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The first negative Hecke eigenvalue of a Siegel cusp form of genus two

by

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1. Introduction and statement of result. Recently there have been several works on sign changes of Fourier coefficients and Hecke eigenvalues of elliptic cusp forms (cf. e.g. [4, 8, 10, 12, 13]).

Notably in [8] it was shown that if f is a normalized Hecke eigenform of integral weight $k \geq 2$ and level $N \in \mathbb{N}$, and $\lambda(n)$ $(n \in \mathbb{N})$ denote its Hecke eigenvalues, then there exists $n \in \mathbb{N}$ with

$$n \ll (k^2 N)^{29/60}$$

such that $\lambda(n) < 0$. Here the constant implied in $\ll$ is absolute and effectively computable. The proof uses convexity estimates for the Hecke L-function of f and exploits the Hecke relations satisfied by the $\lambda(n)$.

Let $S_k(\Gamma_2)$ be the space of Siegel cusp forms of integral weight k on the group $\Gamma_2 := \mathrm{Sp}_2(\mathbb{Z}) \subset \mathrm{GL}_4(\mathbb{Z})$ and let F be a non-zero eigenfunction of all the Hecke operators $T(n)$ $(n \in \mathbb{N})$ (cf. e.g. [2, 7] for details). Denote by $\lambda(n)$ $(n \in \mathbb{N})$ the corresponding eigenvalues.

If k is even and F is contained in the Maass subspace $S_k^*(\Gamma_2) \subset S_k(\Gamma_2)$ (cf. e.g. [5]), it was proved in [3] that $\lambda(n) > 0$ for all n. On the other hand, if either k is odd, or k is even and F is in the orthogonal complement of $S_k^*(\Gamma_2)$, then under the validity of the Ramanujan–Petersson conjecture for F (a proof of which was announced in [15]) it was recently shown in [11] that the sequence $(\lambda(n))_{n\in\mathbb{N}}$ indeed changes sign infinitely often.

In the present paper we shall prove

Theorem. *Let F be a non-zero Siegel–Hecke eigenform in $S_k(\Gamma_2)$ and suppose that either k is odd, or k is even and F is in the orthogonal complement of $S_k^*(\Gamma_2)$. Assume that F satisfies the Ramanujan–Petersson conjecture* (*cf. Sect.* 2). *Denote by $\lambda(n)$ $(n \in \mathbb{N})$ the eigenvalues of F. Then there*

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exists $n \in \mathbb{N}$ *with*

$$n \ll k^2 \log^{20} k$$

such that $\lambda(n) < 0$. *Here the constant implied in* $\ll$ *is absolute and effectively computable.*

We note that the first case where a form F as above exists is $k = 35$ if k is odd and $k = 20$ if k is even.

The proof of the Theorem follows a similar pattern to that in [8], with the Hecke L-function replaced by the spinor zeta function. However, since the Hecke relations for $\lambda(n)$ are more involved in genus 2 than in the elliptic case, exploiting them naturally turns out to be more difficult.

Notations. If in an estimate we write $\ll$, it is always understood that the implied constant is absolute.

2. Preliminaries on Siegel modular forms. For basic facts on Siegel modular forms we refer to [2, 7, 9]. For $n \in \mathbb{N}$ there is a Hecke operator $T(n)$ on $S_k(\Gamma_2)$ given by

$$F|T(n) = \sum_{\gamma \in \Gamma_2 \backslash \mathcal{O}_{2,n}} F|_k\gamma \tag{2.1}$$

where $\mathcal{O}_{2,n}$ is the set of integral symplectic similitudes of size 4 and scale n and

$$(F|_k\gamma)(Z) := (\det\gamma)^{k/2} \det(CZ + D)^{-k} F((AZ + B)(CZ + D)^{-1})$$

for

$$\gamma = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \qquad Z \in \mathcal{H}_2 = \text{Siegel upper half space of genus 2.}$$

Note that our choice of normalization in (2.1) differs from the usual one by the scalar factor $n^{k-3/2}$.

The space $S_k(\Gamma_2)$ has a basis consisting of common eigenfunctions of all the $T(n)$. The Maass subspace $S_k^*(\Gamma_2)$ (k even) is invariant under all Hecke operators.

Let F be a non-zero eigenfunction of all $T(n)$, with $F|T(n) = \lambda(n)F$. Then $\lambda(n)$ is real for all n.

One has

$$\sum_{n \geq 1} \lambda(n) n^{-s} = \frac{1}{\zeta(2s+1)} Z_F(s) \qquad (\sigma := \Re(s) \gg 1) \tag{2.2}$$

where $Z_F(s)$ is the spinor zeta function of F, i.e.

$$Z_F(s) = \prod_p Z_{F,p}(p^{-s})^{-1} \qquad (\sigma \gg 1) \tag{2.3}$$

with

$$(2.4)\qquad Z_{F,p}(X) := (1-\alpha_{0,p}X)(1-\alpha_{0,p}\alpha_{1,p}X)(1-\alpha_{0,p}\alpha_{2,p}X)(1-\alpha_{0,p}\alpha_{1,p}\alpha_{2,p}X)$$

and where $\alpha_{0,p}, \alpha_{1,p}, \alpha_{2,p}$ are "the" Satake p-parameters attached to F. For details we refer to [1].

Note that due to our normalization one has

$$(2.5)\qquad \alpha_{0,p}^2\alpha_{1,p}\alpha_{2,p} = 1.$$

Indeed, in comparison to the "classical" normalization we have replaced the variable s by $s + k - 3/2$.

The function

$$Z_F^*(s) := (2\pi)^{-s}\Gamma(s+k-3/2)\Gamma(s+1/2)Z_F(s)$$

has meromorphic continuation to $\mathbb{C}$ and is $(-1)^k$-invariant under $s \mapsto 1-s$ (see [1]). It is entire if and only if either k is odd, or k is even and F is in the orthogonal complement of $S_k^*(\Gamma_2)$ [6, 14].

In the latter case the Ramanujan–Petersson conjecture says that

$$(2.6)\qquad |\alpha_{1,p}| = |\alpha_{2,p}| = 1 \quad (\forall p)$$

(a proof was announced in [15]). By (2.5) we then also have

$$(2.7)\qquad |\alpha_{0,p}| = 1 \quad (\forall p).$$

3. Convexity estimates. Let $F \in S_k(\Gamma_2)$ be a non-zero Hecke eigenform with normalized eigenvalues $\lambda(n)$ ($n \in \mathbb{N}$). We assume that either k is odd, or k is even and F is not contained in $S_k^*(\Gamma_2)$. We also assume (2.6).

The purpose of this section is to derive estimates uniform with respect to k for $Z_F(s)$ on lines $s = \delta + it$ ($t \in \mathbb{R}$) where $0 < \delta < 1/2$. The arguments will be analogous to those given in Sect. 3 of [12] and therefore we will be brief.

Let us write

$$Z_F(s) = \sum_{n\geq 1} a(n)n^{-s} \quad (\sigma \gg 1).$$

Then from (2.3), (2.4), (2.6) and (2.7) we obtain

$$|a(n)| \leq d_4(n) \quad (n \geq 1)$$

where $d_4(n)$ is the nth coefficient of $\zeta^4(s)$. Since $\zeta^4(s)$ has a pole at $s = 1$ of order 4, a standard Tauberian argument gives

$$(3.1)\qquad \sum_{x_0\leq n\leq x} |a(n)| \ll x\log^3 x \quad (x_0 > 1).$$

Using integration by parts for Stieltjes integrals we deduce from (3.1) in a similar way to [12] that

$$|Z_F(c+it)| \ll 1 + \frac{c}{(c-1)^4} \tag{3.2}$$

whenever $c > 1$.

Next by the functional equation of $Z_F^*(s)$ we get

$$|Z_F(1-s)| = (2\pi)^{2-4\sigma}\left|\frac{\Gamma(s+k-3/2)\Gamma(s+1/2)}{\Gamma(-s+k-1/2)\Gamma(3/2-s)}\right| \cdot |Z_F(s)|.$$

Putting $s = c+it$ and observing that $|\Gamma(z)| = |\Gamma(\overline{z})|$, we in particular obtain

$$|Z_F(1-c-it)| = (2\pi)^{2-4c}\left|\frac{\Gamma(k-3/2+c+it)\Gamma(c+1/2+it)}{\Gamma(k-1/2-c+it)\Gamma(3/2-sc+it)}\right| \cdot |Z_F(c+it)|.$$

We estimate the quotients of Γ-factors in the same way as in [12] to deduce that

$$|Z_F(1-c-it)| \ll |k-1+2it|^{2c-1}|1+it|^{2c-1}|Z_F(c+it)|,$$

hence

$$|Z_F(1-c-it)| \ll k^{2c-1}|1+it|^{4c-2}|Z_F(c+it)|. \tag{3.3}$$

Now put

$$c := 1 + \frac{1}{2\log k}.$$

Then from (3.2) we infer that

$$\left|Z_F\left(1+\frac{1}{2\log k}+it\right)\right| \ll \log^4 k \tag{3.4}$$

and therefore combining with (3.3) it follows that

$$\left|Z_F\left(-\frac{1}{2\log k}+it\right)\right| \ll k\log^4 k \cdot |1+it|^{2+2/\log k}. \tag{3.5}$$

Let us now recall the following “strong convexity” principle, due to Rademacher (cf. e.g. [12, Sect. 3]).

Lemma 1. *Suppose that $g(s)$ is continuous on the closed strip $a \le \sigma \le b$ and holomorphic and of finite order on $a < \sigma < b$. Furthermore suppose that*

$$|g(a+it)| \le E|P+a+it|^{\alpha}, \qquad |g(b+it)| \le F|P+b+it|^{\beta}.$$

Here E and F are positive constants and P, α and β are real constants that satisfy

$$P + a > 0, \qquad \alpha \ge \beta.$$

Then for all $a < \sigma < b$ we have

$$|g(s)| \le (E|P+s|^{\alpha})^{(b-\sigma)/(b-a)}(F|P+s|^{\beta})^{(\sigma-a)/(b-a)}.$$

We apply Lemma 1 to $Z_F(s)$ with

$$a = -\frac{1}{2\log k}, \qquad b = P = 1 + \frac{1}{2\log k}, \qquad E = k\log^4 k, \qquad F = \log^4 k,$$
$$\alpha = 2\left(1 + \frac{1}{\log k}\right), \qquad \beta = 0$$

and $s = \delta + it$ where $0 < \delta < 1/2$. From (3.4) and (3.5) we then obtain easily

PROPOSITION 1. *Let $F \in S_k(\Gamma_2)$ be a non-zero Hecke eigenform with normalized eigenvalues $\lambda(n)$ $(n \in \mathbb{N})$. Assume that either k is odd, or k is even and $F \notin S_k^*(\Gamma_2)$. Let $0 < \delta < 1/2$. Then for all $t \in \mathbb{R}$ one has*

$$|Z_F(\delta + it)| \ll k^{1-\delta}\log^4 k \cdot \left|1 + \frac{1}{2\log k} + \delta + it\right|^{2+1/\log k - 2\delta}. \tag{3.6}$$

4. An upper bound for sums of eigenvalues. We shall prove

PROPOSITION 2. *Let $F \in S_k(\Gamma_2)$ be a non-zero Hecke eigenform with normalized eigenvalues $\lambda(n)$ $(n \in \mathbb{N})$. Assume that either k is odd, or k is even and $F \notin S_k^*(\Gamma_2)$. Also suppose that* (2.6) *holds. Then*

$$\sum_{n \le x} \lambda(n) \log^2\left(\frac{x}{n}\right) \ll k\log^8 k \cdot x^{2/3\log k}.$$

Proof. By Perron's formula and (2.2) we have

$$\sum_{n \le x} \lambda(n) \log^2\left(\frac{x}{n}\right) = \frac{2}{2\pi i} \int_{2-i\infty}^{2+i\infty} \frac{1}{\zeta(2s+1)}\, Z_F(s)\, \frac{x^s}{s^3}\, ds$$

(cf. [12, Sect. 5]).

Let $\frac{1}{2\log k} < \delta < 1/2$. We shift the line of integration to the line $\sigma = \delta$ and recall the well-known estimate (say)

$$\left|\frac{1}{\zeta(\sigma + it)}\right| \ll \beta(t)$$

valid (uniformly) for $\sigma > 1$, where

$$\beta(t) := \begin{cases} 1 & \text{if } |t| \le 10, \\ \log|t| & \text{if } |t| > 10. \end{cases}$$

Applying (3.6) we then obtain in a standard way

$$\sum_{n \le x} \lambda(n) \log^2\left(\frac{x}{n}\right) \ll k^{1-\delta}\log^4 k \cdot \int_{-\infty}^{\infty} \beta(t)\, \frac{\left|1 + \frac{1}{2\log k} + \delta + it\right|^{2+1/\log k - 2\delta}}{|\delta + it|^3}\, dt \cdot x^{\delta}. \tag{4.1}$$

Note that the integral on the right-hand side of (4.1) is absolutely convergent since $2 + 1/\log k - 2\delta < 2$ by hypothesis.

We have to estimate this integral from above uniformly in k. Replacing t by $-t$, it is sufficient to get an upper bound on

$$I_{k,\delta} := \int_0^\infty \beta(t) \frac{\left|1 + \frac{1}{2\log k} + \delta + it\right|^{2+1/\log k - 2\delta}}{|\delta + it|^3}\, dt. \tag{4.2}$$

Note that for $0 < B < A$ one has

$$|A + it| \le \frac{A}{B}\,|B + it| \qquad (\forall t \in \mathbb{R}).$$

Applying this with

$$A := 1 + \frac{1}{2\log k} + \delta, \qquad B := \delta$$

we see that the integrand in (4.2) is bounded from above by

$$C_{k,\delta}\beta(t)|\delta + it|^{-1+1/\log k - 2\delta}$$

where

$$C_{k,\delta} := \left(1 + \frac{1 + \frac{1}{2\log k}}{\delta}\right)^{2+1/\log k - 2\delta}.$$

We split up $I_{k,\delta}$ into an integral from 0 to 10 and an integral from 10 to ∞. The first integral is clearly bounded by

$$\ll C_{k,\delta}\delta^{-1+1/\log k - 2\delta}.$$

The second integral is bounded by

$$\ll C_{k,\delta} \int_{10}^\infty \log t \cdot t^{-1+1/\log k - 2\delta}\, dt \ll C_{k,\delta}\left(\frac{1}{2\delta - \frac{1}{\log k}}\right)^2,$$

where the last estimate follows by partial integration.

We now choose

$$\delta := \frac{2}{3\log k}.$$

We then obtain

$$I_{k,\delta} \ll \log^2 k \cdot (\log k + \log^2 k) \ll \log^4 k.$$

Also $k^{1-\delta} \ll k$ (in fact $k^{1-\delta}$ is of the same order of magnitude as k).

Thus from (4.1) we obtain our assertion.

5. A lower bound for sums of eigenvalues

Proposition 3. *Let $F \in S_k(\Gamma_2)$ be a non-zero Hecke eigenform and assume that either k is odd, or k is even and $F \notin S_k^*(\Gamma_2)$. Suppose that* (2.6) *holds. Let* $\lambda(n)$ $(n \in \mathbb{N})$ *be the normalized eigenvalues of* F *and suppose that* $\lambda(n) \geq 0$ *for* $1 \leq n \leq x$. *Then*

$$\sum_{n \leq x} \lambda(n) \log^2\left(\frac{x}{n}\right) \gg \frac{\sqrt{x}}{\log^2 x} \qquad (x > 1).$$

Proof. Clearly

$$\sum_{n \leq x} \lambda(n) \log^2\left(\frac{x}{n}\right) \gg \sum_{n \leq x/2} \lambda(n),$$

hence it suffices to show that

$$\sum_{n \leq x} \lambda(n) \gg \frac{\sqrt{x}}{\log^2 x} \qquad (x > 1). \tag{5.1}$$

By [1], for each prime p the local spinor polynomial $Z_{F,p}(X)$ given by (2.4) is equal to

$$Z_{F,p}(X) = 1 - \lambda(p)X + (\lambda(p)^2 - \lambda(p^2) - 1/p)X^2 - \lambda(p)X^3 + X^4,$$

hence by (2.2) we have

$$\frac{1 - \frac{1}{p}X^2}{Z_{F,p}(X)} = \sum_{n \geq 0} \lambda(p^n)X^n. \tag{5.2}$$

Clearly (5.2) is equivalent to saying that

$$\begin{aligned} \lambda(p^n) = {} & \lambda(p)\lambda(p^{n-1}) - (\lambda(p)^2 - \lambda(p^2) - 1/p)\lambda(p^{n-2}) \\ & + \lambda(p)\lambda(p^{n-3}) - \lambda(p^{n-4}) \end{aligned} \tag{5.3}$$

for all $n \geq 0$, with the convention that $\lambda(p^n) = 0$ for $n < 0$.

Note that (2.4), (2.6), (2.7) and (5.2) imply that

$$|\lambda(p)|, |\lambda(p^2)|, |\lambda(p^3)| \ll 1. \tag{5.4}$$

To prove (5.1), bearing in mind that $\lambda(n) \geq 0$ for $n \leq x$, let us write

$$\sum_{n \leq x} \lambda(n) \geq \sum_{p,q \leq \sqrt[4]{x}} \lambda(p^2q^2) + \sum_{p,q \leq \sqrt[4]{x}} \lambda(p^2q) + \sum_{p,q \leq \sqrt[4]{x}} \lambda(pq) \tag{5.5}$$

where on the right-hand side p and q run over primes.

Taking $n = 4$ in (5.3) we obtain

$$\lambda(p^4) = \lambda(p^2)^2 + \lambda(p)\lambda(p^3) + \lambda(p^2)(-\lambda(p)^2 + 1/p) + \lambda(p)^2 - 1. \tag{5.6}$$

Similarly, for $n = 3$ we find that

$$\lambda(p^3) = \lambda(p)(2\lambda(p^2) + 1 + 1/p - \lambda(p)^2). \tag{5.7}$$

From (5.6), observing (5.4) we see that

$$\lambda(p^4) \gg \lambda(p^2)^2 - c_1$$

where $c_1 > 0$ is an absolute constant. Thus

$$\sum_{p,q\leq \sqrt[4]{x}} \lambda(p^2q^2) \gg \Big(\sum_{p\leq \sqrt[4]{x}} \lambda(p^2)\Big)^2 - c_1\pi(\sqrt[4]{x}) \tag{5.8}$$

where as usual $\pi(x)\,(x>1)$ denotes the number of primes $p \leq x$.

Next, from (5.7) taking into account (5.4) we see that

$$\lambda(p^3) \gg \lambda(p)\lambda(p^2) - c_2$$

where $c_2 > 0$ is an absolute constant. Hence

$$\sum_{p,q\leq \sqrt[4]{x}} \lambda(p^2q) \gg \Big(\sum_{p\leq \sqrt[4]{x}} \lambda(p^2)\Big)\Big(\sum_{p\leq \sqrt[4]{x}} \lambda(p)\Big) - c_2\pi(\sqrt[4]{x}). \tag{5.9}$$

We finally look at the sum

$$\sum_{p,q\leq \sqrt[4]{x}} \lambda(pq)$$

in (5.5). For $p \leq \sqrt[4]{x}$ the quantities $\lambda(p^3), \lambda(p^2)$ and $\lambda(p)$ are non-negative, hence we deduce from (5.7) for such p that

$$\lambda(p^2) \gg \lambda(p)^2 - c_3$$

where $c_3 > 0$ is an absolute constant. Therefore as before

$$\sum_{p,q\leq \sqrt[4]{x}} \lambda(pq) \gg \Big(\sum_{p\leq \sqrt[4]{x}} \lambda(p)\Big)^2 - c_3\pi(\sqrt[4]{x}). \tag{5.10}$$

Combining (5.8), (5.9) and (5.10) we infer from (5.5) that

$$\sum_{n\leq x} \lambda(n) \gg \Big(\sum_{p\leq \sqrt[4]{x}} \lambda(p^2) + \sum_{p\leq \sqrt[4]{x}} \lambda(p)\Big)^2 - c\pi(\sqrt[4]{x}) \tag{5.11}$$

where $c > 0$ is an absolute constant.

We now claim that $\lambda(p^2)$ and $\lambda(p)$ cannot be simultaneously small for $p \leq \sqrt[4]{x}$. Indeed, otherwise $\lambda(p^3)$ would also be small, by (5.7), and then (5.6) would give a contradiction since $\lambda(p^4) \geq 0$ by hypothesis. Thus there exists an absolute constant $\alpha > 0$ such that

$$\lambda(p^2) + \lambda(p) \geq \alpha \quad (p \leq \sqrt[4]{x}).$$

From (5.11) we now conclude using the prime number theorem that

$$\sum_{n\leq x} \lambda(n) \gg \frac{\sqrt{x}}{\log^2 x}$$

as claimed.

6. Proof of Theorem. Assuming that $\lambda(n) \geq 0$ for $n \leq x$, we infer from Propositions 2 and 3 that

$$\frac{\sqrt{x}}{\log^2 x} \ll k \log^8 k \cdot x^{2/3 \log k} \qquad (x > 1). \tag{6.1}$$

Clearly for x large this is a contradiction.

To get an explicit bound, quoting the more general Lemma 4 in [4] we see that (6.1) implies that

$$x \ll \left(\frac{A}{\delta^2}\right)^{1/\delta} \log^{2/\delta}\left(\frac{A}{\delta^2}\right) \tag{6.2}$$

where

$$A := k \log^8 k, \qquad \delta := \frac{1}{2} - \frac{2}{3 \log k}.$$

We have

$$\frac{1}{\delta} = 2 + \frac{8}{3 \log k - 4}.$$

Hence

$$A^{1/\delta} = (k \log^8 k)^{2+8/(3 \log k - 4)} \ll k^2 \log^{16} k$$

and

$$\log^{2/\delta}\left(\frac{A}{\delta^2}\right) \ll \log^{2/\delta} A \ll \log^4 k.$$

Thus (6.2) implies that

$$x \ll k^2 \log^{20} k.$$

Therefore we obtain the assertion of the Theorem.

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