

# EVOLUTION EQUATIONS: EXISTENCE, REGULARITY AND SINGULARITIES

*Editors of the Volume*

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## PREFACE

The minisemester “Evolution Equations: Existence, Regularity and Singularities” was held at the Banach Center from September 21 to October 2, 1998. The subject of the minisemester was various aspects of the theory of evolution equations. About fifty participants from: the Czech Republic, France, Germany, Great Britain, Greece, Lithuania, the Netherlands, Italy, Japan, Poland, Russia, Slovakia, Spain and the USA gave 40-minute talks on the following main topics:

- existence of global-in-time solutions (H.-D. Alber, P. Biler, L. Boccardo, M. Cannone, B. Ducomet, P. Dubovski, E. Feireisl, M. Fila, J. Gawinecki, J. Goncerzewicz, G. Lieberman, P. Mucha, R. Picard, V. Shutyaev, D. Schmitt, I. Straskraba, M. Tsutsumi, E. Zadrzyńska, W. Zajączkowski),
- blow-up of solutions (P. Biler, M. Fila, T. Nadzieja, G. Rein, D. Schmitt, Ph. Souplet, D. Tzanetis),
- long time behaviour (J. Cholewa, B. Ducomet, K. Gęba, G. Karch, Ph. Laurençot, F. Issard-Roch, D. Tzanetis, Y. Yamada),
- Navier–Stokes and related models in hydrodynamics (M. Cannone, B. Ducomet, G. P. Galdi, J. Malek, G. Łukaszewicz, K. Pileckas, G. Seregin, E. Zadrzyńska, W. Zajączkowski),
- mathematical models in physics and chemistry (H.-D. Alber, R. Balean, P. Biler, P. Dubovski, P. Gwiazda, O. John, P. Mucha, T. Nadzieja, G. Rein, M. Tsutsumi, D. Tzanetis),
- qualitative properties of solutions and singularities (D. Andreucci, H. Drobchenko, S. Ebenfeld, E. Feireisl, G. P. Galdi, B. Gilding, M. Herrero, P. Kaplicky, J. Malek, A. Milani, M. Reissig, J. Stara, Y. Tsutsumi, J. Velázquez),
- free boundary problems and phase transitions (B. Gilding, F. Issard-Roch, P. Rybka, J. Velázquez, E. Zadrzyńska, W. Zajączkowski),
- differential-functional equations (A. Augustynowicz, T. Człapiński, Z. Kamont, H. Leszczyński).

Only 16 participants contributed to these proceedings.

A complete list of lectures is given on p. 8.

*The Editors*

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- D. Andreucci (Italy), *Finite speed of propagation and mass transport in second and higher order parabolic equations.*
- A. Augustynowicz (Poland), *The existence of analytic solutions to the first order pde's with delays.*
- R. Balean (Germany), *Granular avalanche flow down a curved slope: The existence of entropy solutions.*
- P. Biler (Poland), *Global in time and exploding solutions for nonlocal quadratic evolution problems.*
- L. Boccardo (Italy), *Summability of minima of functionals.*
- M. Cannone (France), *A survey of the existence, uniqueness and regularity results for the Navier-Stokes equations in the whole three-dimensional space.*
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- H. Drobchenko (Russia), *On differential systems with maximal monotone multi-valued and continuous single-valued elements.*
- P. Dubovski (Russia), *Scalar conservation laws and infinite linear systems of pde's; Solvability of spatially inhomogeneous Becker-Döring equations.*
- B. Ducomet (France), *Some stability results for 1-d and spherical Navier-Stokes-Poisson system.*
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- E. Feireisl (Czech Republic), *On the qualitative properties of weak solutions to the N-S equations.*
- M. Fila (Slovakia), *Global solutions of a semilinear parabolic equation.*

- G. P. Galdi (Italy), *On the planar, steady-state problem for the Navier-Stokes equations in an exterior domain.*
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- B. H. Gilting (the Netherlands), *Interfaces in the solutions of first and second order conservation laws.*
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- F. Issard-Roch (France), *Asymptotic behaviour of the solutions of Stefan problem with a kinetic term on the free boundary.*
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- P. Kaplicky (Czech Republic),  *$C^{1,\alpha}$ -solutions to a class of nonlinear fluids in 2d.*
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- A. Milani (U.S.A.), *On the diffusion phenomenon for nonlinear hyperbolic waves.*
- P. Mucha (Poland), *Global existence of solutions of Einstein-Boltzmann equations in the case of spherical symmetry; On the diffusion phenomenon for nonlinear hyperbolic waves.*
- T. Nadzieja (Poland), *Poisson-Boltzmann equation.*
- R. Picard (Germany), *Evolution equations as operator equations in lattices of Hilbert spaces.*
- K. Pileckas (Lithuania), *The properties of solutions for Stokes and Navier-Stokes problem in an infinite layer.*
- G. Rein (Germany), *Gravitational collapse of collisionless matter.*

- M. Reissig (Germany),  *$L_p$ - $L_q$  decay estimates and their applications in the theory of nonlinear wave equations.*
- P. Rybka (Poland), *Convergence theorems for equations related to phase transitions.*
- D. Schmitt (France), *Blow-up in reaction-diffusion systems with dissipation of mass.*
- G. Seregin (Russia), *Disjointness of solutions to the modified Navier-Stokes equations.*
- V. Shutyaev (Russia), *Necessary and sufficient conditions for solvability of the initial-boundary value transport problem; Solvability of quasilinear evolution data assimilation problems in a scale of Hilbert spaces.*
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- M. Tsutsumi (Japan), *Well-posedness of a linear heat equation with critical singular potential; On the time dependent Ginzburg-Landau-Maxwell equation.*
- Y. Tsutsumi (Japan), *On the coupled system of nonlinear wave equations with different propagation speeds.*
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- E. Zadrzyńska (Poland), *On stability of equilibrium solutions for free boundary problems for equations of viscous compressible heat conducting fluids.*
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