BULLETIN OF THE POLISH
ACADEMY OF SCIENCES
MATHEMATICS
Online First version

MANIFOLDS AND CELL COMPLEXES

Another example of strongly invertible L-space knot without Khovanov thin surgery

by

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Summary. There are only two known strongly invertible L-space knots whose non-trivial Dehn surgery never yields the double branched cover of a knot or link with thin Khovanov homology. In this paper, we give another example of such an L-space knot.

1. Introduction. An *L-space knot* is a knot that admits a non-trivial surgery yielding an L-space. Initially, all known L-space knots were strongly invertible, but later, Baker and Luecke [5] found infinitely many asymmetric ones.

By using Khovanov homology, Watson [16, Conjecture 30] proposed a characterization of strongly invertible L-space knots. For a knot K with a strong inversion Φ, he introduced an invariant $\varkappa(K,\Phi)$ which is a finite-dimensional vector space with an absolute homological grading h and a relative quantum grading q. He conjectured that a non-trivial knot K with a strong inversion Φ is an L-space knot if and only if $\varkappa(K,\Phi)$ is supported in a single diagonal grading $\delta = q - 2h$. However, Baker, Kegel and McCoy [3] found a pair of counterexamples to this conjecture. They are the knots `t09847` and `o9_30634` in the SnapPy census [1, 2, 7, 8]. These knots are hyperbolic L-space knots with unique strong inversion, but Watson's invariant is supported in two distinct δ-gradings. In addition, no non-trivial Dehn surgery on them yields the double branched cover of a knot or link in the 3-sphere S^3 with thin Khovanov homology.

2020 *Mathematics Subject Classification*: Primary 57K10; Secondary 57K18.
Key words and phrases: L-space knot, Khovanov homology, Watson invariant.
Received 1 May 2026.
Published online 21 May 2026.

DOI: 10.4064/ba260501-8-5

The purpose of the present paper is to give another example of a strongly invertible hyperbolic L-space knot with the same property. Let K be the closure of the 6-braid

$$\beta = [(3,2,4,1,3,5,2,4,3)^3, 3,2,1,1,2,3,2,1,2,2],$$

where the integer i denotes the standard generator of the braid group B_6 of 6 strands. In fact, K is the mirror of `o10_143807` in the census [10].

A non-trivial slope $r \in \mathbb{Q}$ on a knot is said to be (Khovanov) *thin* if it yields the double branched cover of S^3 branched over a knot or link with thin Khovanov homology. (In this paper, we work with reduced Khovanov homology with $\mathbb{Z}_2$ coefficients.)

THEOREM 1.1. *Let K be the knot defined above as the closure of the 6-braid β. Then K has no thin slope.*

We remark that our knot K possesses another interesting property. Its formal semigroup (see [14]) is closed under addition as shown in [13]. In fact, the two knots found in [3] share this property. Thus our knot provides one more positive answer to Question 1.4 of [3]. There are only three known infinite families of hyperbolic L-space knots whose formal semigroups are closed under addition [2, 13]. We expect that all of them would have the same property as discussed in this paper, but it seems to be hard to calculate Watson's invariant.

The argument to prove Theorem 1.1 follows that of [3], but the last part, where we confirm that each symmetry-exceptional slope yields the manifold with the unique description of the double branched cover of S^3, is proved through the Montesinos trick by hand. In [3], it relied on computer calculations. Although the contribution of the present paper is merely the addition of a single example to the existing list, we believe that it represents an interesting phenomenon and will lead to future research.

2. Watson's invariant. In this section, we calculate Watson's invariant for our knot K with (unique) strong inversion Φ. By SnapPy [6], we can confirm that K admits a unique strong inversion. (In fact, the symmetry group of K is $\mathbb{Z}_2$.) Figure 1 shows a strongly invertible position of a link $K_0 \cup C_1 \cup C_2$. We remark that K_0 has writhe 19 in this diagram. By performing (-1)-surgery on C_1 and $1/2$-surgery on C_2, the knot K_0 will be changed into our knot K. See [13].

Take the quotient of the link under the involution around the axis shown there. In Figure 2, we replace the tangles at the quotients of C_1 and C_2 with a (-1)-tangle and a $1/2$-tangle, respectively. Hence the double branched cover of the tangle T, which is the outside of the circle, as shown in Figure 2 (left) recovers the exterior of K. The ∞-tangle filling $T(\infty)$ gives the unknot,

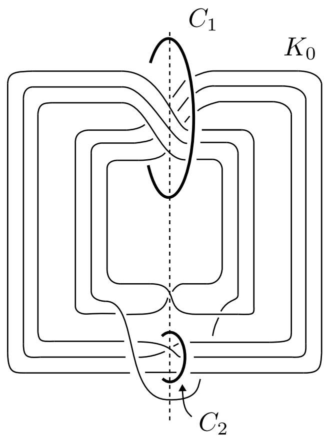

Fig. 1. A strongly invertible position of a link $K_0 \cup C_1 \cup C_2$. After (-1)-surgery on C_1 and $1/2$-surgery on C_2, K_0 is changed into our knot K.

while Figure 2 indicates $T(0)$ with dotted lines. Figures 2–4 show a series of deformations of the tangle T.

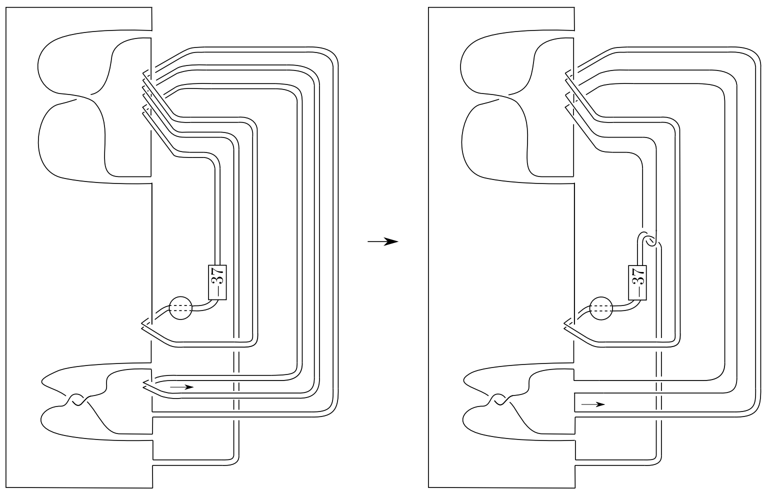

Fig. 2. After taking the quotient under the involution, we replace the tangles at the quotients of C_1 and C_2 with a (-1)-tangle and a $1/2$-tangle, respectively. The outside of the (center) circle defines the tangle T. The box with integer -37 contains 37 (vertical) left-handed half-twists.

If we insert the 0-tangle inside the circle, then the resulting link $T(0)$ gives 0-surgery on K as the double branched cover. In general, for any $r \in \mathbb{Q}$, the surgered manifold $K(r)$ on K is the double branched cover of a knot or link $T(r)$, the filling of T with the rational tangle corresponding to r by the Montesinos trick [11].

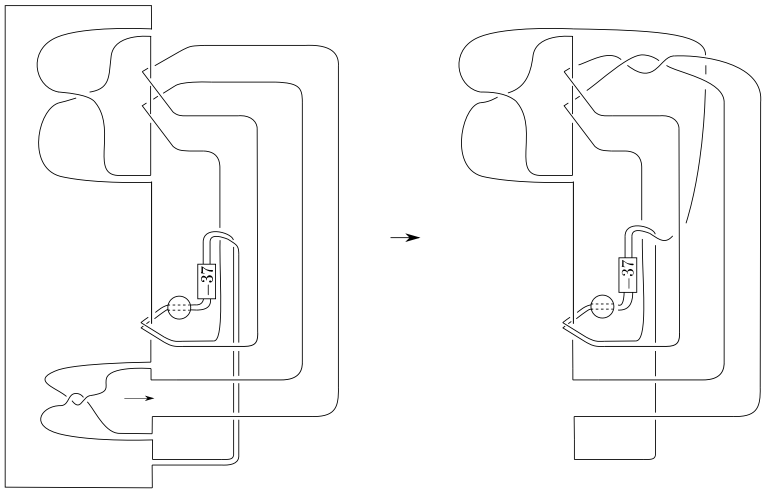

Fig. 3. A deformation of the tangle T

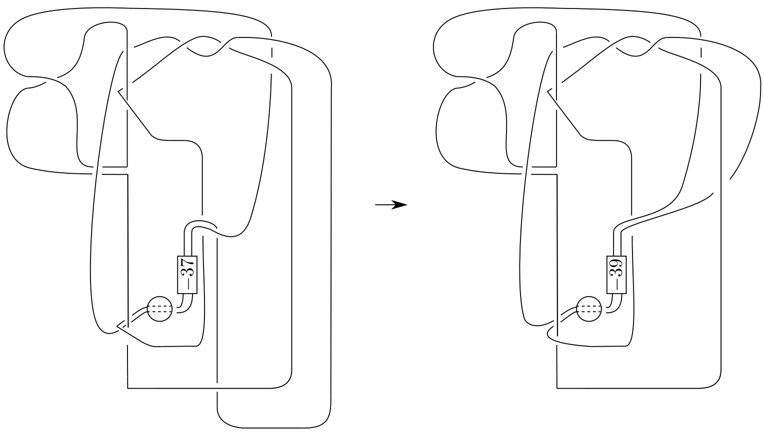

Fig. 4. Further deformation of the tangle T

We quickly review the $\varkappa$-invariant introduced by Watson [16] (see also [3, Section 2]).

For any integer i, let A_i be the reduced Khovanov homology $\mathrm{Kh}(T(i))$ with $\mathbb{Z}_2$ coefficients of a knot or link $T(i)$. (In fact, $T(i)$ is a knot if the integer i is odd, otherwise it is a two-component link.) Owing to the long exact sequence in Khovanov homology associated with a crossing resolution, there is a linear map $f_i\colon A_i \to A_{i-1}$ which preserves the homological grading, but decreases the quantum grading by -1. Let $\mathbb{A}$ be the inverse limit of the system $(A_i, f_i)_{i\in\mathbb{Z}}$. The vector space $\mathbb{A}$ consists of sequences $(x_i)_{i\in\mathbb{Z}}$ such that $x_i \in A_i$ and $f_i(x_i) = x_{i-1}$ for all $i \in \mathbb{Z}$. Let $\mathbb{K}$ be the subspace of $\mathbb{A}$ consisting of sequences (x_i) such that $x_i = 0$ for $i \ll 0$. Then the $\varkappa$-invariant is defined to be the quotient

$$\varkappa(K,\Phi) = \mathbb{A}/\mathbb{K},$$

which is a finite-dimensional vector space. Since the map f_i preserves the homological grading, there is an induced (absolute) grading h on $\varkappa$. Thus $\varkappa(K,\Phi)$ is decomposed into $\bigoplus_{h\in\mathbb{Z}} \varkappa^h(K,\Phi)$, where $\varkappa^h(K,\Phi) \cong \mathbb{A}^h/\mathbb{K}^h$.

Also, since f_i preserves relative quantum gradings, so does $\varkappa$. As in [BKM], we use the convention that the q- and δ-gradings of $\varkappa$ are normalized by agreeing with those of $A_0 = \mathrm{Kh}(T(0))$.

There is a nice property making the calculation of $\varkappa$ very easy. Since $T(\infty)$ is the unknot, $\mathrm{Kh}(T(\infty))$ has dimension 1. Then the exact triangle between A_i, A_{i-1} and $\mathrm{Kh}(T(\infty))$ implies that each map f_i is either (1) injective with cokernel of dimension one or (2) surjective with kernel of dimension 1. Moreover, Watson's result [15, Lemma 4.10] ([1]) guarantees that there exists an integer N such that f_i is surjective for all $i > N+1$ ([2]) and injective for all $i < N$. Thus $\varkappa(K,\Phi)$ is isomorphic to the image of the composition

$$f_N \circ f_{N+1} \colon A_{N+1} \to A_{N-1}.$$

PROPOSITION 2.1. *For the knot K with unique strong inversion Φ, $\varkappa(K,\Phi)$ is supported in two distinct δ-gradings. In fact, it is given as*

h	-6	-5	-4	-3	-2	-1	0	1	2	3	4
$\delta = 41$	1	3	5	7	8	8	7	5	3	1	
$\delta = 39$					1	1	1	2	1	1	1

Moreover, for any $r \in \mathbb{Q}$, the branching set $T(r)$ has Khovanov width 2.

Proof. We computed the reduced Khovanov homology of $T(i)$ for several integers i by using KnotJob [12]. As shown in Figure 5, we found that $N = 40$ as the distinguished value. This immediately gives the values of $\varkappa$. (The δ-grading is also calculated through $T(0)$.)

The rest of the argument follows that of [3, Proposition 3.1]. The computation given in Figure 5 shows that each link $T(i)$ has Khovanov width 2. Then for any $r \in \mathbb{Q}$, $T(r)$ has Khovanov width 2 by [15, Proposition 5.2]. ∎

3. Exceptional cases. A slope r is said to be *exceptional* if $K(r)$ is not hyperbolic, and *symmetry-exceptional* if $K(r)$ is a hyperbolic 3-manifold with a symmetry group larger than that of K [3, 4].

(1) This is not explicitly stated there. See [3, Section 2].

(2) There is a typo in [3].

$i \backslash h$	-6	-5	-4	-3	-2	-1	0	1	2	3	4
44	$\bullet$	$\bullet\bullet\bullet$	$\bullet\bullet\bullet\bullet\bullet$ $\underline{\bullet}$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\underline{\bullet}$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\bullet\underline{\bullet}\circ$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\bullet\circ$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\underline{\bullet}\circ$	$\bullet\bullet\bullet\bullet\bullet$ $\circ\circ$	$\bullet\bullet\bullet\circ$	$\bullet\circ$	$\circ$
43	$\bullet$	$\bullet\bullet\bullet$	$\bullet\bullet\bullet\bullet\bullet$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\underline{\bullet}$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\bullet\underline{\bullet}\circ$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\bullet\circ$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\underline{\bullet}\circ$	$\bullet\bullet\bullet\bullet\bullet$ $\circ\circ$	$\bullet\bullet\bullet\circ$	$\bullet\circ$	$\circ$
42	$\bullet$	$\bullet\bullet\bullet$	$\bullet\bullet\bullet\bullet\bullet$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\bullet\underline{\bullet}\circ$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\bullet\circ$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\underline{\bullet}\circ$	$\bullet\bullet\bullet\bullet\bullet$ $\circ\circ$	$\bullet\bullet\bullet\circ$	$\bullet\circ$	$\circ$
41	$\bullet$	$\bullet\bullet\bullet$	$\bullet\bullet\bullet\bullet\bullet$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\bullet\circ$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\bullet\circ$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\underline{\bullet}\circ$	$\bullet\bullet\bullet\bullet\bullet$ $\circ\circ$	$\bullet\bullet\bullet\circ$	$\bullet\circ$	$\circ$
40	$\bullet$	$\bullet\bullet\bullet$	$\bullet\bullet\bullet\bullet\bullet$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\bullet\circ$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\bullet\circ$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\underline{\bullet}\circ\underline{\circ}$	$\bullet\bullet\bullet\bullet\bullet$ $\circ\circ$	$\bullet\bullet\bullet\circ$	$\bullet\circ$	$\circ$
39	$\bullet$	$\bullet\bullet\bullet$	$\bullet\bullet\bullet\bullet\bullet$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\bullet\circ$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\bullet\circ$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\circ\underline{\circ}$	$\bullet\bullet\bullet\bullet\bullet$ $\circ\circ$	$\bullet\bullet\bullet\circ$	$\bullet\circ$	$\circ$
38	$\bullet$	$\bullet\bullet\bullet$	$\bullet\bullet\bullet\bullet\bullet$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\bullet\circ$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\bullet\circ$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\circ\underline{\circ}$	$\bullet\bullet\bullet\bullet\bullet$ $\circ\circ$	$\bullet\bullet\bullet\circ\underline{\circ}$	$\bullet\circ$	$\circ$
37	$\bullet$	$\bullet\bullet\bullet$	$\bullet\bullet\bullet\bullet\bullet$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\bullet\circ$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\bullet\circ$	$\bullet\bullet\bullet\bullet\bullet$ $\bullet\bullet\circ\underline{\circ}$	$\bullet\bullet\bullet\bullet\bullet$ $\circ\circ$	$\bullet\bullet\bullet\circ\underline{\circ}$	$\bullet\circ\underline{\circ}$	$\circ$

Fig. 5. A_i and the $\varkappa$-invariant of K. A $\bullet$ denotes a copy of $\mathbb{Z}_2$ in relative δ-grading 41, and a $\circ$ denotes a copy of $\mathbb{Z}_2$ in relative δ-grading 39. The generators with an underline do not survive in $\varkappa$. In fact, the image of the composition $f_{40} \circ f_{41} \colon A_{41} \to A_{39}$ gives $\varkappa$.

Lemma 3.1. *The knot K has no exceptional slope.*

Proof. By SnapPy, we can calculate slopes with length at most 6; they are the slopes $38, 39, 40, 41$. However, SnapPy also verifies that none of these yields a non-hyperbolic 3-manifold. ∎

Lemma 3.2. *The symmetry-exceptional slopes of K are* 39, 40 *and* 42.

Proof. By [9, Theorem 7.28], if r is a symmetry-exceptional slope, then

$$L(r) \leq \max\left\{10.1, \sqrt{\frac{2\pi}{\mathrm{sys}(K)} + 58}\right\},$$

where $L(r)$ is the normalized length of r, and $\mathrm{sys}(K)$ is the length of the shortest geodesic in $S^3 - K$. (See also [3, Lemma 4.5].) SnapPy tells us that $\mathrm{sys}(K) > 0.55$, so $L(r) \leq 10.1$. Then such slopes are calculated to be

$$\begin{aligned}
&27, 28, \dots, 52, \quad \frac{a}{2}\ (67 \leq a \leq 91,\ 2 \nmid a),\\
&\frac{b}{3}\ (107 \leq b \leq 128,\ 3 \nmid b), \quad \frac{c}{4}\ (147 \leq c \leq 167,\ 2 \nmid c),\\
&\frac{d}{5}\ (187 \leq d \leq 206,\ 5 \nmid d), \quad \frac{e}{6}\ (229 \leq e \leq 241,\ 2 \nmid e,\ 3 \nmid e),\\
&\frac{f}{7}\ (270 \leq f \leq 281,\ 7 \nmid f).
\end{aligned}$$

Among them, only the slopes $39, 40, 42$ yield a manifold with symmetry group larger than $\mathbb{Z}_2$. (In fact, the symmetry group is $\mathbb{Z}_2^2$ for them.) ∎

PROPOSITION 3.3. *For each symmetry-exceptional slope $r \in \{39, 40, 42\}$, $K(r)$ has the unique description as the double branched cover of S^3. In particular, these slopes are not thin.*

Proof. For each symmetry-exceptional slope $r \in \{39, 40, 42\}$, we examine the description of $K(r)$ as the double branched cover. Since the symmetry group of $K(r)$ is $\mathbb{Z}_2^2$, there are three involutions on $K(r)$. Except for one of them, the quotient manifold is not S^3. This is established by Lemmas 3.4, 3.5 and 3.6 below. Hence for $r \in \{39, 40, 42\}$, the unique description of $K(r)$ as the double branched cover of S^3 is given by that of $T(r)$, which is not thin by Proposition 2.1. ∎

LEMMA 3.4. *For $K(39)$, the quotient manifolds under the three involutions are S^3 and the lens spaces $L(3, 2)$ and $L(13, -2)$.*

Proof. Let L be the mirror of the link `L8a12`, which is the 2-bridge link $C(5, 3)$ in Conway's notation. We first claim that the surgery diagram $L(-5/2, -5/2)$, as shown in Figure 6 (left), represents $K(39)$. Since L is strongly invertible, this is verified by the Montesinos trick [11]. After taking the quotient under the involution, we put the tangle $-5/2$ as in Figure 6 (right). We can confirm that the resulting knot is $T(39)$, which is `K13n2014` in the census. Thus the surgery diagram represents $K(39)$. (Of course, the quotient manifold is S^3 in this case.)

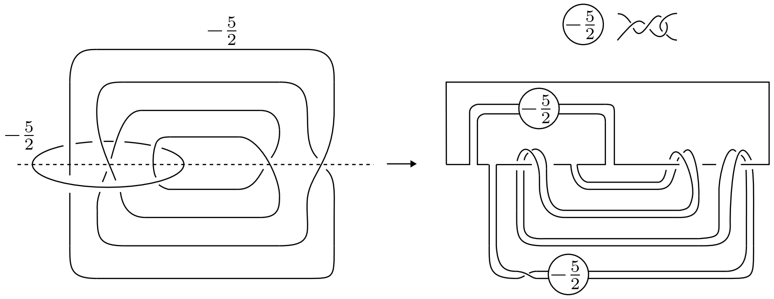

Fig. 6. A surgery diagram $L(-5/2, -5/2)$ in a strongly invertible position. After taking the quotient under the involution, put the tangles. The resulting knot is $T(39)$.

On the other hand, L has cyclic period 2. In fact, there are two such actions. Figure 7 shows them with their axes A_1 and A_2. Take the quotient under the action around A_1. It is straightforward to see that the slope $-5/2$ goes down to $3/2$ on the unknot. Hence the ambient manifold is the lens space $L(3, 2)$. In other words, $K(49)$ is the double branched cover of $L(3, 2)$ branched over the knot $\widetilde{A}_1$, which is the image of A_1. Similarly, the quotient manifold by the action around A_2 is $L(13, -2)$. We omit the details. ∎

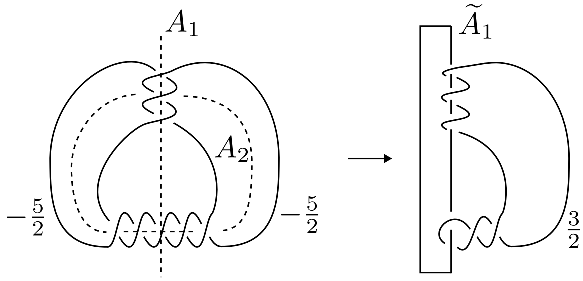

Fig. 7. Two axes A_1 and A_2 of cyclic actions on L. By taking the quotient under the action around A_1, the slope $-5/2$ goes down to $3/2$.

Lemma 3.5. *For $K(40)$, the quotient manifolds under the three involutions are S^3, $L(10, -3)$ and $L(4, -1)$.*

Proof. We use the link L (= `L8a17`), which is the Montesinos link $(-1/4, 3/2, 1/2)$. We claim that the surgery diagram $L(-7/5, -2, -2)$ as shown in Figure 8 represents $K(40)$. After taking the quotient under the involution, we perform tangle replacements.

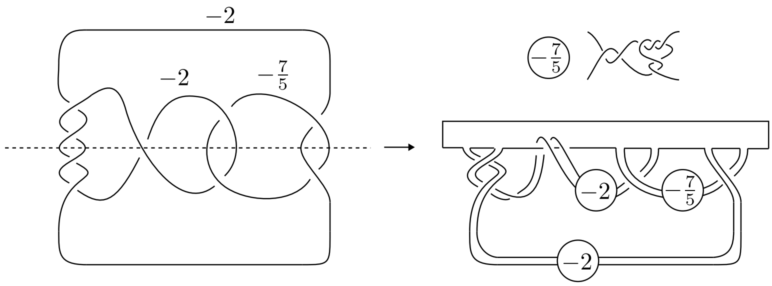

Fig. 8. The surgery diagram $L(-7/5, -2, -2)$ in a strongly invertible position. After taking the quotient under the involution, we perform tangle replacements.

We need to identify the resulting link with $T(40)$. (This can be done by SnapPy, but we confirm it by hand.) The link has an unknotted component, and the other is the knot 7_7. This is the same as $T(40)$. By deforming two links as shown in Figure 9, we see that these two links are the same. Hence the surgery diagram represents $K(40)$.

On the other hand, the link L has other symmetries of order 2. One is shown in Figure 10. It keeps one component of L with slope $-7/5$ invariant while swapping the other two with slope -2. By the Montesinos trick, we see that the quotient manifold is $L(4, -1)$.

The other is a cyclic period of order 2 as shown in Figure 11. As before, the action keeps one component with slope $-7/5$ but swaps the others. We can see that the quotient manifold is $L(10, -3)$. ∎

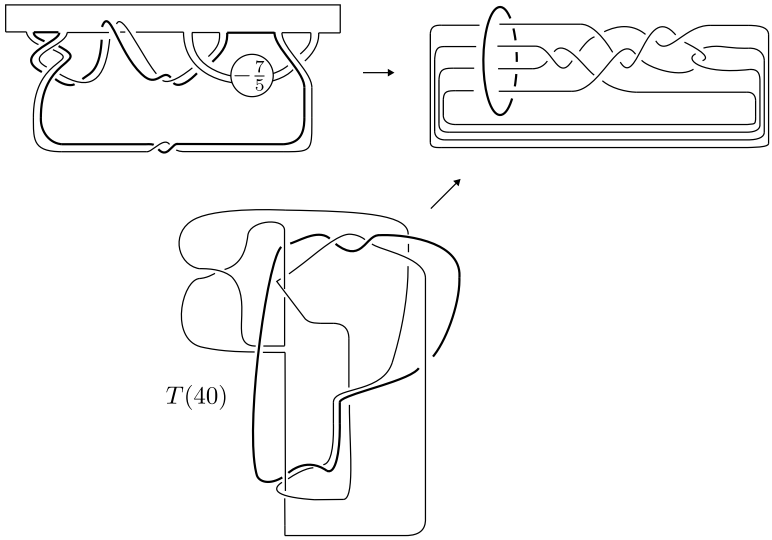

Fig. 9. The resulting link has the unknotted component. This link is equivalent to $T(40)$.

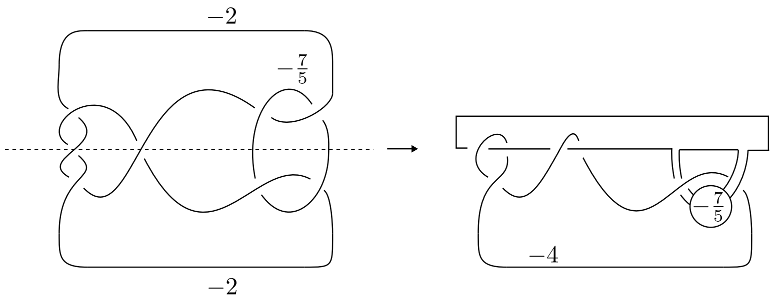

Fig. 10. The quotient manifold is $L(4,-1)$.

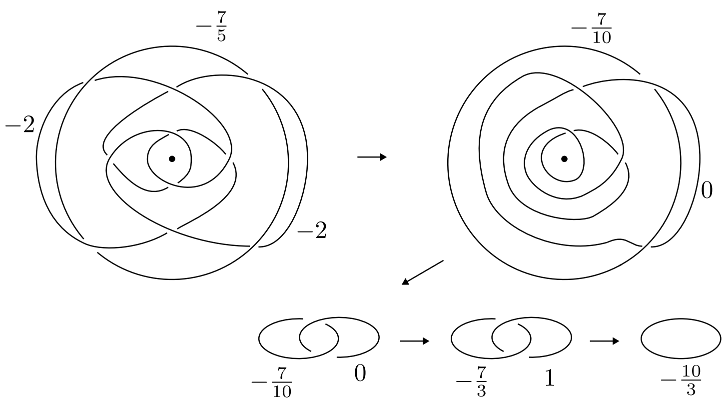

Fig. 11. The dot shows the axis of the cyclic action. The quotient manifold is $L(10,-3)$.

Lemma 3.6. *For $K(42)$, the quotient manifolds under the three involutions are S^3, $L(3,1)$ and $L(7,-4)$.*

Proof. Let L be the link `L12n1226`, which is the pretzel link $P(-3,2,5,-2)$ as shown in Figure 12. We claim that the surgery diagram $L(-7/2,6)$ represents $K(42)$. By the Montesinos trick, it suffices to identify the branch set obtained after the tangle replacements with $T(42)$. As shown in Figure 13, it is equivalent to $T(42)$.

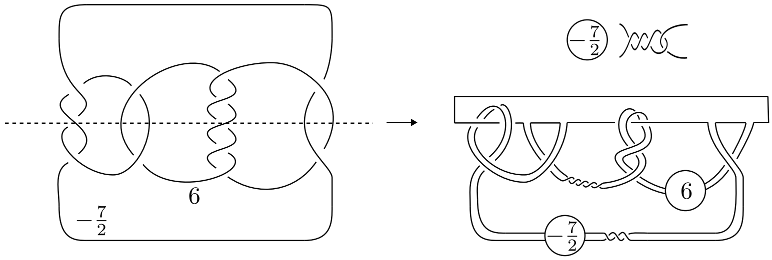

Fig. 12. The surgery diagram $L(-7/2,6)$ in a strongly invertible position.

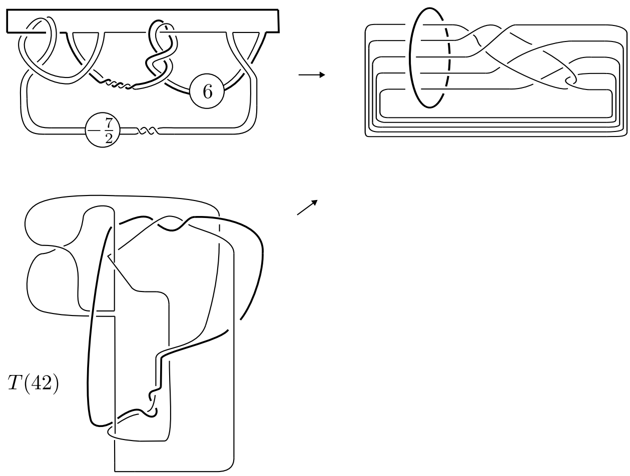

Fig. 13. The branch set is $T(42)$.

The link L has two more symmetries, which are depicted as the involutions around A_1 and A_2 in Figure 14. The quotient manifolds are $L(3,1)$ and $L(7,-4)$, respectively, because the slope 6 goes down to 3 on the unknot for the former, and the slope $-7/2$ to $-7/4$ on the unknot for the latter. ∎

We finish the article by showing that K has no thin slopes.

Proof of Theorem 1.1. Recall that K has the symmetry group $\mathbb{Z}_2$ generated by the unique strong inversion, and that K has no exceptional slope

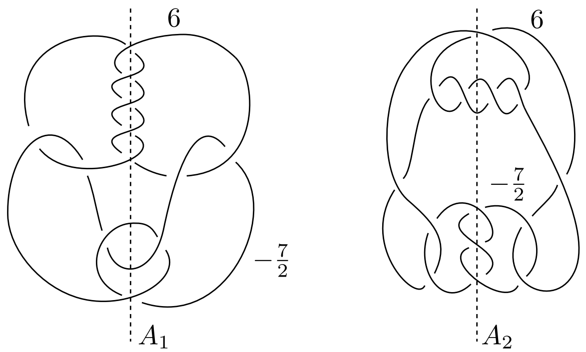

Fig. 14. Two more symmetries for $L(-7/2, 6)$.

by Lemma 3.1. Hence, the symmetry group of $K(r)$ is the same as that of K, apart from symmetry-exceptional slopes. This means that $K(r)$ has the unique description as the double branched cover of a knot or link in S^3. By Proposition 2.1, such branching set $T(r)$ has width 2. On the other hand, any symmetry-exceptional slope of K is not thin by Lemma 3.2 and Proposition 3.3. Thus K has no thin slope. ∎

Acknowledgements. The author would like to thank Marc Kegel for helpful communication.

Funding. The author has been supported by JSPS KAKENHI Grant Number JP25K07004.

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