AN APPLICATION OF THE SPECIAL REISSNER–NORDSTRÖM SPACE-TIME TO DESCRIPTION OF THE UNIVERSE IN THE NEIGHBOURHOOD OF THE PLANCK ERA

Abstract. The description of the special Reissner–Nordström space-time in the Planck era is presented and its comparison with the Robertson–Walker space-time is given.

1. Introduction. The aim of this paper is to present the conjectural description of Universe by means of a special 4-dimensional Reissner–Nordström (briefly R-N) space-time and to compare it with the description given by the 4-dimensional Robertson–Walker (R-W) space-time. Let us recall that according to one of the cosmological conjectures the history of Universe began with Big Bang [6]. We will start our description at time $t_1 = 10^{-44}$ sec, i.e. at the end of the Planck era, and conclude it at time $t_2 = 10^{-34}$ sec [6].

Definition 1. The generalized 4-dimensional R-N space-time is defined by the metric tensor

$$(1) \quad \text{diag}(-E^{a+1}, E^{a-1}, r^2, r^2 \sin^2 \theta),$$

where

$$E = 1 - \frac{r_0}{r} + \frac{K r_0^2}{r^2}, \quad a \in [0, 1],$$

$$K \in \left(\frac{1}{4}, \frac{9}{32}\right), \quad r_0 = \text{const} > 0,$$

$$(x^1, x^2, x^3, x^4) = (t, r, \theta, \varphi).$$

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For $a = 1$ the metric tensor (1) has the form

\[\text{diag}(-E^2, 1, r^2, r^2 \sin^2 \theta).\]

**Definition 2.** The *special R-N space-time* is one with metric tensor (2) and parameter $K = (1 + \varepsilon)/4$, where $\varepsilon > 0$ is close to zero.

Diagram (9) of [4] shows the graph of scalar curvatures $T = T(r)$ in the model of 4-dimensional generalized R-N space-time in dependence on the radius $r$ of the Universe. We will determine the radii $r'$ and $r''$ for the scalar curvatures $T_1$, $T_2$ corresponding to $t_1$ and $t_2$. In [4] the scalar curvature $T_\varepsilon$ of the special R-N space-time ($a = 1$) was determined in the form

\[T_\varepsilon = \frac{4(1 + \varepsilon)r_0^2}{r^2 \left[4 \left(r - \frac{r_0}{2}\right)^2 + \varepsilon r_0^2\right]} \tag{3}\]

or equivalently

\[r^2 - \frac{r_0}{2} r - \frac{r_0}{\sqrt{T_\varepsilon}} = 0. \tag{4}\]

The solutions of (4) are

\[\hat{r}_1 = \frac{r_0}{4} - \frac{1}{2} \sqrt{\frac{r_0^2}{4} + \frac{4r_0^2}{\sqrt{T_\varepsilon}}}, \quad \hat{r}_2 = \frac{r_0}{4} + \frac{1}{2} \sqrt{\frac{r_0^2}{4} + \frac{4r_0^2}{\sqrt{T_\varepsilon}}} \tag{5}\]

It follows from (3) that

\[\lim_{r \to 0^+} T = +\infty, \quad \lim_{r \to r_0/2} T = +\infty, \tag{6}\]

so we can assume, for $\varepsilon$ close to zero, that $4r_0/\sqrt{T_\varepsilon} = 0$.

The radius $r$ is close to zero so the approximation $\sqrt{r} \approx \frac{1}{2}x$ allows one to obtain the following values:

\[r' = \frac{r_0}{4} - \frac{r_0^2}{16}, \quad r'' = \frac{r_0}{4} + \frac{r_0^2}{16}. \tag{7}\]

The above data are illustrated in diagram (7) below.

We can summarize this in the following theorem.

**Theorem 1.** *In the special 4-dimensional R-N space-time with metric tensor (2) there exist two Big Bangs for $r = 0$ and $r = r_0/2$ given in diagram (7).*
Definition 3. The 4-dimensional Robertson–Walker (R-W) space-time is defined by the following metric tensor ([2, p. 190]):

\[
\text{diag} \left( -1, \frac{R^2}{1 - kr^2}, R^2r^2, R^2r^2 \sin^2 \theta \right),
\]

where \( R = R(t) \) denotes the dimension free scale factor and \( k \in [-1, 1] \) is a parameter related with space curvature.

There are three Friedmann models of this space-time [2] described as follows:

We are interested in the flat Friedmann model \((k = 0)\). In this model the
scale factor $R(t)$ has the form [2]

$$R(t) = \left(\frac{3A}{2}\right)^{2/3} t^{2/3}, \quad A > 0, \quad A = 8\pi G_0 R_0^3/3$$

and inflation occurs.

The inflation model can be described by the diagram [1]

In the inflation model, immediately after Big Bang there occurs a rapid and impetuous growth of the Universe. The inflation process ends at time $t_2 = 10^{-34}$ sec [6].

**Theorem 2.** In the flat model of the R-W space-time with metric tensor (8) there occurs a Big Bang for $r = 0$ and inflation for $r = r_0/2$.

**Corollary 1.** In the flat model of the R-W space-time inflation occurs exactly at the moment when the second Big Bang occurs in the special R-N space-time.

**Corollary 2.** The graph of the scalar curvature of the special R-N space-time is the curve (1) in diagram (7).

Observe that in the generalized R-N space-time the graph of the scalar curvature tends for $a \to 0^+$ to the limit curvature $T = 0$ for $r \neq r_0/2$ and to $T = +\infty$ for $r = r_0/2$ (see (2) in diagram (7)).

In [3, p. 223, (2.14)] we have presented a formula for the Weyl curvature. Now we can give the following complement.

**Corollary 3.** The neighbourhood of a Big Bang or the neighbourhood of a Black Hole has for $0 < a < 1$ the structure of a generalized R-N space-time (see (2) in diagram (7)).

**Remark.** It follows from the above corollaries that in the model of the Universe described by the flat R-W space-time, inflation occurs at time $t_2$ while in the model described by the special R-N space-time the second Big Bang occurs.
It is a problem of experimental cosmology to investigate which of these conjectures is valid [5].

References


Institute of Mathematics
Technical University of Szczecin
Al. Piastów 17
70-310 Szczecin, Poland
E-mail: glanc@arcadia.tuniv.szczecin.pl

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