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Corrigendum to "On density modulo 1 of some expressions containing algebraic integers"

(Acta Arith. 127 (2007), 217–229)

by

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0.1. Main theorem. In the proof of [1, Theorem 1.6] there is an incorrect statement. First of all, the matrix $\mathcal{J}_{\kappa,\iota}$ on p. 227 should be defined as $\left(\begin{smallmatrix}\kappa\,\mathrm{Id} & 0\\ 0 & \iota\,\mathrm{Id}\end{smallmatrix}\right)$. The set $\widetilde{X^{\mathrm{ac}}_\alpha}$ should be defined as $\mathcal{J}_{\kappa,\kappa}X^{\mathrm{ac}}_\alpha$, i.e., with $\kappa = \iota$. Then on p. 228 it is claimed that the set $\pi_1(S)$ is Σ_1-invariant. However, it is not, and so we cannot apply the ergodic argument (Lemma 5.18) to conclude that $\pi_1(S) = \mathbb{T}$. Therefore, actually we have only proved that for every pair (ξ_1, ξ_2), with not both $\xi_i = 0$, there exists a natural number κ such that $\mathbb{T} = \kappa\pi_1(S)$. Hence, we are not able to deduce the density modulo 1 of $\lambda_1^n\mu_1^m\xi_1 + \lambda_2^n\mu_2^m\xi_2$ but only of $\lambda_1^n\mu_1^m\kappa\xi_1 + \lambda_2^n\mu_2^m\kappa\xi_2$.

Hence the corrected version of [1, Theorem 1.6] is as follows.

THEOREM 0.1 ([1, Theorem 1.6 (corrected)]). *Let λ_1, μ_1 and λ_2, μ_2 be two distinct pairs of multiplicatively independent real algebraic integers of degree 2. Assume that*

(i) $|\lambda_i|, |\mu_i| > 1$, $i = 1, 2$, *and the absolute values of their conjugates, $\tilde\lambda_i, \tilde\mu_i$, are also greater than* 1.
(ii) $\mu_i = g_i(\lambda_i)$ *for some* $g_i \in \mathbb{Z}[x]$, $i = 1, 2$.
(iii) *In each pair (λ_i, μ_i), at least one element has all non-negative powers irrational.*
(iv) *There exist $k, l, k', l' \in \mathbb{N}$ such that*

$$\min\{|\lambda_2|^k|\mu_2|^l, |\tilde\lambda_2|^k|\tilde\mu_2|^l\} > \max\{|\lambda_1|^k|\mu_1|^l, |\tilde\lambda_1|^k|\tilde\mu_1|^l\}$$

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and

$$\min\{|\lambda_1|^{k'}|\mu_1|^{l'}, |\tilde{\lambda}_1|^{k'}|\tilde{\mu}_1|^{l'}\} > \max\{|\lambda_2|^{k'}|\mu_2|^{l'}, |\tilde{\lambda}_2|^{k'}|\tilde{\mu}_2|^{l'}\}.$$

Then for any real numbers ξ_1, ξ_2 *with at least one* $\xi_i \neq 0$, *there exists a natural number* κ *such that the set*

$$\{\lambda_1^n \mu_1^m \kappa\xi_1 + \lambda_2^n \mu_2^m \kappa\xi_2 : n, m \in \mathbb{N}\}$$

is dense modulo 1.

0.2. Proposition 5.7. There are some mistakes in the proof of [1, Proposition 5.7]. For example, we cannot assume (5.11) of [1] because multiplying by the matrix $\left(\begin{smallmatrix} q_1 \mathrm{Id} & 0 \\ 0 & q_2 \mathrm{Id} \end{smallmatrix}\right)$ we can produce other rational points on the axes. Hence, some modification of the argument is required. In particular, we have to construct a sequence of points on the axes, not just one.

A similar mistake occurs in the proof of [2, Proposition 4.5], where the more general case of algebraic numbers (not necessarily algebraic integers) was considered. Since the principal idea of the proof is the same in both papers we refer the reader to [3] for the details.

0.3. Some other mistakes.

- In (1.13), ξ_2 should be deleted.
- In Lemma 4.1 the congruence should be modulo $q\mathbb{Z}^d$ (and similarly in the proof of Lemma 4.2).
- In the proof of Lemma 4.1 one should write "we conclude that $\det \bar{\sigma}$ is invertible in $\mathbb{Z}/q\mathbb{Z}$" instead of "we conclude that $\det \bar{\sigma} \neq 0$".
- Lemma 4.4 (unused) can be omitted.

References

[1] R. Urban, *On density modulo 1 of some expressions containing algebraic integers*, Acta Arith. 127 (2007), 217–229.

[2] —, *Algebraic numbers and density modulo 1*, J. Number Theory 128 (2008), 645–661.

[3] —, *Corrigendum to "Algebraic numbers and density modulo 1"*, ibid. 129 (2009), 2879–2881.

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