0)\}$, i.e. Y_q is the union of those fibers of s which are fibers of q (and thus are contained in some fiber of p).

Y_q is closed in Y. To prove this we show that $\mathcal{S} \cap W^{-1}(t_0)$ is closed in $\mathcal{C}(Y)$. (Note that $\mathcal{S}$ may fail to be closed.) Let $\{A_k\}_{k=1}^{\infty} \subset \mathcal{S} \cap W^{-1}(t_0)$ converge to some $A \in \mathcal{C}(Y)$. Then $A \in W^{-1}(t_0)$ since $W^{-1}(t_0)$ is closed in $\mathcal{C}(Y)$. Each A_k is contained in some fiber B_k of p, and we may assume that $\{B_k\}$ converges in $\mathcal{C}(Y)$ to some continuum B. Clearly $A \subset B$ and as p is continuous, B is contained in some fiber of p. Hence A is a fiber of q and is contained in a fiber of p so $A \in \mathcal{S}$ and $\mathcal{S} \cap W^{-1}(t_0)$ is closed.

We claim that

(iv) $\dim s(Y \setminus Y_q) \le 1$.

Indeed, $Y \setminus Y_q$ is a union of fibers of s which are also fibers of p (but are not fibers of q). Hence the decomposition of $Y \setminus Y_q$ induced by the map $s|_{Y \setminus Y_q}$ is identical to the decomposition induced by $p|_{Y \setminus Y_q}$. Thus $s(Y \setminus Y_q)$ and $p(Y \setminus Y_q)$ are homeomorphic. It follows that $\dim s(Y \setminus Y_q) = \dim p(Y \setminus Y_q) \le \dim V$ and $\dim V \le 1$ since $\varphi : V \to Q$ is light and $\dim Q = 1$.

We also have $\dim f(Y_q) = 1$. Indeed, let $A \in \mathcal{S} \cap W^{-1}(t_0)$. Then A is a fiber of q which is contained in a fiber B of p. Moreover, A is not a singleton and as f is light both $f(A)$ and $f(B)$ are nontrivial continua in X. Recall that $\psi \circ f = \varphi \circ p$. Hence $\psi(f(B)) = \varphi(p(B))$ and as B is a fiber of p, $\varphi(p(B))$ is a singleton and ψ is constant on $f(B)$. It follows that $f(B)$ is contained in $\psi^{-1}(Q_1)$ (which is the union of all fibers of ψ with dimension > 0) and also that $f(Y_q) \subset \psi^{-1}(Q_1)$ and as $\dim \psi^{-1}(Q_1) \le 1$, we have $\dim f(Y_q) \le 1$.

Set $Y_0 = \bigcup\{E : E \in W^{-1}(t), E \subset Y_q\}$. Thus Y_0 consists of those fibers of h which are contained in Y_q. Note that the decomposition $W^{-1}(t_0)$ strictly refines $W^{-1}(t)$, so if $E \in W^{-1}(t)$ then E is a union of fibers of q.

(v) Y_0 is closed in Y

since $\mathcal{D} = \{E : E \in W^{-1}(t), E \subset Y_q\}$ is closed in $\mathcal{C}(Y)$. The latter holds since if $E_k \in \mathcal{D}$ and $E_k \to E$ in $\mathcal{C}(Y)$ then $E \in W^{-1}(t)$ and $E \subset Y_q$ as $W^{-1}(t)$ is closed in $\mathcal{C}(Y)$ and Y_q is closed in Y.

And as $f(Y_0) \subset f(Y_q)$ we also have

(vi) $\dim f(Y_0) \le 1$.

(Note that as f is light, $\dim Y_q \le 1$ too.)

We claim that $\dim g(Y_0) > n$. Once we show this we are done. Indeed, $g(Y_0) = \{f(h(y)) : y \in Y_0\}$. For $y \in Y_0$, $h(y) \in W^{-1}(t)$ is contained in Y_0

and it follows that $g(Y_0) \subset \mathcal{C}(f(Y_0))$. This implies that $\dim \mathcal{C}(f(Y_0)) > n$. Hence $f(Y_0)$ (which is compact by (v) and 1-dimensional by (vi)) must contain a 1-dimensional component T_n with $\dim \mathcal{C}(T_n) > n$.

Aiming at a contradiction assume $\dim g(Y_0) \le n$. Then we may apply Lemma 2.4. Let $Z_1, \ldots, Z_m \subset Y_0$ be from the conclusion of Lemma 2.4 for $\mathcal{K} = W^{-1}(t)$. Then

(vii) the sets $s(Z_i)$, $1 \le i \le m$, are mutually disjoint.

Indeed, the map s is a factor of g. By this we mean that the fibers of s are contained in those of g. Hence $g(Z_i) \cap g(Z_j) = \emptyset$ implies $s(Z_i) \cap s(Z_j) = \emptyset$. If for some $i < j$, $g(Z_i) \cap g(Z_j) \neq \emptyset$ then by Lemma 2.4, $\operatorname{dist}(Z_i, Z_j) \ge \alpha(\varepsilon, n)$. By (iii) each fiber of q has diameter $< \alpha(\varepsilon, n)$, which implies that $q(Z_i) \cap q(Z_j) = \emptyset$ and as s is a factor of q too, $s(Z_i) \cap s(Z_j) = \emptyset$.

(viii) $s(F_1) \cap s(F_2) = \emptyset$.

This holds since q and hence s are ε-maps (by (iii)) and $\varepsilon = \frac{1}{3} \operatorname{dist}(F_1, F_2)$. The same argument combined with the fact that $\operatorname{diam} Z_i < \varepsilon$ also implies that

(ix) for every $1 \le i \le m$, $s(Z_i)$ intersects at most one of the sets $s(F_1)$ and $s(F_2)$.

Set $H_1 = s(F_1) \cup (\bigcup\{s(Z_i) : s(F_1) \cap s(Z_i) \neq \emptyset\})$ and $H_2 = s(F_2) \cup (\bigcup\{s(Z_i) : s(F_1) \cap s(Z_i) = \emptyset\})$. By (ix), $H_1 \cap H_2 = \emptyset$. By (iv), $\dim s(Y \setminus Y_q) \le 1$ hence there exists a closed subset L of $S = s(Y)$ which separates between H_1 and H_2 in S such that $\dim L \cap s(Y \setminus Y_q) = 0$. Then L also separates $s(F_1)$ from $s(F_2)$ and

(x) $L \cap s(\bigcup_{i=1}^m Z_i) = \emptyset$.

By (i), $s^{-1}(L)$ has a component M with $\operatorname{diam} M > r$. Then $M \cap (Y \setminus Y_q) = \emptyset$.

Indeed, by (ii) fibers of s have diameter $< r$. Hence $s(M)$ is a nontrivial continuum in L. If $y \in M \cap (Y \setminus Y_q)$ then $w = s(y) \in L \cap s(Y \setminus Y_q)$. Since Y_q is a union of fibers of s (those fibers which are also fibers of q) we have $s(Y \setminus Y_q) = s(Y) \setminus s(Y_q)$ and hence $w \in L \setminus s(Y_q) = L \cap s(Y \setminus Y_q)$. As $s(Y_q)$ is closed and $\dim L \setminus s(Y_q) = \dim L \cap s(Y \setminus Y_q) = 0$, $\{w\}$ is a component of L and hence $s(M) \subset \{w\}$, which contradicts the fact that $s(M)$ is nontrivial.

It follows that $M \subset Y_q$. Let $A \in W^{-1}(t)$ be such that $A \cap M \neq \emptyset$. As $\operatorname{diam} A < r$, we have $A \subset M \subset s^{-1}(L)$ (by (ii)). So $A \subset Y_0$ and $s(A) \subset L$. By Lemma 2.4, $\bigcup_{i=1}^m Z_i$ intersects A and hence $s(\bigcup_{i=1}^m Z_i)$ intersects L, contradicting (x). This contradiction implies $\dim g(Y_0) > n$ and concludes the proof. ■

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