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An algorithmic construction of cyclic p-extensions of fields, with characteristic different from p, not containing the pth roots of unity

by

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Since the nineteenth century, when Kummer theory was first developed, we know how to build the cyclic p-extensions of fields E/F containing sufficiently many roots of unity, more precisely when F contains the p^nth roots where $p^n = [E : F]$ is the degree of E/F (cf. [6, p. 289]). In 1989, Karpilovsky [5, p. 389] set the problem of finding an explicit description of all cyclic p-extensions. In 2002, the author [7] gave an algorithmic construction of any cyclic p-extension of fields with characteristic different from p, containing only the pth roots of unity. The next and final step is to eliminate any primitive pth root of unity in the extension. This is what is done in the theorem stated below. The method uses the notion of a Galois average introduced in [8] (see also [4]). As a corollary for $p = 3$, we exhibit an algorithmic computable primitive element for any cyclic 3-extension.

Here, the notations of [7] have been changed to be more algorithmic and functional in the cyclotomic descent (so, in this aesthetic sense also, this paper is an improvement of [7]).

To state the theorem we have to recall briefly the definition of a p-Galois average [8, Sect. 2]. Let p be a prime number and F be a field of characteristic not p. Denote by E/F a finite Galois extension where the top field E contains the group μ_p of pth roots of unity. Let G_p be the characteristic subgroup of $G := \mathrm{Gal}(E/F)$ generated by all the p-Sylow subgroups of G (with $G_p = \mathbf{1}$ whenever $p \nmid |G|$). A *p-Galois average* of E/F is an endomorphism of the ($\mathbb{F}_p := \mathbb{Z}/p\mathbb{Z}$)-vector space

$$(E^\times/E^{\times p})^{G_p} = \{\overline{x} \in E^\times/E^{\times p} \mid \forall\gamma \in G_p\ \gamma(\overline{x}) = \overline{x}\}.$$

Precisely, for each $\varphi \in \mathrm{Hom}(G/G_p, \mathbb{F}_p^\times)$, the *$p$-Galois average* of E/F for φ,

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which we denote $\mathrm{ga}^{\varphi}_{E/F}$, is defined by

$$\mathrm{ga}^{\varphi}_{E/F} : (E^{\times}/E^{\times p})^{G_p} \to (E^{\times}/E^{\times p})^{G_p}, \qquad \overline{x} \mapsto \Big(\prod_{\overline{\gamma}\in G/G_p} \gamma(\overline{x})^{\varphi(\overline{\gamma}^{-1})} \Big)^{d^{-1}},$$

where d is the order of G/G_p and d^{-1} its inverse in $\mathbb{F}_p^{\times}$.

Theorem. *Let p be an odd prime number. Let D_0 be a field, of characteristic different from p, not containing the pth roots of unity: $D_0 \cap \mu_p = \{1\}$. Let D_m/D_0 be a cyclic p-extension of degree p^m ($m \in \mathbb{N} \setminus \{0\}$). For each $n \in \{0, \dots, m\}$, denote by D_n the subfield of D_m with degree $[D_n : D_0] = p^n$, and C_n its pth cyclotomic translation: $C_n = D_n(\mu_p)$.*

(1) *Let ζ_p be a primitive pth root of unity. There exists a field $D_{m+1} \supset D_m$ such that D_{m+1}/D_0 is a cyclic p-extension of degree p^{m+1} if and only if there exists an element $\xi \in C_m$ with norm ζ_p over C_0: $N_{C_m/C_0}(\xi) = \zeta_p$.*

(2) *Assume that in (1) the field D_{m+1} exists. Then:*

(2.1) *A primitive element x_{m+1} of $C_{m+1} := D_{m+1}(\mu_p)$ over C_m is given by*

$$x^p_{m+1} = c_{0(m+1)} y_m$$

where, in succession for each $n \in \{0, \dots, m\}$, a primitive element x_{n+1} of C_{n+1} over C_n is given by

$$x^p_{n+1} = c_{0(n+1)} y_n.$$

All of this with the following definitions:

- *$c_{02}, \dots, c_{0(n+1)}, \dots, c_{0(m+1)}$ are fixed elements of $C_0^{\times}$;*
- *$y_0 \in C_0^{\times}/C_0^{\times p}$;*
- *for all $n \in \{1, \dots, m\}$,*

$$y_n := x_n \prod_{i=1}^{p-1} \sigma_n^{ip^{n-1}}(z_n^i), \quad z_n := \prod_{j=0}^{p^{n-1}-1} \sigma_n^j(N_{C_m/C_n}(\xi));$$

- *σ_n is the generator of the cyclic group $\mathrm{Gal}(C_n/C_0)$ defined by*

$$\frac{\sigma_n(x_n)}{x_n} = N_{C_m/C_{n-1}}(\xi), \qquad \forall n \in \{2, \dots, m\},\ \sigma_{n|C_{n-1}} = \sigma_{n-1}.$$

(2.2) *The trace over D_{m+1} of x_{m+1} in (2.1) provides a primitive element of D_{m+1} over D_m:*

$$D_{m+1} = D_m(\mathrm{Tr}_{D_{m+1}/D_m}(x_{m+1})).$$

(3) *Conversely, assume that there exists an element $\xi \in C_m$ with norm $N_{C_m/C_0}(\xi) = \zeta_p$ (cf. (1)). As in (2), the datum of the cyclic p-extension D_m/D_0 allows us to define algorithmically the elements*

$$y_0, y_1, \dots, y_{m-1}, x_1, \dots, x_m, \sigma_1, \dots, \sigma_m.$$

(3.1) *$(C_m(y_m^{1/p})/C_0)$ is a cyclic p-extension of degree p^{m+1}.*

(3.2) *Let* $\overline{\eta} \in \mathrm{Hom}(\mathrm{Gal}(C_0/D_0), \mathbb{F}_p^\times)$ *be the "cyclotomic homomorphism" defined by*

$$\forall \tau \in \mathrm{Gal}(C_0/D_0)\ \forall \zeta \in \mu_p \quad \tau(\zeta) = \zeta^{\overline{\eta}(\tau)}.$$

For any element x_{m+1} *(in an algebraic closure of* D_0 *containing* C_m*) such that*

$$\overline{x_{m+1}^p} = \mathrm{ga}_{C_m/D_0}^{\overline{\eta}}(\overline{y}_m),$$

the following properties hold for the field $C_{m+1} := C_m(x_{m+1})$:

- C_{m+1}/C_0 *is a cyclic* p*-extension of degree* p^{m+1};
- C_{m+1}/D_0 *is a Galois extension*;
- $\mathrm{Gal}(C_{m+1}/C_0)$ *admits a unique complement, say* T_{m+1}, *in* $\mathrm{Gal}(C_{m+1}/D_0)$;
- $\mathrm{Gal}(C_{m+1}/D_0)$ *splits into the direct product*

$$\mathrm{Gal}(C_{m+1}/D_0) = \mathrm{Gal}(C_{m+1}/C_0) \times T_{m+1}.$$

(3.3) *Let* D_{m+1} *be the fixed field of* T_{m+1} *in* C_{m+1}: $D_{m+1} := C_{m+1}^{T_{m+1}}$. *Necessarily* D_{m+1} *contains* D_m, *and* D_{m+1}/D_0 *is a cyclic* p*-extension of degree* p^{m+1}.

(3.4) *Finally, a primitive element of* D_{m+1} *over* D_m *is the following*:

$$D_{m+1} = D_m\Big(\sum_{\tau \in T_{m+1}} \tau(x_{m+1})\Big)$$

where x_{m+1} *is any element in* (3.2).

Proof. (1) Cf. [8, Prop. 7.3].

(2) (2.1) Since C_1/C_0 is a Kummer extension of degree p, the element y_0 exists. From y_0, we deduce x_1 by $x_1^p = y_0$ and σ_1 such that

$$\sigma_1(x_1)/x_1 = \zeta_p = N_{C_m/C_0}(\xi).$$

From x_1 and σ_1, we define

$$y_1 := x_1 \prod_{i=1}^{p-1} \sigma_1^i(z_1^i), \qquad z_1 := N_{C_m/C_1}(\xi).$$

Then we deduce x_2, σ_2, and so on until we find x_n and σ_n. Assume now that y_n is defined as in the statement of the Theorem. We have

$$\frac{\sigma_n(y_n)}{y_n} = N_{C_m/C_{n-1}}(\xi) \prod_{i=1}^{p-1} \sigma_n^{ip^{n-1}}\left(\left(\frac{\sigma_n(z_n)}{z_n}\right)^i\right)$$

where

$$\frac{\sigma_n(z_n)}{z_n} = \frac{\sigma_n^{p^{n-1}}(N_{C_m/C_n}(\xi))}{N_{C_m/C_n}(\xi)}.$$

Then, by a straightforward calculation, we get

$$\frac{\sigma_n(y_n)}{y_n} = (N_{C_m/C_n}(\xi))^p.$$

At this point, instead of the proof of [7], it is more convenient to use

Lemma. *Let L/K be a cyclic p-extension with $\mu_p \subset K$, and $\langle\sigma\rangle = \mathrm{Gal}(L/K)$. Let M/L be a cyclic extension of degree p. For M/K to be a cyclic extension, it is necessary and sufficient that for any $x \in L$ such that $M = L(\sqrt[p]{x})$, there exists $\lambda \in L$ for which*

$$\sigma(x)/x = \lambda^p$$

with the norm $N_{L/K}(\lambda) \neq 1$.

Proof. Standard fact from Galois theory (see [9, p. 15]). ∎

Here, since $N_{C_n/C_0}(N_{C_m/C_n}(\xi)) = N_{C_m/C_0}(\xi) = \zeta_p \neq 1$, the Lemma ensures that $C_n(y_n^{1/p})/C_0$ is a cyclic extension (of degree p^{n+1}). But so is C_{n+1}/C_0 (by translation of D_{n+1}/D_0). Let us write $C_{n+1} = C_n(a_n^{1/p})$ with $a_n \in C_n$. By the Lemma again, there exist $\lambda_n \in C_n$ and $i \in \mathbb{F}_p^\times$ for which

$$\sigma_n(a_n)/a_n = \lambda_n^p, \quad N_{C_n/C_0}(\lambda_n N_{C_m/C_n}(\xi)^{-i}) = 1.$$

Then we apply the Hilbert Theorem 90: there exist $b_n \in C_n^\times$, and consequently $a_0 \in C_0^\times$, such that

$$a_n = a_0 b_n^p y_n^i.$$

To complete the proof of (2.1), it suffices to choose $i' \in \mathbb{N}$ with $ii' = 1 + qp$ ($q \in \mathbb{N}$); indeed, for

$$x_{n+1} := b_n^{-i'} y_n^{-q} a_n^{i'/p}, \quad c_{0n} := a_0^{i'} \in C_0^\times,$$

we have $C_{n+1} = C_n(x_{n+1})$ and $x_{n+1}^p = c_{0n} y_n$.

(2.2) Standard fact from number theory: in our situation see [11, Thm. 3.2(2)] or more generally [2, p. 245, Thm. 5.3.5(2)].

(3) (3.1) By the same calculation as in (2.1), we get

$$\sigma_m(z_m)/z_m = \sigma_m^{p^{m-1}}(\xi)/\xi, \quad \sigma_m(y_m) = \xi^p y_m.$$

Then it suffices to apply the Lemma.

(3.2) A direct application of [8, Thm. 5.4(1.2)].

(3.3) Here we fill a gap in the proof of [8, Thm. 5.4(2.1)]. Indeed, why does D_{m+1} necessarily have to contain D_m? We have the Galois parallelogram [10]

$$[D_0, C_0, C_{m+1}, D_{m+1}].$$

By construction, the following restriction homomorphisms exist:

$$\begin{aligned} r_{C_{m+1},C_0} &: \mathrm{Gal}(C_{m+1}/D_0) \to \mathrm{Gal}(C_0/D_0),\\ r_{C_{m+1},C_m} &: \mathrm{Gal}(C_{m+1}/D_0) \to \mathrm{Gal}(C_m/D_0),\\ r_{C_m,C_0} &: \mathrm{Gal}(C_m/D_0) \to \mathrm{Gal}(C_0/D_0), \end{aligned}$$

and clearly

$$r_{C_{m+1},C_0} = r_{C_m,C_0} \circ r_{C_{m+1},C_m}.$$

From $[D_0, C_0, C_{m+1}, D_{m+1}]$, we deduce $|T_{m+1}| = [C_0 : D_0] \mid p-1$, and

$$\begin{aligned} |T_{m+1}| &= |r_{C_{m+1},C_0}(T_{m+1})| = |r_{C_m,C_0}(r_{C_{m+1},C_m}(T_{m+1}))|\\ &\le |r_{C_{m+1},C_m}(T_{m+1})| \le |T_{m+1}|; \end{aligned}$$

then

$$|r_{C_{m+1},C_m}(T_{m+1})| = |T_{m+1}| \mid p-1.$$

Since $[C_m : C_0] = p^n$, we get

$$r_{C_{m+1},C_m}(T_{m+1}) \cap \mathrm{Gal}(C_m/C_0) = \mathbf{1}.$$

Consequently, the image $r_{C_{m+1},C_m}(T_{m+1})$ is a complement of $\mathrm{Gal}(C_m/C_0)$ into $\mathrm{Gal}(C_m/D_0)$. But so is $\mathrm{Gal}(C_m/D_m)$ which is a normal subgroup. Then necessarily, by the Hauptsatz of Zassenhaus [3, p. 127, 18.2],

$$r_{C_{m+1},C_m}(T_{m+1}) = \mathrm{Gal}(C_m/D_m);$$

and so

$$D_m = C_m^{\mathrm{Gal}(C_m/D_m)} = C_m^{r_{C_{m+1},C_m}(T_{m+1})} \le C_{m+1}^{T_{m+1}} = D_{m+1}.$$

To conclude the proof, it now suffices to apply point (2) of Theorem 5.4 in [8]. ∎

Corollary. *When $p = 3$, the conclusion of the Theorem holds for*:

(3.2) $\overline{x_{m+1}^3} = \overline{N_{C_m/D_m}(y_m)\, y_m}$;
(3.4) $D_{m+1} = D_m(x_{m+1} + x_{m+1}^2/y_m)$.

Proof. Indeed, in the definition of the Galois average $\mathrm{ga}^{\overline{\eta}}_{C_m/D_0}$, we have $G_3 = \mathrm{Gal}(C_m/C_0)$, whence $G/G_3 = \mathrm{Gal}(C_0/D_0)$ and $d = 2$ so $d^{-1} = 2$ (into $\mathbb{F}_3^\times$). Let us write $\mathrm{Gal}(C_0/D_0) =: \{1, \tau_0\}$ where $\overline{\eta}(\tau_0) = 2$. The Galois parallelogram $[D_0, C_0, C_m, D_m]$ implies that $\mathrm{Gal}(C_m/D_m) =: \{1, \tau_m\}$ with $\overline{\tau}_m = \tau_0$. Therefore,

$$\mathrm{ga}^{\overline{\eta}}_{C_m/D_0}(\overline{y}_m) = (\overline{y}_m \tau_m(\overline{y}_m)^2)^2 = \overline{y}_m^2 \tau_m(\overline{y}_m) = N_{C_m/D_m}(\overline{y}_m)\, \overline{y}_m.$$

Now, for any element x_{m+1} such that

$$\overline{x_{m+1}^3} = \overline{N_{C_m/D_m}(y_m) y_m},$$

let $C_{m+1} := C_m(x_{m+1})$ and $D_{m+1} := C_{m+1}^{T_{m+1}}$ where T_{m+1} is the unique complement of $\mathrm{Gal}(C_{m+1}/C_0)$ into $\mathrm{Gal}(C_{m+1}/D_0)$ (cf. Thm. (3.2)). Following

[8, Thm. 5.4(2.1)], we have the Galois parallelograms

$$[D_0, C_0, C_{m+1}, D_{m+1}], \qquad [D_m, C_m, C_{m+1}, D_{m+1}].$$

For $\mathrm{Gal}(C_{m+1}/D_{m+1}) =: \{1, \tau_{m+1}\}$, we then have

$$\overline{\tau}_{m+1} = \tau_{m+1|C_m} = \tau_m.$$

Moreover, following [8, Prop. 2.5(4)],

$$\overline{x_{m+1}^3} \in \mathrm{Im}(\mathrm{ga}_{C_m/D_0}^{\overline{\eta}}) = \mathrm{Nor}^{\eta}(C_m/D_0),$$

with $\eta(\tau_m) = \overline{\eta}(\overline{\tau}_m) = \overline{\eta}(\tau_0) = 2$. Indeed, for $x_{m+1}^3 = y_m^2 \tau_m(y_m)$,

$$\frac{\tau_m(x_{m+1}^3)}{(x_{m+1}^3)^{\eta(\tau_m)}} = \left(\frac{\tau_{m+1}(x_{m+1})}{x_{m+1}^2}\right)^3 = \frac{1}{y_m^3}$$

and there exists $\nu \in \mathbb{F}_3$ such that

$$\frac{\tau_{m+1}(x_{m+1})}{x_{m+1}^2} = \frac{\zeta_3^{\nu}}{y_m}.$$

But $\tau_{m+1}^2 = 1$. It now suffices to apply $x_{m+1} = \tau_{m+1}^2(x_{m+1})$ to get $\nu = 0$. This completes the proof. ■

Example. Let us take $D_0 = \mathbb{Q}_3$, the local field of 3-adic numbers. Then $C_0 = \mathbb{Q}_3(j)$ where $j := e^{2i\pi/3}$. For $y_0 = 4 + 6j$, we have

$$\overline{x_1^3} = \mathrm{ga}_{C_0/D_0}^{\overline{\eta}}(\overline{4+6j}) = \overline{28}\;\overline{4+6j}.$$

But, following [13, p. 219, Prop. 9], there exists a unique cubic root of 28 in $\mathbb{Q}_3$; we denote it by $\sqrt[3]{28}$. Then we can take $C_1 = C_0(x_1)$ with

$$x_1 = (4+6j)^{1/3}$$

(any fixed cubic root of $4 + 6j$). Furthermore,

$$\left(\frac{\tau_1(x_1)}{x_1^2}\right)^3 = \frac{\tau_0(4+6j)}{(4+6j)^2} = \left(\frac{\sqrt[3]{28}}{4+6j}\right)^3 \Leftrightarrow \left(\exists \nu \in \mathbb{F}_3 \quad \tau_1(x_1) = j^{\nu}\,\frac{\sqrt[3]{28}}{4+6j}\,x_1^2\right).$$

But $\tau_1^2 = 1$ and $\tau_1(\sqrt[3]{28}) = \sqrt[3]{28}$ (since $\sqrt[3]{28} \in \mathbb{Q}_3$). Therefore $\nu = 0$; and by our last assertion (3.4), the extension D_1/D_0 is cyclic of degree 3 with

$$D_1 = D_0(x_1 + \tau_1(x_1)) = \mathbb{Q}_3\left((4+6j)^{1/3} + \frac{\sqrt[3]{28}}{4+6j}\,(4+6j)^{2/3}\right).$$

Is it possible now to apply the Theorem one step further? Indeed, we can by using its first point (1), as we deduce from the Hilbert–Artin–Tate symbol [1, p. 163, Thm. 9], for the wild place $1-j$, that $\langle 4+6j, j\rangle_{(1-j)} = 1$; whence, there exists $\xi \in C_1$ with norm $N_{C_1/C_0}(\xi) = j$. To compute such a ξ, we observe, for the valuation $\mathrm{ord}_{(1-j)}$, that

$$\mathrm{ord}_{(1-j)}\left(1 - \frac{11-6j}{2+3j}\right) = \mathrm{ord}_{(1-j)}(-9+9j) = 5 > 3.$$

Therefore, following [13, *loc. cit.*] or [14] with defect theory (generalized in [12]), there exists $\theta \in \mathbb{Q}_3(j)$ with $\theta^3 = (11-6j)/(2+3j)$. This allows us to take

$$\xi := \frac{2+x_1}{(1-j)x_1(\theta+x_1)}.$$

We have just proved that there exists a field $D_2 > D_1$ inducing a cyclic extension D_2/D_0 of degree 9.

To build such a field, it now suffices to apply the Theorem for $m = 1$. Clearly

$$z_1 = \xi, \qquad y_1 = x_1\sigma_1(\xi)\sigma_1^2(\xi^2)$$

where σ_1 is the generator of the cyclic group $\mathrm{Gal}(C_1/C_0)$ defined by

$$\frac{\sigma_1(x_1)}{x_1} = j = N_{C_1/C_0}(\xi).$$

This gives

$$y_1 = \frac{jx_1(2+jx_1)(2+j^2x_1)^2}{(1-j)^3(4+6j)(\theta+jx_1)(\theta+j^2x_1)^2}.$$

The extension $C_1(y_1^{1/3})/\mathbb{Q}_3(j)$ is cyclic of degree 9, but $C_1(y_1^{1/3})$ is not Galois over $\mathbb{Q}_3$. To get such a field, we have to take the Galois average of $\overline{y}_1$:

$$\mathrm{ga}^{\overline{\eta}}_{C_1/D_0}(\overline{y}_1) = \overline{y}_1^2\tau_1(\overline{y}_1)$$

(cf. Cor. (3.2)), where:

- classes are mod $C_1^{\times 3}$ (for instance, $\overline{4+6j} = \overline{x_1^3} = \overline{1}$);
- $\mathrm{Gal}(C_i/D_i) = \{1, \tau_i\}$ $(i = 0, 1)$, $\tau_1|_{C_0} = \tau_0$, $\tau_0(j) = j^2$

(in particular, for $\theta = m + nj$, $m, n \in \mathbb{Q}_3$, $\tau_1(\theta) = \tau_0(\theta) = m + nj^2$). Since $\tau_1(x_1) = \sqrt[3]{28}\,x_1^2/(4+6j)$, we get

$$\tau_1(\overline{y}_1) = \overline{\frac{j^2\sqrt[3]{28}\,x_1^2(8+12j+j^2\sqrt[3]{28}\,x_1^2)(8+12j+j\sqrt[3]{28}\,x_1^2)^2}{((4+6j)\tau_0(\theta)+j^2\sqrt[3]{28}\,x_1^2)((4+6j)\tau_0(\theta)+j\sqrt[3]{28}\,x_1^2)^2}}.$$

Finally, following (3.4) in the Corollary (or the Theorem), for any x_2 (in an algebraic closure of $\mathbb{Q}_3$ containing $C_1 = \mathbb{Q}_3(j, (4+6j)^{1/3})$) such that

$$\overline{x_2^3} = \overline{y}_1^2\tau_1(\overline{y}_1) = \overline{\frac{j\sqrt[3]{28}\,x_1(2+j^2x_1)(\theta+jx_1)}{(2+jx_1)(\theta+j^2x_1)}} \times \overline{\frac{(8+12j+j^2\sqrt[3]{28}\,x_1^2)((4+6j)\tau_0(\theta)+j\sqrt[3]{28}\,x_1^2)}{(8+12j+j\sqrt[3]{28}\,x_1^2)((4+6j)\tau_0(\theta)+j^2\sqrt[3]{28}\,x_1^2)}},$$

the field

$$D_2 = D_1(x_2 + x_2^2/y_1)$$

induces a cyclic extension $D_2/\mathbb{Q}_3$ of degree 9.

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