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A NEW APPROACH TO THE CONSTRAINED CONTROLLABILITY PROBLEM

Abstract. We consider the problem of internal regional controllability with output constraints. It consists in steering a hyperbolic system to a final state between two prescribed functions only on a subregion of the evolution system domain. This problem is solved by characterizing the optimal control in terms of a subdifferential associated with the minimized functional.

1. Introduction. Various real problems can be formulated within certain concepts of distributed systems analysis. These include notions like controllability, observability etc., which enable a better knowledge and understanding of the system under study.

For distributed parameter systems, controllability consists in steering a system to a prescribed state defined on a spatial domain Ω. This concept has been much studied and widely developed [3, 2]. Later the notion of regional controllability was introduced and studied [5], and interesting results have been obtained, in particular the possibility to reach a state only on a subregion ω interior to Ω. These results have been applied to the case where ω is a part of the boundary $\partial\Omega$ of Ω [9]. A situation that is very important in practical applications is that of controllability with hard constraints on controls and states [10]. Here we are interested in steering the system from an initial state to a final one between two prescribed functions given only on a subregion ω of the geometric domain where the system is considered.

There are many reasons for studying this kind of problem. One of them is the mathematical model of a real system which is obtained from measures or from approximation techniques and is very often affected by perturbations,

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so if the solution for such a system is approximately known then the control problem subject to output constraints is more realistic and more adapted to system analysis than the classical one [6]. Also this kind of problem corresponds to real industrial preoccupations where the desired state is required only to be between two desired profiles.

The aim of this paper is to explore the notion of hyperbolic systems with constraints and to give an approach which leads to interesting results characterizing the optimal control that satisfies the output constraints.

2. Constrained controllability. Let Ω be an open bounded and regular subset of $\mathbb{R}^n$ $(n = 1, 2, 3)$ with boundary $\partial\Omega$. For $T > 0$, let $Q = \Omega\times]0, T[$ and $\Sigma = \partial\Omega \times]0, T[$. We consider the following hyperbolic system:

$$(2.1)\qquad \begin{cases} \dfrac{\partial^2 y}{\partial t^2}(x,t) - Ay(x,t) = Bu(t) & \text{in } Q, \\ y(x,0) = y_0(x),\ \dfrac{\partial y}{\partial t}(x,0) = y_1(x) & \text{in } \Omega, \\ y(\xi,t) = 0 & \text{on } \Sigma, \end{cases}$$

where A is a second-order elliptic linear operator, $B \in \mathcal{L}(\mathbb{R}^p, L^2(\Omega))$, $u \in U = L^2(0, T, \mathbb{R}^p)$ (p depends on the number of actuators considered) and $(y_0, y_1) \in H_0^1(\Omega) \times L^2(\Omega)$. We denote by $Z_u(\cdot) = \big(y_u(\cdot), \frac{\partial y_u}{\partial t}(\cdot)\big)$ the solution of (2.1) such that the final state $Z_u(T)$ is in $H_0^1(\Omega) \times L^2(\Omega)$.

If we denote

$$\bar{A} = \begin{pmatrix} 0 & I \\ A & 0 \end{pmatrix}, \qquad z = \begin{bmatrix} y \\ \dfrac{\partial y}{\partial t} \end{bmatrix}, \qquad \bar{B}u = \begin{bmatrix} 0 \\ Bu \end{bmatrix},$$

then system (2.1) can be written as follows:

$$(2.2)\qquad \begin{cases} \dfrac{\partial z}{\partial t}(x,t) + \bar{A}z(x,t) = \bar{B}u(t) & \text{in } Q, \\ z(0) = (y_0, y_1) & \text{in } \Omega. \end{cases}$$

The operator $\bar{A}$ is closed and linear, with dense domain in $H_0^1(\Omega) \times L^2(\Omega)$. Hence system (2.2) admits a unique solution which is expressed using the semigroup $(\bar{S}(t))_{t\geq 0}$ generated by $\bar{A}$ [7] as follows:

$$z(t) = \bar{S}(t)z_0 + \int_0^t \bar{S}(t-\tau)\bar{B}u(\tau)\, d\tau.$$

Let ω be an open subset of Ω assumed to be of positive Lebesgue measure and consider the restriction operator to ω defined as follows:

$$\chi_\omega : L^2(\Omega) \times L^2(\Omega) \to L^2(\omega) \times L^2(\omega), \qquad (z_1, z_2) \mapsto (z_1, z_2)_{|\omega},$$

and its adjoint operator χ_ω^* defined from $L^2(\omega)\times L^2(\omega)$ to $L^2(\Omega)\times L^2(\Omega)$ by

$$\chi_\omega^*(z_1,z_2)(x) = \begin{cases} (z_1,z_2)x, & x\in\omega,\\ 0, & x\in\Omega\setminus\omega.\end{cases}$$

Moreover, let

$$\tilde{\chi}_\omega : L^2(\Omega)\to L^2(\omega), \qquad z\mapsto z_{|\omega}.$$

Let $\alpha_i(\cdot)$ and $\beta_i(\cdot)$ $(i=1,2)$ be functions in $L^2(\omega)$ such that $\alpha_i(\cdot)\le\beta_i(\cdot)$ a.e. in ω. Throughout the paper we set

$$[\alpha_i(\cdot),\beta_i(\cdot)] = \{y_i\in L^2(\omega) \mid \alpha_i(\cdot)\le y_i\le\beta_i(\cdot) \text{ a.e. in } \omega\} \quad (i=1,2).$$

We recall that an actuator is conventionally defined by a couple (D,f), where $D\subset\bar{\Omega}$ is the geometric support of the actuator and f is the spatial distribution of the action on the support D [4]. In the case of a pointwise actuator (internal or boundary) $D=\{b\}$ and $f=\delta(b-\cdot)$, where δ is the Dirac mass concentrated at b, and the actuator is then denoted by (b,δ_b). For definitions and properties of strategic actuators we refer to [5, 9].

We also recall that system (2.1) is said to be ω *exactly* (resp. ω *approximately*) *controllable* if for all $(p^d,v^d)\in L^2(\omega)\times L^2(\omega)$ (resp. for all $\epsilon>0$) there exists a control $u\in U$ such that $\tilde{\chi}_\omega y_u(T)=p^d$ and $\tilde{\chi}_\omega\frac{\partial y_u}{\partial t}(T)=v^d$ (resp. $\|\tilde{\chi}_\omega y_u(T)-p^d\|_{L^2(\omega)}+\left\|\tilde{\chi}_\omega\frac{\partial y_u}{\partial t}(T)-v^d\right\|_{L^2(\omega)}<\epsilon$) [11].

Let H be the operator from U to $L^2(\Omega)\times L^2(\Omega)$ defined by

$$Hu=\int_0^T \bar{S}(T-\tau)\bar{B}u(\tau)\,d\tau \qquad \text{for } u\in U.$$

Definition 2.1. We say that system (2.1) is $[\alpha_1(\cdot),\beta_1(\cdot)]\times[\alpha_2(\cdot),\beta_2(\cdot)]$-*controllable* in ω if

$$(\operatorname{Im}\chi_\omega H)\cap([\alpha_1(\cdot),\beta_1(\cdot)]\times[\alpha_2(\cdot),\beta_2(\cdot)])\neq\emptyset.$$

Remark 2.2. Equivalently, system (2.1) is $[\alpha_1(\cdot),\beta_1(\cdot)]\times[\alpha_2(\cdot),\beta_2(\cdot)]$-controllable in ω at time T if there exists $u\in U$ such that

$$\alpha_1(\cdot)\le\tilde{\chi}_\omega y_u(T)\le\beta_1(\cdot) \quad\text{and}\quad \alpha_2(\cdot)\le\tilde{\chi}_\omega\frac{\partial y_u}{\partial t}(T)\le\beta_2(\cdot) \text{ a.e. in } \omega.$$

Definition 2.3. The actuator (D,f) is said to be $[\alpha_1(\cdot),\beta_1(\cdot)]\times[\alpha_2(\cdot),\beta_2(\cdot)]$-*strategic* in ω if the excited system is $[\alpha_1(\cdot),\beta_1(\cdot)]\times[\alpha_2(\cdot),\beta_2(\cdot)]$-controllable in ω.

Remark 2.4. 1. If system (2.1) is $[\alpha_1(\cdot),\beta_1(\cdot)]\times[\alpha_2(\cdot),\beta_2(\cdot)]$-controllable in ω_1, then it is $[\alpha_1(\cdot),\beta_1(\cdot)]\times[\alpha_2(\cdot),\beta_2(\cdot)]$-controllable for any $\omega_2\subseteq\omega_1$.

2. Let

$$\mathcal{J}(u)=\frac{1}{2}\int_0^T\|u(t)\|^2_{\mathbb{R}^p}\,dt$$

be the transfer cost, $(p^d, v^d) \in I := [\alpha_1(\cdot), \beta_1(\cdot)] \times [\alpha_2(\cdot), \beta_2(\cdot)]$, and consider the sets

$$\mathcal{W}_\omega = \left\{ u \in L^2(0,T,\mathbb{R}^p) \;\middle|\; \left(y_u(T), \frac{\partial y_u}{\partial t}(T) \right) = (p^d, v^d) \text{ in } \omega \right\},$$

$$\mathcal{W}_I = \left\{ u \in L^2(0,T,\mathbb{R}^p) \;\middle|\; \alpha_1(\cdot) \le y_u(T) \le \beta_1(\cdot) \text{ and } \alpha_2(\cdot) \le \frac{\partial y_u}{\partial t}(T) \le \beta_2(\cdot) \text{ a.e. in } \omega \right\}.$$

We have $\mathcal{W}_\omega \subseteq \mathcal{W}_I$, so

$$\inf_{\mathcal{W}_I} \mathcal{J}(u) \le \inf_{\mathcal{W}_\omega} \mathcal{J}(u).$$

This means that the cost of steering the system in $[\alpha_1(\cdot), \beta_1(\cdot)] \times [\alpha_2(\cdot), \beta_2(\cdot)]$ is less than steering it to a fixed desired state (p^d, v^d) in ω.

The $[\alpha_1(\cdot), \beta_1(\cdot)] \times [\alpha_2(\cdot), \beta_2(\cdot)]$-controllability in ω may be characterized by the following result:

PROPOSITION 2.5. *System* (2.1) *is* $[\alpha_1(\cdot), \beta_1(\cdot)] \times [\alpha_2(\cdot), \beta_2(\cdot)]$*-controllable in* ω *if and only if*

$$(\operatorname{Ker} \chi_\omega + \operatorname{Im} H) \cap ([\alpha_1(\cdot), \beta_1(\cdot)] \times [\alpha_2(\cdot), \beta_2(\cdot)]) \ne \emptyset.$$

Proof. We suppose that there exist $z \in ([\alpha_1(\cdot), \beta_1(\cdot)] \times [\alpha_2(\cdot), \beta_2(\cdot)])$ and $u \in U$ such that $\chi_\omega Z_u(T) = \chi_\omega z$. Set $z_1 = z - Z_u(T)$ and $z_2 = Z_u(T)$. Then $z = z_1 + z_2$ where $z_1 \in \operatorname{Ker} \chi_\omega$ and $z_2 \in \operatorname{Im} H$, which proves that $z \in \operatorname{Ker} \chi_\omega + \operatorname{Im} H$.

Conversely, if $(\operatorname{Ker} \chi_\omega + \operatorname{Im} H) \cap ([\alpha_1(\cdot), \beta_1(\cdot)] \times [\alpha_2(\cdot), \beta_2(\cdot)]) \ne \emptyset$ then there exists $z \in ([\alpha_1(\cdot), \beta_1(\cdot)] \times [\alpha_2(\cdot), \beta_2(\cdot)])$ such that $z \in \operatorname{Ker} \chi_\omega + \operatorname{Im} H$, so $z = z_1 + z_2$, where $\chi_\omega z_1 = 0$ and $z_2 = Hu$ for some $u \in U$. It follows that there exist $z \in [\alpha_1(\cdot), \beta_1(\cdot)] \times [\alpha_2(\cdot), \beta_2(\cdot)]$ and $u \in U$ such that $\chi_\omega Z_u(T) = z$. ■

3. Subdifferential approach. The purpose of this section is to apply the subdifferential approach [1] to the optimal control problem for a hyperbolic equation excited by an internal zone actuator which steers system (2.1) from the initial state $(y_0, y_1) \in H_0^1(\Omega) \times L^2(\Omega)$ to a final state (p^d, v^d) such that $\alpha_1(\cdot) \le p^d \le \beta_1(\cdot)$ and $\alpha_2(\cdot) \le v^d \le \beta_2(\cdot)$ in a subregion ω.

More precisely we are interested in the following minimization problem:

$$\begin{cases} \inf \mathcal{J}(u), \\ u \in U_{\text{ad}}, \end{cases} \tag{3.1}$$

where

$$U_{\mathrm{ad}} = \left\{ u \in U \;\middle|\; \left(y_u(T), \frac{\partial y_u}{\partial t}(T) \right) \in [\alpha_1(\cdot), \beta_1(\cdot)] \times [\alpha_2(\cdot), \beta_2(\cdot)] \right\}$$

is the set of admissible controls.

The following result ensures the existence and uniqueness of solution of problem (3.1).

PROPOSITION 3.1. *If system* (2.1) *is* $[\alpha_1(\cdot), \beta_1(\cdot)] \times [\alpha_2(\cdot), \beta_2(\cdot)]$*-controllable in* ω *then problem* (3.1) *has a unique solution* u^*.

Proof. If system (2.1) is $[\alpha_1(\cdot), \beta_1(\cdot)] \times [\alpha_2(\cdot), \beta_2(\cdot)]$-controllable in ω then $U_{\mathrm{ad}} \neq \emptyset$. Since the mapping $u \mapsto \frac{1}{2}\|u\|^2$ is strictly convex, coercive, proper and lower semicontinuous in U which is reflexive, we only have to verify that U_{ad} is a closed convex subset of U.

The mapping $u \mapsto \big(y_u(T), \frac{\partial y_u}{\partial t}(T)\big)$ is linear, so U_{ad} is convex.

To prove that U_{ad} is closed, we consider a sequence $(u_n)_n$ in U_{ad} such that $u_n \to u$ strongly in U.

Since $\chi_\omega H$ is continuous, $\chi_\omega H u_n$ converges strongly to $\chi_\omega H u$ in $L^2(\omega) \times L^2(\omega)$, but $\big(\chi_\omega y_{u_n}(T), \chi_\omega \frac{\partial y_{u_n}}{\partial t}(T)\big) \in [\alpha_1(\cdot), \beta_1(\cdot)] \times [\alpha_2(\cdot), \beta_2(\cdot)]$ which is closed, so $\big(\chi_\omega y_u(T), \chi_\omega \frac{\partial y_u}{\partial t}(T)\big) \in [\alpha_1(\cdot), \beta_1(\cdot)] \times [\alpha_2(\cdot), \beta_2(\cdot)]$; this means that $u \in U_{\mathrm{ad}}$ is closed for the topology induced on $L^2(0, T, \mathbb{R}^p)$. Thus (3.1) has a unique solution. ∎

Let $\varGamma_0(U)$ denote the set of proper semicontinuous and convex functions $f : U \to \tilde{\mathbb{R}} =]-\infty, +\infty]$.

For $f \in \varGamma_0(U)$,

$$\mathrm{dom}(f) = \{u \in U \mid f(u) < \infty\},$$

and f^* is the polar function of f given by

$$f^*(v^*) = \sup_{u \in \mathrm{dom}(f)} \{\langle v^*, u\rangle - f(u)\} \quad \forall v^* \in U.$$

For $v_0 \in \mathrm{dom}(f)$ the set

$$\partial f(v_0) = \{u^* \in U \mid f(u) \geq f(v_0) + \langle u^*, u - v_0\rangle \ \forall u \in U\}$$

is the *subdifferential* of f at v_0.

For a nonempty subset G of U,

$$\psi_G(u) = \begin{cases} 0 & \text{if } u \in G, \\ \infty & \text{otherwise,} \end{cases}$$

denotes the indicator functional of G.

Then problem (3.1) is equivalent to the problem

$$\begin{cases} \inf\big(\frac{1}{2}\|u\|^2 + \psi_{U_{\mathrm{ad}}}(u)\big), \\ u \in U_{\mathrm{ad}}. \end{cases} \tag{3.2}$$

And the solution of problem (3.2) may be characterized by the following result.

PROPOSITION 3.2. *If system* (2.1) *is* $[\alpha_1(\cdot),\beta_1(\cdot)]\times[\alpha_2(\cdot),\beta_2(\cdot)]$*-controllable in* ω *then* u^* *is the solution of* (3.2) *if and only if*

$$u^* \in U_{\mathrm{ad}} \quad \textit{and} \quad \psi^*_{U_{\mathrm{ad}}}(-u^*) = -\|u^*\|^2. \tag{3.3}$$

Proof. Let $f_\sigma(u^*) = \frac{1}{2}\|u^*\|^2$. Then u^* is the solution of (3.2) if and only if $0 \in \partial(f_\sigma + \psi_{U_{\mathrm{ad}}})(u^*)$. But $f_\sigma \in \Gamma_0(U)$, and since U_{ad} is closed, convex and nonempty, we have $\psi_{U_{\mathrm{ad}}} \in \Gamma_0(U)$. Moreover $[\alpha_1(\cdot),\beta_1(\cdot)]\times[\alpha_2(\cdot),\beta_2(\cdot)]$-controllability in ω implies $\mathrm{dom}(f_\sigma)\cap\mathrm{dom}(\psi_{U_{\mathrm{ad}}}) \neq \emptyset$. Since f_σ is continuous we obtain

$$\partial(f_\sigma + \psi_{U_{\mathrm{ad}}})(u^*) = \partial f_\sigma(u^*) + \partial\psi_{U_{\mathrm{ad}}}(u^*).$$

It follows that u^* is the solution of (3.2) if and only if $0 \in \partial f_\sigma(u^*) + \partial\psi_{U_{\mathrm{ad}}}(u^*)$. Furthermore f_σ is Fréchet differentiable, so $\partial f_\sigma(u^*) = \{u^*\}$ and u^* is the solution of (3.2) if and only if $-u^* \in \partial\psi_{U_{\mathrm{ad}}}(u^*)$ which is equivalent to $u^* \in U_{\mathrm{ad}}$, $\psi_{U_{\mathrm{ad}}}(u^*) + \psi^*_{U_{\mathrm{ad}}}(-u^*) = -\|u^*\|^2$ and gives $u^* \in U_{\mathrm{ad}}$, $\psi^*_{U_{\mathrm{ad}}}(-u^*) = -\|u^*\|^2$. ∎

Set

$$\begin{aligned} a(\cdot) &= (\alpha_1(\cdot),\alpha_2(\cdot)) - \chi_\omega S(T)(y_0,y_1),\\ b(\cdot) &= (\beta_1(\cdot),\beta_2(\cdot)) - \chi_\omega S(T)(y_0,y_1). \end{aligned}$$

Then $U_{\mathrm{ad}} = \{u \in U \mid \chi_\omega H u \in [a(\cdot),b(\cdot)]\}$ and we have the following result:

PROPOSITION 3.3. *If system* (2.1) *is* $[\alpha_1(\cdot),\beta_1(\cdot)]\times[\alpha_2(\cdot),\beta_2(\cdot)]$*-controllable in* ω *then* u^* *is the solution of* (3.2) *if and only if*

$$\min\{\langle(\chi_\omega H)^\dagger a(\cdot), u^*\rangle, \langle(\chi_\omega H)^\dagger b(\cdot), u^*\rangle\} = \|u^*\|^2, \tag{3.4}$$

where $(\chi_\omega H)^\dagger$ *is the pseudo-inverse of* $\chi_\omega H$.

Proof. We have $U_{\mathrm{ad}} = (\chi_\omega H)^\dagger[a(\cdot),b(\cdot)]$ and from Proposition 3.2, u^* is the solution of (3.2) if and only if $u^* \in U_{\mathrm{ad}}$, $\psi^*_{U_{\mathrm{ad}}}(-u^*) = -\|u^*\|^2$.

Then

$$\begin{aligned} \psi^*_{U_{\mathrm{ad}}}(v^*) &= \sup_{v\in U_{\mathrm{ad}}} \langle v^*, v\rangle \qquad \forall v^* \in U\\ &= \sup_{v\in(\chi_{\omega H})^\dagger([a(\cdot),b(\cdot)])} \langle v^*, v\rangle = \sup_{y\in[a(\cdot),b(\cdot)]} \langle v^*, (\chi_{\omega H})^\dagger y\rangle\\ &= \sup_{\lambda\in[0,1]} \langle((\chi_{\omega H})^\dagger)^* v^*, \lambda a(\cdot) + (1-\lambda)b(\cdot)\rangle. \end{aligned}$$

Since the map $\lambda \mapsto \langle((\chi_{\omega H})^\dagger)^* v^*, \lambda a(\cdot) + (1-\lambda)b(\cdot)\rangle$ from $[0,1]$ into $\mathbb{R}$ is convex and continuous, the Krein–Milman theorem [8] yields

$$\psi^*_{U_{\mathrm{ad}}}(v^*) = \sup_{\lambda\in[0,1]} \langle((\chi_{\omega H})^\dagger)^* v^*, \lambda a(\cdot) + (1-\lambda)b(\cdot)\rangle.$$

It follows that $\psi^*_{U_{\mathrm{ad}}}(v^*) = \max\{\langle v^*, (\chi_{\omega H})^\dagger a(\cdot)\rangle, \langle v^*, (\chi_{\omega H})^\dagger b(\cdot)\rangle\}$ and from (3.3) we obtain (3.4). ■

REMARK 3.4. If $\alpha_1(\cdot) = \beta_1(\cdot) = \{p_d\}$ and $\alpha_2(\cdot) = \beta_2(\cdot) = \{v_d\}$ we find the notion of regional controllability [11] then $a(\cdot) = b(\cdot) = (p_d(\cdot), v_d(\cdot)) - \chi_\omega S(T)(y_0, y_1)$ and

$$U_{\mathrm{ad}} = (\chi_\omega H)^\dagger((p_d, v_d) - \chi_\omega S(T)(y_0, y_1))$$

and the solution of (3.1) is given by

$$u^*(t) = (\chi_\omega H)^\dagger((p_d, v_d) - \chi_\omega S(T)(y_0, y_1)).$$

4. Conclusion. We have developed an extension of the notion of controllability for hyperbolic systems with constraints and we characterized the optimal control using the subdifferential approach. However, numerical simulations cannot be carried out without difficulty. Future work will aim to overcome this difficulty by giving an approach based on Lagrangian multipliers and also to extend this notion of internal regional controllability with constraints to the case where ω is a part of the boundary of the system evolution domain Ω.

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