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W. Antoniak (Warszawa)
M. Kałuszka (Łódź)

ON OPTIMAL CREDIBILITY PREMIUMS IN MULTIPERIOD INSURANCE

Abstract. This paper focuses on the problem of optimal arrangement of a stream of premiums in a multiperiod credibility model. On the basis of a given claim history (*screening*) and some individual information unknown to the insurance company (*signaling*), we derive the optimal streams in the case when the coverage period is not necessarily fixed, e.g., because of lapses, renewals, deaths, total losses, etc.

1. Introduction. Credibility theory is based on the assumption that each policyholder belongs to some predefined class of risk characterized by a risk profile θ, which is unknown to the insurance company. Let us consider a client who, during the period t, generates a claim modeled by a random variable X_t. Let $f(\,\cdot\,|\,\theta)$ be the conditional probability density function (with respect to some σ-finite measure) of X_t when $\Theta = \theta$. In this paper f is independent of t, and Θ has density function $\pi(\theta)$. Let us assume that $\mathrm{Cov}(X_i, X_j \,|\, \Theta = \theta) = 0$ for $i \neq j$. For simplicity, we put $\mu(\theta) = \mathsf{E}(X_i \,|\, \Theta = \theta)$ and $\sigma^2(\theta) = \mathrm{Var}(X_i \,|\, \Theta = \theta)$ for all i. Setting $m = \mathsf{E}\mu(\Theta)$, $s^2 = \mathsf{E}\sigma^2(\Theta)$ and $a^2 = \mathrm{Var}\,\mu(\Theta)$, we assume that, based on the claim history, the insurer knows the exact values of the coefficients m, s^2 and a^2. The main problem of the insurance company at time n is to establish the adequate net premium P_{n+1} for a given client. The pricing process should take into account the claims $X_1, \ldots, X_n$ resulting from periods $1, \ldots, n$, respectively. If the insurer knew the client's class of risk θ, then the premium would be settled as $\widehat{P}_{n+1} = \mathsf{E}(X_{n+1} \,|\, \Theta = \theta)$. Another solution of this problem is to use the optimal linear predictor $P^*_{n+1} = \mathsf{E}(X_{n+1} \,|\, X_1, \ldots, X_n)$,

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where $P_1^* = \mathsf{E}X_1 = m$. Unfortunately, in order to derive an explicit formula for the premium P_{n+1}^*, it is essential to know the exact conditional distributions, which are known only in some cases, e.g. in exponential models. Bühlmann proposed to apply the linear predictor of the random variable X_{n+1}, i.e. the predictor $P_{n+1} = a_0 + \sum_{i=1}^n a_i X_i$ which minimizes the mean squared error $\mathsf{E}(P_{n+1} - X_{n+1})^2$, where $P_1 = \mathsf{E}X_1$. The solution of this problem is the credibility premium

$$P_{n+1}^{\mathrm{Cr}} = z_n \bar{X}_n + (1 - z_n)m, \tag{1}$$

where $\bar{X}_n = (X_1 + \cdots + X_n)/n$ and

$$z_n = \frac{a^2 n}{s^2 + a^2 n} \tag{2}$$

is the credibility coefficient (see [1]–[3] and [6]).

Recently, the credibility premium has been thoroughly investigated. Various changes have been proposed, which extend or adjust the Bühlmann approach (see [5], [7]–[16]). Most of the research is devoted to various modifications of the loss function.

In this paper we propose a new method of premium calculation. Our enhancement is based on the assumption that some clients know their class of risk. We show that the new premium corresponds better to future losses. Our considerations begin with one-period insurance contract. Then the results obtained are applied to multiperiod insurance contracts.

2. One-period model. An application-motivated generalization of the Bühlmann model is the Bühlmann–Straub model. Its construction begins with independent random vectors

$$(X_{i,1}, \ldots, X_{i,n_i+1}), \quad i = 1, \ldots, N,$$

which describe losses generated by one (ith) of the N clients. They can belong to different classes of risk specified by risk profiles θ. Let us assume that the risk profile of the ith client, characterized by a random variable Θ_i, is unknown to the insurer, and that $\Theta_1, \ldots, \Theta_N$ are independent identically distributed random variables. Furthermore, let us assume that for all i and $s \neq t$, we have

$$\mathsf{E}(X_{i,t} \mid \Theta_i = \theta_i) = \mu(\theta_i), \quad \mathrm{Var}(X_{i,t} \mid \Theta_i = \theta_i) = \frac{\sigma^2(\theta_i)}{w_{i,t}},$$
$$\mathrm{Cov}(X_{i,t}, X_{i,s} \mid \Theta_i = \theta) = 0,$$

where μ and σ are some functions, while $w_{i,t}$ are known weights. In this paper we assume that the insurer has information about the coefficients

$$m = \mathsf{E}\mu(\Theta_i), \quad s^2 = \sigma^2(\Theta_i), \quad a^2 = \mathrm{Var}\,\mu(\Theta_i).$$

Let P denote the net premium for the ith client established on the basis of the known claims $(X_{j,1},\dots,X_{j,n_j})$, where $j=1,\dots,N$.

In our approach the insurer minimizes not only the discrepancy between the premium P and X_{i,n_i+1}, but also the discrepancy between the average premium for a given client and the average claim of this client, i.e. the optimal premium $P=a_0+\sum_{j=1}^{n_i}a_jX_{i,j}$ should minimize the function

$$I_i=\mathsf{E}(P-X_{i,n_i+1})^2+\gamma^2\mathsf{E}\big(\mathsf{E}(P\,|\,\Theta_i)-\mathsf{E}(X_{i,n_i+1}\,|\,\Theta_i)\big)^2,$$

where $\gamma\ge 0$ is a fixed number. The claims $X_{t,j}$ for $t\ne i$ are not included in the premium formula since their independence from $X_{i,j}$ implies that the optimal coefficients are equal to zero.

Our approach is similar to the Markowitz optimal portfolio selection in the sense that both methods take into consideration the average individual inadequacy of the premium. The coefficient γ describes information unknown to the insurance company which can be provided by a client during the acquisition of the insurance policy (*signaling*).

First, note that

$$I_i=\operatorname{Var}\Big(\sum_{j=1}^{n_i}a_jX_{i,j}-X_{i,n_i+1}\Big)+\Big(a_0-\Big(1-\sum_{j=1}^{n_i}a_j\Big)m\Big)^2 + \gamma^2\Big(a_0-\Big(1-\sum_{j=1}^{n_i}a_j\Big)m\Big)^2+\gamma^2\Big(1-\sum_{j=1}^{n_i}a_j\Big)^2a^2. \tag{3}$$

Hence, the optimal coefficient is equal to

$$\widehat{a}_0=\Big(1-\sum_{j=1}^{n_i}a_j\Big)m. \tag{4}$$

Furthermore, for all i,j and $s\ne t$ we have

$$\operatorname{Var}X_{i,j}=\operatorname{Var}(\mathsf{E}(X_{i,j}\,|\,\Theta_i))+\mathsf{E}\operatorname{Var}(X_{i,j}\,|\,\Theta_i)=a^2+\frac{s^2}{w_{i,j}},$$

$$\operatorname{Cov}(X_{i,t},X_{i,s})=\operatorname{Cov}(\mathsf{E}(X_{i,t}\,|\,\Theta_i),\mathsf{E}(X_{i,s}\,|\,\Theta_i))+\mathsf{E}\operatorname{Cov}(X_{i,t},X_{i,s}\,|\,\Theta_i)=a^2.$$

Combining this with (3) and (4) implies

$$\min_{(a_j)}I_i=\sum_{t=1}^{n_i}\sum_{s=1}^{n_i}a_ta_s\operatorname{Cov}(X_{i,t},X_{i,s})-2\operatorname{Cov}\Big(\sum_{j=1}^{n_i}a_jX_{i,j},X_{i,n_i+1}\Big) + \operatorname{Var}X_{i,n_i+1}+\gamma^2\Big(1-\sum_{j=1}^{n_i}a_j\Big)^2a^2$$

$$
\begin{aligned}
&= \sum_{j=1}^{n_i} a_j^2 \Big(a^2 + \frac{s^2}{w_{i,j}}\Big) + \sum_{t\neq s} a_s a_t a^2 - 2\sum_{j=1}^{n_i} a_j a^2 \\
&\quad + \Big(a^2 + \frac{s^2}{w_{i,n_i+1}}\Big) + \gamma^2\Big(1 - \sum_{j=1}^{n_i} a_j\Big)^2 a^2 \\
&= \Big(\sum_{j=1}^{n_i} a_j\Big)^2 a^2 + \sum_{j=1}^{n_i} a_j^2 \frac{s^2}{w_{i,j}} - 2a^2 \sum_{j=1}^{n_i} a_j + \gamma^2\Big(1 - \sum_{j=1}^{n_i} a_j\Big)^2 a^2 \\
&\quad + a^2 + \frac{s^2}{w_{i,n_i+1}} \\
&= a^2\Big(\sum_{j=1}^{n_i} a_j - 1\Big)^2 (1+\gamma^2) + s^2 \sum_{j=1}^{n_i} a_j^2 \frac{1}{w_{i,j}} + \frac{s^2}{w_{i,n_i+1}}.
\end{aligned}
$$

Applying the Cauchy–Schwarz inequality we have

$$
\Big(\sum_{j=1}^{n_i} a_j\Big)^2 = \Big(\sum_{j=1}^{n_i} \frac{a_j}{\sqrt{w_{i,j}}}\sqrt{w_{i,j}}\Big)^2 \le \sum_{j=1}^{n_i} \frac{a_j^2}{w_{i,j}} \sum_{j=1}^{n_i} w_{i,j}
$$

and equality holds if and only if there exists a constant c such that $a_j = cw_{i,j}$ for all j. Thus

$$
\min_{(a_j)} I_i = \min_{c\in\mathbb{R}} [a^2(cw_i - 1)^2(1+\gamma^2) + s^2c^2 w_i] + \frac{s^2}{w_{i,n_i+1}},
$$

where $w_i = \sum_{j=1}^{n_i} w_{i,j}$. The minimum value is attained when

$$
c = \frac{a^2(1+\gamma^2)}{a^2(1+\gamma^2)w_i + s^2}. \tag{5}
$$

Summarizing, the optimal premium is given by the formula

$$
P = z\sum_{j=1}^{n_i} \frac{w_{i,j}}{w_i} X_{i,j} + (1-z)m, \tag{6}
$$

where

$$
z = \frac{a^2(1+\gamma^2)w_i}{a^2(1+\gamma^2)w_i + s^2}.
$$

As in the Bühlmann–Straub model, we have $P \to m$ as $a \to 0$, and $P - \bar{X}_n \to 0$ as $s^2 \to 0$. Furthermore, in the case of $w_{ij} = 1$ for all i, j, the premium P converges to the individual net premium $\mathsf{E}(X_1 \mid \Theta = \theta)$ as $n_i \to \infty$, and the convergence rate is greater for larger γ's. If a client is conscious that he is good, then he can set a bigger value of γ. The average premium $\mu(\theta)$ for good clients is smaller than m.

3. Multiple period model. For the sake of simplicity, further studies will be limited to the Bühlmann model, but they can be easily generalized to the Bühlmann–Straub model. Let us assume that the random variable X_t describes the ith client loss generated during period t, where $t = 1, 2, \ldots$. The premium P_t for an insurance policy which covers the claim X_t at the end of period t is receivable at the beginning of the period. The premium P_t is derived taking into consideration the claim history, i.e. the losses $X_1, \ldots, X_{t-1}$, where $n \le t \le T$ and T is a random variable. The optimal stream of premiums (P_t) is given by the formula

$$P_t = a_{0,t} + \sum_{i=1}^{t-1} a_{i,t} X_i, \quad a_{i,j} \in \mathbb{R},$$

where the coefficients $(a_{i,t})$ are set in such a way that they minimize the function

$$\mathsf{E}\Big[\sum_{t=n}^{T} \big((P_t - X_t)^2 + \gamma_t^2 \big(\mathsf{E}(X_t|\Theta) - \mathsf{E}(P_t|\Theta)\big)^2\big)\Big], \tag{7}$$

where (γ_t) is a sequence of nonnegative numbers.

Let us assume that the random variables T and (X_i) are independent, and T and Θ are independent, e.g., T is the future life of an owner of an insured real estate. Hence minimization of (7) comes down to minimization of

$$\sum_{t=n}^{\infty} \big(\mathsf{E}(P_t - X_t)^2 + \gamma_t^2 \mathsf{E}\big(\mathsf{E}(X_t \,|\, \Theta) - \mathsf{E}(P_t \,|\, \Theta)\big)^2\big)\mathsf{P}(T \ge t),$$

where we seek the optimum sequences $a_{i,t} \in \mathbb{R}$. It can be further simplified to the following minimalization problems: for all $t \ge n$

$$\min_{(a_{0,t},\ldots,a_{t-1,t}) \in \mathbb{R}^t} \big(\mathsf{E}(P_t - X_t)^2 + \gamma_t^2 \mathsf{E}(\mathsf{E}(P_t - X_t \,|\, \Theta))^2\big).$$

This is a one-period problem, solved in Section 1. Thus the optimal stream of premiums is given by the formula

$$P_t(\gamma_t) = z_t(\gamma_t)\bar{X}_{t-1} + (1 - z_t(\gamma_t))m, \quad t = n, \ldots, T, \tag{8}$$

where

$$z_t(\gamma_t) = \frac{a^2(1+\gamma_t^2)(t-1)}{a^2(1+\gamma_t^2)(t-1) + s^2}. \tag{9}$$

The premium (8) satisfies the net premium principle, i.e. $\mathsf{E}P_t(\gamma_t) = \mathsf{E}X_t$ for all $\gamma_t \ge 0$, and it can be rewritten as

$$P_t(\gamma_t) = (1 - \beta_t(\gamma_t))\bar{X}_{t-1} + \beta_t(\gamma_t)P_t^{\mathrm{Cr}}, \tag{10}$$

where P_t^{Cr} is the credibility premium (1) and

$$\beta_t(\gamma_t) = \frac{1 - z_t(\gamma_t)}{1 - z_t(0)},$$

in which $0 < \beta_t(\gamma_t) \le 1$. Let

$$U_t(\theta) := \mathsf{E}\big(P_t(\gamma_t) - P_t^{\mathrm{Cr}} \mid \Theta = \theta\big) = (1 - \beta_t(\gamma_t))(1 - z_t(0))(\mu(\theta) - m).$$

If an insured person is a good client ($\mu(\theta) < m$), then $U_t(\theta) < 0$, while if he/she is bad ($\mu(\theta) > m$), then $U_t(\theta) > 0$. In comparison to the credibility premium, the good client pays on average less and the bad client pays on average more. Furthermore, we have

$$U_{t+1}(\theta) - U_t(\theta) = \frac{[(\beta_t - \beta_{t+1})(s^2 + ta^2) - (1 - \beta_{t+1})a^2]s^2}{(s^2 + ta^2)(s^2 + (t-1)a^2)}(\mu(\theta) - m),$$

where $\beta_t = \beta_t(\gamma_t)$. If the sequence (γ_t) of numbers is set so that

$$(\beta_t - \beta_{t+1})(s^2 + ta^2) > (1 - \beta_{t+1})a^2,$$

then the difference between the average premium $P_t(\gamma_t)$ paid by the good client and the average premium P_t^{Cr} rises with time.

The stream (8) is also optimal when the objective is to minimize

$$J = \mathsf{E}\Big[\sum_{t=n}^{T} \big(\mathsf{E}(P_t - X_t)^2 + \gamma_t^2 \mathsf{E}\big(\mathsf{E}(X_t|\Theta) - \mathsf{E}(P_t|\Theta)\big)^2\big)\Big]$$

provided the stopping moment T is chosen in such a way that $T \ge n$ and the probability $\mathsf{P}(T \ge t)$ is not a function of the coefficients $a_{i,t}$. This follows immediately from the identity

$$J = \sum_{t=n}^{\infty} \big[\mathsf{E}(P_t - X_t)^2 + \gamma_t^2 \mathsf{E}\big(\mathsf{E}(X_t \mid \Theta) - \mathsf{E}(P_t \mid \Theta)\big)^2\big] \mathsf{P}(T \ge t).$$

Examples of such stopping moments include:

1. $T_1 = \inf\{t \ge n\colon X_1 + \cdots + X_t > c_t\}$, where $c_t > 0$ is any sequence of real numbers and $\inf \emptyset = \infty$. It corresponds to the case when the insurer does not renew the insurance contract because the client's aggregate loss exceeds the predefined thresholds c_t.

2. $T_2 = \inf\{t \ge n\colon \max_{1 \le k \le t-1} X_k < c_t X_t\}$. The insurance company does not renew the insurance contract when an extraordinary claim appears (many times bigger than the previous claims). A similar case is when $T_2' = \min(t \ge n\colon X_1 \le c_1, \ldots, X_{t-1} \le c_{t-1}, X_t > c_t)$.

3. $T_3 = \inf\{t \ge n\colon \rho_t(X_1, \ldots, X_t) > c_t\}$, where ρ_t is any risk measure, e.g., $\rho_t = \sum_{i=1}^{t} \alpha_i X_{i:t}$, in which $X_{i:t}$ is the ith order statistic from the sequence $X_1, \ldots, X_t$.

4. $T_4 = \min\{T_k, T\}$, where the stopping moments T_k, $k = 1, 2, 3$, are previously defined and T is the expected future lifetime or the contract boundaries.

As far as we know, the first generalization of credibility theory to multiperiod models was proposed by Gajek et al. [4]. We will summarize the results of that paper. Let $X_{-n}, \dots, X_{-1}$ be losses incurred before time 0, when the insurance policy lasting T years is taken out. The premiums $P_1, \dots, P_T$ cover the random losses $X_1, \dots, X_T$. It is assumed that the client cannot resign from the contract. Thus T is fixed. Gajek et al. [4] proposed the following generalization of the credibility premium:

$$P_t^{\mathrm{GMS}} = \alpha_t\Big(\sum_{i=1}^{t-1} X_i + (T-t+1)P_t^{\mathrm{Cr}}\Big), \quad t = 1, \dots, T, \tag{11}$$

where $\sum_{i=1}^{0} X_i = 0$ and $\alpha_t \geq 0$ are numbers which minimize some two distance functions. Furthermore, let

$$P_t^{\mathrm{Cr}} = z_t \frac{X_{-n} + \dots + X_{-1} + \sum_{i=1}^{t-1} X_i}{t-1+n} + (1-z_t)m,$$

in which

$$z_t = \frac{a^2(t-1+n)}{s^2 + a^2(t-1+n)}.$$

Note that the family of premiums (11) does not include all linear functions of $X_1, \dots, X_{t-1}$. The authors present an analysis which recommends the sequence $\alpha_t = 1/T$. In this case, the recommended premiums are

$$\widehat{P}_t^{\mathrm{GMS}} = \frac{1}{T}\Big(\sum_{i=1}^{t-1} X_i + (T-t+1)P_t^{\mathrm{Cr}}\Big), \quad t = 1, \dots, T \tag{12}$$

(see [4, pp. 230–232]). It is also shown that the modified premiums are more adequate than the credibility premiums because

$$\begin{aligned} U_t(\theta) &= \mathsf{E}[\widehat{P}_t^{\mathrm{GMS}} - P_t^{\mathrm{Cr}} \mid \Theta = \theta] \\ &= \frac{s^2(t-1)}{T(s^2 + a^2(t-1+n))}(\mu(\theta) - m) < 0 \end{aligned}$$

for good risks. Thus good clients pay on average less than the credibility premium. Moreover, for all $t < T$ and for $\mu(\theta) \neq m$,

$$\frac{U_{t+1}(\theta) - U_t(\theta)}{\mu(\theta) - m} = \frac{s^2(s^2 + na^2)}{T(s^2 + (n+t)a^2)(s^2 + (n+t-1)a^2)} \geq 0,$$

which indicates that the surplus of a good client rises with time. This property seems to be desired by good clients and in the opinion of the authors it will make potential clients take out the insurance policy. At the same time, bad clients will prefer the credibility premium.

However, calculating the premiums (12) point to a problem. Let us assume that the good client is considering buying a T year insurance contract. He believes that he is good, because $X_{-n} = \cdots = X_{-1} = 0$, but he allows for the possibility that the only positive claim occurs in the first period, i.e. $X_1 > 0$ and $X_2 = \cdots = X_T = 0$. Hence

$$\widehat{P}_1^{\mathrm{GMS}} = P_1^{\mathrm{Cr}}, \quad \widehat{P}_2^{\mathrm{GMS}} = \frac{1}{T}X_1 + \frac{T-1}{T}P_1^{\mathrm{Cr}}, \ \ldots\ , \widehat{P}_T^{\mathrm{GMS}} = \frac{1}{T}X_1 + \frac{1}{T}P_T^{\mathrm{Cr}},$$

and the sum of the whole stream of premiums is equal to

$$\sum_{t=1}^{T} \widehat{P}_t^{\mathrm{GMS}} = \frac{T-1}{T}X_1 + P_1^{\mathrm{Cr}} + \frac{T-1}{T}P_2^{\mathrm{Cr}} + \cdots + \frac{1}{T}P_T^{\mathrm{Cr}}.$$

In spite of the decrease of premiums, the total sum is slightly smaller than the single claim X_1, thus the insurance policy does not provide necessary insurance coverage.

Note that the problem appeared because the first component of the sum (12) does not include the losses $X_{-n}, \ldots, X_{-1}$. We propose the following adjustment of the stream (12):

$$\widehat{P}_t = (1-\beta_t)\frac{1}{t-1+n}\sum_{i=-n}^{t-1} X_i + \beta_t P_t^{\mathrm{Cr}}, \tag{13}$$

where

$$\beta_t = \frac{T-t+1}{T}.$$

Note that

$$U_t(\theta) = \mathsf{E}[\widehat{P}_t - P_t^{\mathrm{Cr}} \,|\, \Theta = \theta] = \mathsf{E}[\widehat{P}_t^{\mathrm{GMS}} - P_t^{\mathrm{Cr}} \,|\, \Theta = \theta]$$

for all $t = 1, \ldots, T$ and θ. In other words, the stream of premiums (13) differentiates good and bad clients in the same way as (11). On the other hand, from (13) and (10) it follows that the stream (13) minimizes the sum

$$\mathsf{E}\Big[\sum_{t=1}^{T} \big((P_t - X_t)^2 + \gamma_t^2(\mathsf{E}(P_t - X_t \,|\, \Theta))^2\big)\Big], \tag{14}$$

where

$$P_t = a_{0,t} + \sum_{i=-n}^{-1} a_{i,t}X_{i,t} + \sum_{j=1}^{t-1} a_{j,t}X_{j,t}, \quad a_{i,t} \in \mathbb{R},$$

and

$$\gamma_t^2 = \frac{(t-1)(s^2 + a^2(t-1+n))}{(T-t+1)a^2(t-1+n)}, \quad t = 1, \ldots, T.$$

The stream $\widehat{P}_t$ satisfies the net premium principle $\mathsf{E}\widehat{P}_t = \mathsf{E}X_t$, $t = 1, \ldots, T$. Hence the weak Axiom of Solvency is satisfied, i.e. $\mathsf{E}\sum_{s=1}^{t} \widehat{P}_s \geq \mathsf{E}\sum_{s=1}^{t} X_s$

for all t and $\mathsf{E}\sum_{t=1}^{T} P_t = \mathsf{E}\sum_{t=1}^{T} X_t$ (see [4]). Example 1 shows differences between (12) and (13).

Example 1. Let $T = 5$ years, $n = 10$ years, $X_{-10} = X_{-9} = \cdots = X_{-1} = 0$, $X_1 = 20\,000$, $X_2 = \cdots = X_5 = 0$. The stream of premiums (12) is is of the form

$$P_1^{\mathrm{GMS}} = P_1^{\mathrm{Cr}}, \quad P_2^{\mathrm{GMS}} = 4\,000 + \tfrac{4}{5}P_1^{\mathrm{Cr}}, \quad P_3^{\mathrm{GMS}} = 4\,000 + \tfrac{3}{5}P_1^{\mathrm{Cr}},$$
$$P_4^{\mathrm{GMS}} = 4\,000 + \tfrac{2}{5}P_1^{\mathrm{Cr}}, \quad P_5^{\mathrm{GMS}} = 4\,000 + \tfrac{1}{5}P_1^{\mathrm{Cr}}.$$

The total sum of premiums during 5 years is equal to $16\,000 + 3P_1^{\mathrm{Cr}}$ and it corresponds to a single claim equal to 20 000. Using the formula (13) we have

$$P_1 = P_1^{\mathrm{Cr}}, \quad P_2 = 363 + \tfrac{4}{5}P_1^{\mathrm{Cr}}, \quad P_3 = 666 + \tfrac{3}{5}P_1^{\mathrm{Cr}},$$
$$P_4 = 923 + \tfrac{2}{5}P_1^{\mathrm{Cr}}, \quad P_5 = 1\,142 + \tfrac{1}{5}P_1^{\mathrm{Cr}},$$

and the sum is equal to $P_1 + \cdots + P_5 = 3\,094 + 3P_1^{\mathrm{Cr}}$.

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W. Antoniak
Warsaw School of Economics
Collegium of Economic Analysis
Aleja Niepodległości 162
Warszawa, Poland
E-mail: antoniakwojtek@gmail.com

M. Kałuszka
Institute of Mathematics
Technical University of Łódź
Wólczańska 215
90-005 Łódź, Poland
E-mail: kaluszka@p.lodz.pl

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