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# QUASI-DIFFUSION SOLUTION OF A STOCHASTIC DIFFERENTIAL EQUATION

*Abstract.* We consider the stochastic differential equation

$$X_t = X_0 + \int_0^t (A_s + B_s X_s)\,ds + \int_0^t C_s\,dY_s, \tag{0.1}$$

where $A_t$, $B_t$, $C_t$ are nonrandom continuous functions of $t$, $X_0$ is an initial random variable, $Y = (Y_t,\, t \ge 0)$ is a Gaussian process and $X_0$, $Y$ are independent. We give the form of the solution $(X_t)$ to (0.1) and then basing on the results of Plucińska [Teor. Veroyatnost. i Primenen. 25 (1980)] we prove that $(X_t)$ is a quasi-diffusion proces.

**1. Stochastic integral.** In this section, we define stochastic integrals

$$\int_a^b g(t)\,d_tY(t,\omega)$$

following [2] (see also [1] and [6]) in a way that will be useful in Section 2. Here $g : \mathbb{R} \to \mathbb{R}$ is a deterministic continuous function and $Y$ is a Gaussian process with an absolutely continuous covariance function. For abbreviation, we write the above integral in the form

$$\int_a^b g(t)\,dY_t.$$

Let $Y = (Y_t,\, t \ge 0)$ be a zero mean process with an absolutely continuous covariance function. We assume that there exists a function $\kappa : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ such that

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$$E[(Y_{t_r} - Y_{t_{r-1}})(Y_{t_s} - Y_{t_{s-1}})] = \int_{t_{r-1}}^{t_r} \int_{t_{s-1}}^{t_s} \kappa(u,v)\,du\,dv \tag{1.1}$$

for all $r \le s$, $t_r \le t_s$, $r, s = 1, \ldots, n$ and fixed n. The function $\kappa$ may also be a Dirac delta function. We call $\kappa$ a *quasi spectral density*.

To define the integral $\int_a^b g(t)\,dY_t$ consider first the case when $g$ is piecewise constant, i.e. there exist $t_0 < t_1 < \cdots < t_n$ such that

$$g(t) = g_k \quad \text{for } t_{k-1} \le t < t_k,\ k = 1, \ldots, n,$$

where the $g_k$ are constants, $t_0 = a$, $t_n = b$. Let

$$\Delta_k Y = Y_{t_k} - Y_{t_{k-1}}.$$

Then

$$\int_a^b g(t)\,dY_t = \sum_{k=1}^n g_k \Delta_k Y.$$

LEMMA 1.1. *If $g_1$ and $g_2$ are piecewise constant functions, then*

$$E\Big[\int_a^b g_1(t)\,dY_t \int_a^b g_2(t)\,dY_t\Big] = \iint_{S_1} g_1(u)g_2(v)\kappa(u,v)\,du\,dv + \iint_{S_2} g_1(v)g_2(u)\kappa(u,v)\,du\,dv,$$

*where $S_1 = \{(u,v) : a < u < b, u \le v < b\}$ and $S_2 = \{(u,v) : a < v < b, v < u < b\}$.*

*Proof.* Assume that $a = t_0 \le t_1 < \cdots < t_n = b$ are such that the functions $g_1$ and $g_2$ are constant in every $(t_{k-1}, t_k)$ for $k = 1, \ldots, n$ and equal respectively to $g_{1k}$, $g_{2k}$. Then

$$\begin{aligned}
E\Big[\int_a^b g_1(t)\,dY_t \int_a^b g_2(t)\,dY_t\Big] &= E\Big[\sum_{k=1}^n g_{1k}\Delta_k Y \sum_{k=1}^n g_{2k}\Delta_k Y\Big] \\
&= E\Big[\sum_{1\le i\le n}\sum_{i\le j\le n} g_{1i}g_{2j}(Y_{t_i} - Y_{t_{i-1}})(Y_{t_j} - Y_{t_{j-1}}) \\
&\quad + \sum_{1<i\le n}\sum_{1\le j<i} g_{1i}g_{2j}(Y_{t_i} - Y_{t_{i-1}})(Y_{t_j} - Y_{t_{j-1}})\Big] \\
&= \sum_{1\le i\le n}\sum_{i\le j\le n} g_{1i}g_{2j}E[(Y_{t_i} - Y_{t_{i-1}})(Y_{t_j} - Y_{t_{j-1}})] \\
&\quad + \sum_{1<i\le n}\sum_{1\le j<i} g_{1i}g_{2j}E[(Y_{t_i} - Y_{t_{i-1}})(Y_{t_j} - Y_{t_{j-1}})]
\end{aligned}$$

$$= \sum_{1\le i\le n}\sum_{i\le j\le n} g_{1i}g_{2j}\int_{t_{i-1}}^{t_i}\int_{t_{j-1}}^{t_j}\kappa(u,v)\,du\,dv$$

$$+ \sum_{1\le j<n}\sum_{j<i\le n} g_{1i}g_{2j}\int_{t_{j-1}}^{t_j}\int_{t_{i-1}}^{t_i}\kappa(u,v)\,dv\,du$$

$$= \iint_{S_1} g_1(u)g_2(v)\kappa(u,v)\,du\,dv + \iint_{S_2} g_1(v)g_2(u)\kappa(u,v)\,du\,dv. \blacksquare$$

LEMMA 1.2. *If* $(Y_t,\, t\ge 0)$ *is a Gaussian process and* $I_t = \int_0^t g(s)\,dY_s$ *with* $g$ *piecewise constant, then* $(I_t,\, t\ge 0)$ *is also Gaussian.*

*Proof.* This is obvious. ∎

We now approximate $g$ by piecewise constant functions. Let $g$ be a function such that

$$\int_a^b\int_a^b g(u)g(v)\kappa(u,v)\,du\,dv < \infty.$$

There exists a sequence $\{g_n\}$ of piecewise constant functions [2] such that

$$\lim_{n,m\to\infty}\int_a^b\int_a^b (g(u)-g_n(u))(g(v)-g_m(v))\kappa(u,v)\,du\,dv = 0, \qquad n,m = 0,1,\ldots.$$

Then we define the integral $\int_a^b g(t)\,dY_t$ as

$$\int_a^b g(t)\,dY_t = \underset{n\to\infty}{\mathrm{l.i.m.}}\int_a^b g_n(t)\,dY_t, \tag{1.2}$$

the mean square limit of the integrals $\int_a^b g_n(t)\,dY_t$.

One can prove [2] that this limit does not depend on the approximating sequence $g_n$. Furthermore, the properties of the integral (1.2) are analogous to the properties of integrals of piecewise constant functions. In particular, $\int_a^b g(t)\,dY_t$ is a Gaussian process as a limit of Gaussian processes [4].

**2. Quasi-diffusion solution of the stochastic differential equation.** Let $(\Omega,\mathcal{F},\mathbb{P})$ be a probability space. Let $Y=(Y_t,\, t\ge 0)$ be a Gaussian process which satisfies (1.1). Let $X_0$ be a random variable, independent of $Y$, whose distribution is absolutely continuous.

LEMMA 2.1. *Consider the stochastic differential equation*

$$X_t = X_0 + \int_0^t (A_s + B_sX_s)\,ds + \int_0^t C_s\,dY_s, \tag{2.1}$$

*where* $A_t, B_t, C_t : [0,\infty) \to \mathbb{R}$ *are nonrandom continuous functions of* $t$ *and* $X_0$ *is an initial random variable. Then*

(a) *there is a unique solution to* (2.1) *given by the formula*

$$X_t = e^{\int_0^t B_s ds}\Big[X_0 + \int_0^t e^{-\int_0^s B_u du} A_s\,ds + \int_0^t e^{-\int_0^s B_u du} C_s\,dY_s\Big], \tag{2.2}$$

(b) *the covariance of the solution is equal to*

$$\mathrm{Cov}(X_{t_1}, X_{t_2}) = e^{\int_0^{t_1} B_s\,ds + \int_0^{t_2} B_s\,ds}\Big(E[X_0 - EX_0]^2 + \int_0^{t_1}\int_0^{t_2} e^{-\int_0^u B_s\,ds - \int_0^v B_s\,ds} C_u C_v \kappa(u,v)\,du\,dv\Big).$$

*Proof.* (a) We use the method of successive approximations from [3]. Let $X_t^0 = X_0$ for all $t$ and

$$X_t^n = X_0 + \int_0^t (A_s + B_s X_s^{n-1})\,ds + \int_0^t C_s\,dY_s \quad (n \ge 1).$$

Define

$$\alpha_t = \int_0^t A_s\,ds, \quad \beta_t = \int_0^t B_s\,ds, \quad J_t = \int_0^t C_s\,dY_s.$$

Let

$$(Lh)(t) = \int_0^t B_s h_s\,ds.$$

This is a Volterra operator on $L^2[0,T]$. Then

$$X_t^1 = X_0(1+\beta_t) + \alpha_t + J_t$$

and for any $n \ge 1$,

$$X_t^{n+1} = X_0[1 + \beta_t + \cdots + L^n\beta_t] + [\alpha_t + L\alpha_t + \cdots + L^n\alpha_t] + \int_0^t \Big[1 + (\beta_t - \beta_s) + \frac{1}{2!}(\beta_t-\beta_s)^2 + \cdots + \frac{1}{n!}(\beta_t - \beta_s)^n\Big] C_s\,dY_s.$$

Since $L$ is a Volterra operator,

$$\alpha_t + L\alpha_t + \cdots + L^n\alpha_t \to (I-L)^{-1}\alpha_t,$$

$$(I-L)^{-1}\alpha_t = e^{\beta_t}\int_0^t e^{-\beta_s} A_s\,ds,$$

$$[1 + \beta_t + \cdots + L^n\beta_t] \to 1 + (I-L)^{-1}\beta_t = 1 + e^{\beta_t}\int_0^t e^{-\beta_s} B_s\,ds = e^{\beta_t}.$$

Let

$$f_n(t,s) = 1 + (\beta_t - \beta_s) + \frac{1}{2!}(\beta_t - \beta_s)^2 + \cdots + \frac{1}{n!}(\beta_t - \beta_s)^n.$$

Then

$$E\Big[\int_0^t f_n(t,s)C_s\,dY_s - \int_0^t e^{\beta_t - \beta_s} C_s\,dY_s\Big]^2 \to 0 \quad \text{as } n \to \infty.$$

Hence a solution $X_t$ is given by

$$X_t = e^{\int_0^t B_s\,ds}\Big[X_0 + \int_0^t e^{-\int_0^s B_u\,du} A_s\,ds + \int_0^t e^{-\int_0^s B_u\,du} C_s\,dY_s\Big].$$

Now we prove uniqueness. Let $X_t^1$ and $X_t^2$ be two solutions of (2.1). Then

$$X_t^1 - X_t^2 = \int_0^t B_s(X_s^1 - X_s^2)\,ds.$$

Because $B_s$ is bounded by a constant $b$ on any finite interval, we have

$$|X_t^1 - X_t^2| \le b\int_0^t |X_s^1 - X_s^2|\,ds,$$

and by Gronwall's lemma ([4, pp. 287–288])

$$|X_t^1 - X_t^2| = 0,$$

which completes the proof of uniqueness.

(b) Now we calculate the covariance:

$$\begin{aligned}
\operatorname{Cov}(X_{t_1}, X_{t_2}) &= E(X_{t_1} - EX_{t_1})(X_{t_2} - EX_{t_2}) \\
&= e^{\int_0^{t_1} B_s\,ds + \int_0^{t_2} B_s ds}\Big(E[X_0 - EX_0]^2 \\
&\quad + E\Big[\int_0^{t_1} e^{-\int_0^s B_u\,du} C_s\,dY_s \int_0^{t_2} e^{-\int_0^s B_u\,du} C_s dY_s\Big]\Big) \\
&= e^{\int_0^{t_1} B_s\,ds + \int_0^{t_2} B_s\,ds}\Big(E[X_0 - EX_0]^2 + E\Big[\int_0^{t_1} e^{-\int_0^s B_u\,du} C_s\,dY_s\Big]^2 \\
&\quad + E\Big[\int_0^{t_1} e^{-\int_0^s B_u\,du} C_s\,dY_s \int_{t_1}^{t_2} e^{-\int_0^s B_u\,du} C_s\,dY_s\Big]\Big)
\end{aligned}$$

$$= e^{\int_0^{t_1} B_s\,ds + \int_0^{t_2} B_s\,ds}\Big(E[X_0 - EX_0]^2$$

$$+ \int_0^{t_1}\int_0^{t_1} e^{-\int_0^u B_s\,ds - \int_0^v B_s\,ds} C_u C_v \kappa(u,v)\,du\,dv$$

$$+ \iint_{S_1} e^{-\int_0^u B_s\,ds - \int_0^v B_s\,ds} C_u C_v \kappa(u,v)\,du\,dv$$

$$+ \iint_{S_2} e^{-\int_0^u B_s\,ds - \int_0^v B_s\,ds} C_u C_v \kappa(u,v)\,du\,dv\Big),$$

where $S_1 = \{(u,v) : 0 < u < t_1,\ u + t_1 \le v < t_2\}$, $S_2 = \{(u,v) : t_1 < v < t_2,\ v - t_1 < u < t_1\}$. Hence

$$\mathrm{Cov}(X_{t_1}, X_{t_2}) = e^{\int_0^{t_1} B_s ds + \int_0^{t_2} B_s\,ds}\Big(E[X_0 - EX_0]^2$$

$$+ \int_0^{t_1}\int_0^{t_1} e^{-\int_0^u B_s\,ds - \int_0^v B_s\,ds} C_u C_v \kappa(u,v)\,du\,dv$$

$$+ \int_0^{t_1}\int_{t_1}^{t_2} e^{-\int_0^u B_s\,ds - \int_0^v B_s\,ds} C_u C_v \kappa(u,v)\,du\,dv\Big)$$

$$= e^{\int_0^{t_1} B_s\,ds + \int_0^{t_2} B_s\,ds}\Big(E[X_0 - EX_0]^2$$

$$+ \int_0^{t_1}\int_0^{t_2} e^{-\int_0^u B_s\,ds - \int_0^v B_s\,ds} C_u C_v \kappa(u,v)\,du\,dv\Big). \blacksquare$$

Let $Z = (Z_t, t \ge 0)$ be a Gaussian process, and $X_0$ be a random variable with density $f(t_0, x_0)$, expected value $m_{X_0} = EX_0$ and variance $\sigma_{X_0}^2 = \mathrm{Var}\, X_0$. Moreover, let $X_0$ and $Z$ be independent.

We put

$$X_t = X_0 g(t) + Z_t, \tag{2.3}$$

where $g \in C^1$, $g(t_0) = 1$.

Let $t_0 < t_1 < \cdots < t_n$, $m(t) = EZ_t$, $K_{ij} = K(t_i, t_j) = \mathrm{Cov}(Z_{t_i}, Z_{t_j})$, where $i, j = 0, \ldots, n$ and $Z_{t_0} = 0$.

Notice that the equation (2.2) can also be written in the form (2.3).

LEMMA 2.2. *For every $n > 1$ the conditional distribution*

$$X_{t_n} \mid X_{t_{n-1}}, \ldots, X_{t_0}$$

*is Gaussian.*

*Proof.* It is easy to verify that the $(n+1)$-dimensional distribution of $(X_{t_0}, \ldots, X_{t_n})$ has the form

$$
\begin{aligned}
&f_{n+1}(t_0,\dots,t_n,x_0,\dots,x_n)\\
&= \frac{1}{(2\pi)^{n/2}\sqrt{\mathcal{K}^{(n)}}}\exp\bigg\{-\frac{1}{2}\sum_{k=1}^{n}\frac{1}{\sigma_k^2}\bigg[x_k - x_0 g_k - m(t_k)\\
&\qquad - \frac{1}{\mathcal{K}_{kk}^{(k)}}\sum_{i=1}^{k-1}\mathcal{K}_{ik}^{(k)}(x_i - x_0 g_i - m(t_i))\bigg]^2\bigg\} f(t_0,x_0),
\end{aligned}
$$

where $\mathcal{K}^{(n)} = \det[k_{ij}]_{i,j=1}^{n}$, $\mathcal{K}_{ij}^{(n)}$ is the cofactor of $k_{ij}$ in the matrix $[k_{ij}]_{i,j=1}^{n}$, and $\sigma_k^2 = \mathcal{K}_{kk}^{(k)}/\mathcal{K}_{k-1,k-1}^{(k)}$.

The conditional density of $X_{t_n} \mid X_{t_{n-1}},\dots,X_{t_0}$ is

$$
\begin{aligned}
f(t_n,x_n \mid t_0,\dots,t_{n-1},x_0,\dots,x_{n-1}) &= \frac{f_{n+1}(t_0,\dots,t_n,x_0,\dots,x_n)}{f_n(t_0,\dots,t_{n-1},x_0,\dots,x_{n-1})}\\
&= \frac{\sqrt{\mathcal{K}^{(n-1)}}}{\sqrt{2\pi}\sqrt{\mathcal{K}^{(n)}}}\exp\bigg\{-\frac{1}{2\sigma_n^2}\bigg[x_n - x_0 g_n - m(t_n)\\
&\qquad - \frac{1}{\mathcal{K}_{nn}^{(n)}}\sum_{i=1}^{n-1}\mathcal{K}_{in}^{(n)}(x_i - x_0 g_i - m(t_i))\bigg]^2\bigg\}. \ \blacksquare
\end{aligned}
$$

Let $t_0 < t_1 < \dots < t_{n+k}$, $\mathbf{t}_n = (t_1,\dots,t_n)$, $\Delta t_n = t_{n+1} - t_n$, $\mathbf{x}_n = (x_1,\dots,x_n) \in \mathbb{R}^n$, and let $A_1,\dots,A_k$ be Borel sets in $\mathbb{R}$. We will use the following notations:

$$
\begin{aligned}
&\mathbb{P}(X_{t_{n+1}} \in A_1,\dots,X_{t_{n+k}} \in A_k \mid X_{t_1},\dots,X_{t_n})\\
&\qquad =: \mathbb{P}^{(n)}(\mathbf{t}_n, X_{t_1},\dots,X_{t_n}, t_{n+1},\dots,t_{n+k}, A_1,\dots,A_k),\\
&\frac{1}{\Delta \mathbf{t}_n}\int_A (x - x_n)^i \mathbb{P}^{(n)}(\mathbf{t}_n,\mathbf{x}_n; t_{n+1}, dx) =: Q_i(\mathbf{t}_{n+1},\mathbf{x}_n, A), \qquad i = 0,1,2.
\end{aligned}
$$

DEFINITION 2.1. We say that $X = (X_t, t \ge 0)$ is a *quasi-diffusion process* [7] if for all $n \ge 1$, $\varepsilon > 0$, $\mathbf{t}_n$, $\mathbf{x}_n$,

$$
\lim_{\Delta t_n \to 0^+} Q_0(\mathbf{t}_{n+1},\mathbf{x}_n, V_\varepsilon(x_n)) = 0 \tag{2.4}
$$

and the following limits exist:

$$
\lim_{\Delta t_n \to 0^+} Q_i(\mathbf{t}_{n+1},\mathbf{x}_n, U_\varepsilon(x_n)) = a_i(\mathbf{t}_n,\mathbf{x}_n), \qquad i = 1,2, \tag{2.5}
$$

where

$$
U_\varepsilon(x_n) = \{x : |x - x_n| < \varepsilon\}, \qquad V_\varepsilon(x_n) = \mathbb{R} \setminus U_\varepsilon(x_n).
$$

THEOREM 2.1. *The process $(X_t)$ defined by (2.2) is a quasi-diffusion process.*

*Proof.* Condition (2.4) is proved in [7] for Gaussian processes. An analysis of the paper [7] shows that it is also true for conditional Gaussian processes considered in [5].

Now we calculate the drift coefficient $a_1(\mathbf{t}_n, \mathbf{x}_n)$ and the diffusion coefficient $a_2(\mathbf{t}_n, \mathbf{x}_n)$. We shall write $m(t) = E(X_t)$ and $k_{ij} = K(t_i, t_j) = \mathrm{Cov}(X_{t_i}, X_{t_j})$. The expected value of the process $X$ is equal to

$$m(t_i) = e^{\int_0^{t_i} B_s\,ds}\Big[EX_0 + \int_0^{t_i} e^{-\int_0^s B_u\,du} A_s\,ds\Big].$$

Notice that by continuity of the functions $A_s$, $B_s$, $C_s$ the following limits exist:

$$\lim_{\Delta t_n \to 0} \frac{1}{\Delta t_n}\,[K(t_i, t_{n+1}) - K(t_i, t_n)] := h(t_i, t_n),$$

$$\lim_{\Delta t_n \to 0} \frac{1}{\Delta t_n}[K(t_{n+1}, t_{n+1}) - K(t_n, t_{n+1})] := h(t_n).$$

We shall denote by $K_{ij}^{(n)}$ the cofactor of the corresponding element of the matrix $[k_{ij}]_{i,j=1}^n$, $K^{(n)}$ the determinant of that matrix, $h_{ij} = h(t_i, t_j)$, $h_n = h(t_n)$,

$$L^{(n+1)} = \begin{vmatrix} k_{11} & \dots & k_{1n} & h_{1n} \\ \dots & \dots & \dots & \dots \\ k_{n1} & \dots & k_{nn} & h_{nn} \\ k_{n+1,1} & \dots & k_{n+1,n} & 1 \end{vmatrix},$$

and $L_{ij}^{(n+1)}$ the cofactor of the corresponding element of $L^{(n+1)}$.

Following [7] we can show that

$$a_1(\mathbf{t}_n, \mathbf{x}_n) = m(t_n) + \frac{1}{K^{(n)}} \sum_{i=1}^n L_{n+1,i}^{(n+1)}(x_i - m(t_i)), \tag{2.6}$$

$$a_2(\mathbf{t}_n, \mathbf{x}_n) = \lim_{\Delta t_n \to 0} \frac{1}{\Delta t_n}\,(k_{n+1,n+1} - 2k_{n,n+1} + k_{nn}) = h_n - h_{nn}. \tag{2.7}$$

This completes the proof. ∎

Example 1. If $Y$ is the Wiener process then $\kappa(u, v) = \delta(u - v)$ is the Dirac delta function. The functions $h(t_i, t_n)$ and $h(t_n)$ for the process $X$ have the form

$$h(t_i, t_n) = e^{\int_0^{t_i} B_s\,ds + \int_0^{t_n} B_s\,ds} B_{t_n}\Big([E(X_0 - EX_0)]^2 + \int_0^{t_i} e^{-2\int_0^v B_s\,ds} C_v^2\,dv\Big),$$

$$h(t_n) = 1 + e^{2\int_0^{t_n} B_s\,ds} B_{t_n}\Big([E(X_0 - EX_0)]^2 + \int_0^{t_n} e^{-2\int_0^v B_s\,ds} C_v^2\,dv\Big).$$

Therefore $a_1(\mathbf{t}_n, \mathbf{x}_n)$ is of the form (2.6) and $a_2(\mathbf{t}_n, \mathbf{x}_n) = 1$.

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