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CORRECTION TO "THE VALUE FUNCTION IN ERGODIC CONTROL OF DIFFUSION PROCESSES WITH PARTIAL OBSERVATIONS II"

(APPLICATIONES MATH. 27 (2000), 455–464)

In the above article [1], there is an error in the proof of Lemma 3.1: The claim that it follows from (3.3) that $\overline{V}(x,y) = V(x) + V(y)$ is a stochastic Lyapunov function for $\overline{X}(\cdot) = \overline{X}_1(\cdot) + \overline{X}_2(\cdot)$ is not correct. One needs the following modification:

From (3.1) and the argument that follows it, it follows that the invariant probability distributions under arbitrary Markov controls remain tight. It then follows as in [2] that there exists a C^2 stochastic Lyapunov function $V : \mathbb{R} \to \mathbb{R}$ and an $\ell(\cdot) \in C(\mathbb{R})$ such that $\lim_{\|x\|\to\infty} \ell(x) = \infty$ and

$$\max_u L_u V(x) \le -\ell(x), \qquad x \in \mathbb{R}. \tag{1}$$

With (1) in place of (3.3), the claim concerning $\overline{V}(\cdot)$ is true and the rest of the argument proceeds as before with just one extra step: One redefines $\mathcal{P}_e(\mathbb{R})$ as $\{\pi \in \mathcal{P}(\mathbb{R}) : \pi(V) < \infty\}$. Now note that for $\tau_r := \inf\{t \ge 0 : |X(t)| \ge r\}$, the optional sampling theorem leads to

$$\begin{aligned} E[V(X(t \wedge \tau_r))] &= E[V(X(0))] + E\Big[\int_0^{t\wedge\tau_r} L_{u(s)} V(X(s))\, ds\Big] \\ &\le E[V(X(0))] + Ct \end{aligned}$$

for a suitable constant $C < \infty$. Letting $r \to \infty$, we get

$$E[V(X(t))] = E[\pi_t(V)] \le Ct + E[V(X(0))] = Ct + E[\pi_0(V)].$$

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Thus $\pi_0 \in \mathcal{P}_e(\mathbb{R}) \Rightarrow \pi_t \in \mathcal{P}_e(\mathbb{R})$ a.s. This proves the claim at the end of Section 3, replacing the original argument. The rest remains as before.

References

[1] V. S. Borkar, *The value function in ergodic control of diffusion processes with partial observations II*, Appl. Math. (Warsaw) 27 (2000), 455–464.

[2] —, *Uniform stability of controlled Markov processes*, in: *System Theory*: *Modeling*, *Analysis and Control*, E. Djaferis and I. C. Schick (eds.), Kluwer, Boston, MA, 107–120.

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