ANNALES
POLONICI MATHEMATICI
83.1 (2004)

Natural affinors on the (r, s, q)-cotangent bundle of a fibered manifold

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Abstract. For natural numbers r, s, q, m, n with $s \geq r \leq q$ we describe all natural affinors on the (r, s, q)-cotangent bundle $T^{r,s,q*}Y$ over an (m, n)-dimensional fibered manifold Y.

Introduction. Let r, s, q, m, n be natural numbers with $s \geq r \leq q$.

Let $T^{r,s,q*} : \mathcal{FM}_{m,n} \to \mathcal{VB}$ be the (r, s, q)-cotangent bundle functor from the category $\mathcal{FM}_{m,n}$ of (m, n)-dimensional fibered manifolds (m is the base dimension and n is the fiber dimension) and their fibered local diffeomorphisms into the category of vector bundles and their homomorphisms (see [4], [8]).

An $\mathcal{FM}_{m,n}$-natural affinor on $T^{r,s,q*}$ is an $\mathcal{FM}_{m,n}$-invariant system of affinors $A : TT^{r,s,q*}Y \to TT^{r,s,q*}Y$ on $T^{r,s,q*}Y$ for any $\mathcal{FM}_{m,n}$-object Y.

In the present note we describe completely all $\mathcal{FM}_{m,n}$-natural affinors on $T^{r,s,q*}$. In the proof we will use the classification of all natural affinors on the r-cotangent bundle functor T^{r*} over n-manifolds by the first author [6].

At the end of the paper we record a similar result for $T^{r,s*}$ in place of $T^{r,s,q*}$.

Natural affinors can be used to study torsions of connections (see [5]). That is why they have been classified in many papers ([1]–[4], [6]–[8], etc).

The standard (m, n)-fibered manifold $\mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}^m$ will be denoted by $\mathbb{R}^{m,n}$. The usual coordinates on $\mathbb{R}^{m,n}$ will be denoted by $x^1, \ldots, x^m, y^1, \ldots,$ $\ldots, y^n$.

All manifolds and maps are assumed to be smooth, i.e. of class $\mathcal{C}^\infty$.

1. The (r, s, q)-cotangent bundle $T^{r,s,q*}$. Let r, s, q, m, n be natural numbers with $s \geq r \leq q$.

2000 *Mathematics Subject Classification*: 58A20, 53A55.
Key words and phrases: natural bundles, bundle functors, natural affinors.

The concept of r-jets can be generalized as follows (see [3]). Let $Y \to M$ and $Z \to N$ be fibered manifolds. We recall that two fibered maps $f, g : Y \to Z$ with base maps $\underline{f}, \underline{g} : M \to N$ determine the same (r,s,q)-jet $j^{r,s,q}_y f = j^{r,s,q}_y g$ at $y \in Y_x$, $x \in M$, if $j^r_y f = j^r_y g$, $j^s_y(f|Y_x) = j^s_y(g|Y_x)$ and $j^q_x \underline{f} = j^q_x \underline{g}$. The space of all (r,s,q)-jets of Y into Z is denoted by $J^{r,s,q}(Y,Z)$. The composition of fibered maps induces the composition of (r,s,q)-jets [3, p. 126].

The vector r-cotangent bundle functor $T^{r*} = J^r(\cdot, \mathbb{R})_0 : \mathcal{M}f_m \to \mathcal{VB}$ can be generalized as follows (see [4], [7]). Let $\mathbb{R}^{1,1} = \mathbb{R} \times \mathbb{R}$ be the trivial bundle over $\mathbb{R}$. The space $T^{r,s,q*} = J^{r,s,q}(Y, \mathbb{R}^{1,1})_0$, $0 \in \mathbb{R}^2$, has an induced structure of a vector bundle over Y. Every $\mathcal{FM}_{m,n}$-map $f : Y \to Z$ induces a vector bundle map $T^{r,s,q*}f : T^{r,s,q*}Y \to T^{r,s,q*}Z$ covering f, $T^{r,s,q*}f(j^{r,s,q}_y\gamma) = j^{r,s,q}_{f(y)}(\gamma \circ f^{-1})$ for $\gamma : Y \to \mathbb{R}^{1,1}$ with $\gamma(y) = 0$. The correspondence $T^{r,s,q*} : \mathcal{FM}_{m,n} \to \mathcal{VB}$ is a vector bundle functor in the sense of [3]. We call it the (r,s,q)-*cotangent bundle functor*.

2. Natural affinors on $T^{r,s,q*}$. An $\mathcal{FM}_{m,n}$-*natural affinor* on $T^{r,s,q*}$ is an $\mathcal{FM}_{m,n}$-invariant system of affinors

$$A : TT^{r,s,q*}Y \to TT^{r,s,q*}Y$$

on $T^{r,s,q*}Y$ for any $\mathcal{FM}_{m,n}$-object Y. The invariance means that for any $\mathcal{FM}_{m,n}$-morphism $f : Y \to Z$ we have $TT^{r,s,q*}f \circ A = A \circ TT^{r,s,q*}f$.

We present some examples of natural affinors on $T^{r,s,q*}$.

Example 1. There is the identity affinor Id on $T^{r,s,q*}Y$ for any fibered manifold Y from $\mathcal{FM}_{m,n}$.

To present next examples we need some observations.

(a) There are two canonical 1-forms θ^τ, $\tau = 1, 2$, on $T^{r,s,q*}Y$ given by

$$\theta^\tau_{j^{r,s,q}_y\gamma} = d_y(\gamma^V_\tau),$$

where $y \in Y$, $\gamma = (\gamma_1, \gamma_2) : Y \to \mathbb{R}^{1,1}$ is a fibered map with $\gamma(y) = 0$, and $f^V = f \circ \pi : T^{r,s,q*}Y \to \mathbb{R}$ is the vertical lifting of $f : Y \to \mathbb{R}$ to $T^{r,s,q*}Y$.

(b) For $a = 1, \ldots, q$ there is a canonical vertical vector field $L^{[a]}$ on $T^{r,s,q*}Y$ given by

$$L^{[a]} j^{r,s,q}_y\gamma = (j^{r,s,q}_y\gamma, j^{r,s,q}_y(\gamma^a_1, 0)) \in \{j^{r,s,q}_y\gamma\} \times T^{r,s,q*}_y Y \cong V_{j^{r,s,q}_y\gamma} T^{r,s,q*}Y,$$

where $y \in Y$ and $\gamma = (\gamma_1, \gamma_2) : Y \to \mathbb{R}^{1,1}$ is a fibered map with $\gamma(y) = 0$.

(c) For non-negative integers b and c with $1 \le b + c \le r$ and $b \ne 0$ there is a canonical vertical vector field $L^{[b,c]}$ on $T^{r,s,q*}Y$ given by

$$\begin{aligned} L^{[b,c]} j^{r,s,q}_y\gamma &= (j^{r,s,q}_y\gamma, j^{r,s,q}_y(0, \gamma^b_1\gamma^c_2)) \\ &\in \{j^{r,s,q}_y\gamma\} \times T^{r,s,q*}_y Y \cong V_{j^{r,s,q}_y\gamma} T^{r,s,q*}Y, \end{aligned}$$

where $y \in Y$, $\gamma = (\gamma_1, \gamma_2) : Y \to \mathbb{R}^{1,1}$ is a fibered map with $\gamma(y) = 0$, and γ_1^a is the ath power of γ_1.

(d) For $e = 1, \dots, s$ there is a canonical vertical vector field $L^{\langle e\rangle}$ on $T^{r,s,q*}Y$ given by

$$L^{\langle e\rangle} j_y^{r,s,q}\gamma = (j_y^{r,s,q}\gamma, j_y^{r,s,q}(0, \gamma_2^e)) \in \{j_y^{r,s,q}\gamma\} \times T_y^{r,s,q*}Y \cong V_{j_y^{r,s,q}\gamma}T^{r,s,q*}Y,$$

where $y \in Y$ and $\gamma = (\gamma_1, \gamma_2) : Y \to \mathbb{R}^{1,1}$ is a fibered map with $\gamma(y) = 0$.

(e) If θ is the canonical 1-form on $T^{r,s,q*}Y$ and L is the canonical vector field on $T^{r,s,q*}Y$, then $\theta \otimes L$ is the canonical affinor on $T^{r,s,q*}Y$.

EXAMPLE 2. For $a = 1, \dots, q$ and $\tau = 1, 2$ there is an $\mathcal{FM}_{m,n}$-natural affinor

$$A^{[a,\tau]} = \theta^\tau \otimes L^{[a]}$$

on $T^{r,s,q*}Y$ for any fibered manifold Y in $\mathcal{FM}_{m,n}$.

EXAMPLE 3. For non-negative integers b and c with $1 \le b + c \le r$ and $b \ne 0$ and $\tau = 1, 2$ there is an $\mathcal{FM}_{m,n}$-natural affinor

$$A^{[b,c,\tau]} = \theta^\tau \otimes L^{[b,c]}$$

on $T^{r,s,q*}Y$ for any fibered manifold Y in $\mathcal{FM}_{m,n}$.

EXAMPLE 4. For $e = 1, \dots, s$ and $\tau = 1, 2$ there is an $\mathcal{FM}_{m,n}$-natural affinor

$$A^{\langle e,\tau\rangle} = \theta^\tau \otimes L^{\langle e\rangle}$$

on $T^{r,s,q*}Y$ for any fibered manifold Y in $\mathcal{FM}_{m,n}$.

The main result of the present paper is the following classification theorem.

THEOREM 1. *Let m, n, r, s, q be integers such that $m \ge 2$, $n \ge 2$, $r \ge 1$ and $s \ge r \le q$. The vector space of all $\mathcal{FM}_{m,n}$-natural affinors on $T^{r,s,q*}$ is $(r^2 + r + 2s + 2q + 1)$-dimensional. The natural affinors from Examples 1–4 form an $\mathbb{R}$-basis of this vector space.*

The proof of Theorem 1 will occupy the rest of the paper.

From now on we consider an $\mathcal{FM}_{m,n}$-natural affinor A on $T^{r,s,q*}$ and assume that m, n, s, r, q are as in Theorem 1.

Since the natural affinors from Examples 1–4 are linearly independent, it is sufficient to prove that A is their linear combination.

3. A decomposition lemma

LEMMA 1. *There is a real number α such that $A - \alpha\,\mathrm{Id}$ is of vertical type, i.e. $\mathrm{im}(A - \alpha\,\mathrm{Id}) \subset VT^{r,s,q*}Y$ for any Y in $\mathcal{FM}_{m,n}$.*

Proof. Let $\pi : T^{r,s,q*}Y \to Y$ be the bundle projection. We define α by

$$\alpha = d_0 y^1\Big(T\pi \circ A\Big(\Big(\frac{\partial}{\partial x^1}\Big)^C_{j_0^{r,s,q}(0)}\Big)\Big),$$

where X^C is the complete lifting of a projectable vector field on Y to $T^{r,s,q*}Y$ and $x^1, \ldots, x^m, y^1, \ldots, y^n$ are the usual coordinates on $\mathbb{R}^{m,n}$.

Using the invariance of $T\pi \circ (A - \alpha\,\mathrm{Id})_{|VT^{r,s,q}\mathbb{R}^{m,n}}$ with respect to the homotheties $t^{-1}\mathrm{id}_{\mathbb{R}^{m,n}}$ for $t > 0$ and next letting $t \to 0$ we deduce that

$$T\pi \circ (A - \alpha\,\mathrm{Id})_{|VT^{r,s,q}\mathbb{R}^{m,n}} = 0.$$

It remains to prove that $T\pi \circ (A - \alpha\,\mathrm{Id})(X^C_u) = 0$ for any constant vector field X on $\mathbb{R}^{m,n}$ and any $u = j_0^{r,s,q}\gamma \in T_0^{r,s,q*}\mathbb{R}^{m,n}$.

Because of the invariance of A we can assume that $X = \partial/\partial x^1$. Write

$$T\pi \circ (A - \alpha\,\mathrm{Id})\Big(\Big(\frac{\partial}{\partial x^1}\Big)^C_u\Big) = \sum_{i=1}^m \alpha_i \frac{\partial}{\partial x^i}_{\,0} + \sum_{j=1}^n \beta_j \frac{\partial}{\partial y^j}_{\,0}.$$

Using the invariance of $A - \alpha\,\mathrm{Id}$ with respect to $(x^1, t^{-1}x^2, \ldots, t^{-1}x^m, t^{-1}y^1, \ldots, t^{-1}y^n)$ for $t > 0$ and then letting $t \to 0$ we deduce that $\alpha_2 = \ldots = \alpha_m = \beta_1 = \ldots = \beta_n = 0$. Then using the invariance of $A - \alpha\,\mathrm{Id}$ with respect to $t^{-1}\mathrm{id}_{\mathbb{R}^{m,n}}$ for $t > 0$ and then letting $t \to 0$ and using the definition of α we deduce that

$$T\pi \circ (A - \alpha\,\mathrm{Id})\Big(\Big(\frac{\partial}{\partial x^1}\Big)^C_u\Big) = T\pi \circ (A - \alpha\,\mathrm{Id})\Big(\Big(\frac{\partial}{\partial x^1}\Big)^C_0\Big) = 0. \ \blacksquare$$

Because of Lemma 1, replacing A by $A - \alpha\,\mathrm{Id}$ we can and do assume that A is of vertical type.

4. A reducibility lemma

LEMMA 2. *Assume that*

$$A\Big(\Big(\frac{\partial}{\partial x^1}\Big)^C_{j_0^{r,s,q}(x^1,y^1)}\Big) = A\Big(\Big(\frac{\partial}{\partial y^1}\Big)^C_{j_0^{r,s,q}(x^1,y^1)}\Big) = 0$$

and

$$A\Big(\frac{d}{dt}_{0}\Big(j_0^{r,s,q}(x^1,y^1) + tj_0^{r,s,q}(x^2,0)\Big)\Big) = A\Big(\frac{d}{dt}_{0}(j_0^{r,s,q}(x^1,y^1) + tj_0^{r,s,q}(0,y^2))\Big) = 0.$$

Then $A = 0$.

Proof. It is sufficient to show that $A(w) = 0$ for any $w \in T_u^{r,s,q*}Y$.

By the fibered version of the rank theorem, $j_0^{r,s,q}(x^1,y^1)\in T_0^{r,s,q*}\mathbb{R}^{m,n}$ has dense orbit in $T^{r,s,q*}Y$ with respect to $\mathcal{FM}_{m,n}$-maps. Therefore we can assume $u=j_0^{r,s,q}(x^1,y^1)$.

Because of the fiber linearity of A we can assume that either $w=X_u^C$ for $u=j_0^{r,s,q}(x^1,y^1)$ and a constant vector field X on $\mathbb{R}^{m,n}$, or $w=\frac{d}{dt}_0(j_0^{r,s,q}(x^1,y^1)+tj_0^{r,s,q}\gamma)$ for a fibered map $\gamma=(\gamma_1,\gamma_2):\mathbb{R}^{m,n}\to\mathbb{R}^{1,1}$ with $\gamma(0)=0$.

In the first case because of the invariance of A with respect to linear $\mathcal{FM}_{m,n}$-maps we can assume that $X=\partial/\partial x^1$ or $X=\partial/\partial y^1$. In the second case by the fibered version of the rank theorem we can assume that $\gamma=(x^2,y^2)$. Then by the fiber linearity of A we can assume that $\gamma=(x^2,0)$ or $\gamma=(0,y^2)$.

Now, applying the assumptions of the lemma completes the proof. ■

Lemma 2 means that A is determined by the four vectors

$$A\Big(\Big(\frac{\partial}{\partial x^1}\Big)^C_{j_0^{r,s,q}(x^1,y^1)}\Big),\quad A\Big(\Big(\frac{\partial}{\partial y^1}\Big)^C_{j_0^{r,s,q}(x^1,y^1)}\Big),$$
$$A\Big(\frac{d}{dt}_0(j_0^{r,s,q}(x^1,y^1)+tj_0^{r,s,q}(x^2,0))\Big),$$
$$A\Big(\frac{d}{dt}_0(j_0^{r,s,q}(x^1,y^1)+tj_0^{r,s,q}(0,y^2))\Big).$$

We study these vectors in the next sections.

5. An inessential assumption. We can write

$$A\Big(\Big(\frac{\partial}{\partial x^1}\Big)^C_{j_0^{r,s,q}(x^1,y^1)}\Big)=\frac{d}{dt}_0(j_0^{r,s,q}(x^1,y^1)+tj_0^{r,s,q}\gamma)$$

for a fibered map $\gamma=(\gamma_1,\gamma_2):\mathbb{R}^{m,n}\to\mathbb{R}^{1,1}$ with $\gamma(0)=0$. Using the invariance of A with respect to $(x^1,tx^2,\dots,tx^m,y^1,ty^2,\dots,ty^n)$ for $t>0$ and then letting $t\to 0$ we get

$$\begin{aligned}A\Big(\Big(\frac{\partial}{\partial x^1}\Big)^C_{j_0^{r,s,q}(x^1,y^1)}\Big)&=\sum_{a=1}^{q}\alpha_a^1\frac{d}{dt}_0(j_0^{r,s,q}(x^1,y^1)+tj_0^{r,s,q}((x^1)^a,0))\\&\quad+\sum_{1\le b+c\le r,\,b\neq 0}\beta^1_{b,c}\frac{d}{dt}_0(j_0^{r,s,q}(x^1,y^1)+tj_0^{r,s,q}(0,(x^1)^b(y^1)^c))\\&\quad+\sum_{e=1}^{s}\delta_e^1\frac{d}{dt}_0(j_0^{r,s,q}(x^1,y^1)+tj_0^{r,s,q}(0,(y^1)^c))\end{aligned}$$

for some real numbers α_a^1, $\beta^1_{b,c}$ and δ^1_e.

Similarly, we get

$$A\bigg(\Big(\frac{\partial}{\partial y^1}\Big)^C_{j_0^{r,s,q}(x^1,y^1)}\bigg) = \sum_{a=1}^{q}\alpha_a^2 \frac{d}{dt}_0 (j_0^{r,s,q}(x^1,y^1) + tj_0^{r,s,q}((x^1)^a,0))$$
$$+ \sum_{1\le b+c\le r,\, b\neq 0} \beta^2_{b,c} \frac{d}{dt}_0 (j_0^{r,s,q}(x^1,y^1) + tj_0^{r,s,q}(0,(x^1)^b(y^1)^c))$$
$$+ \sum_{e=1}^{s}\delta_e^2 \frac{d}{dt}_0 (j_0^{r,s,q}(x^1,y^1) + tj_0^{r,s,q}(0,(y^1)^c))$$

for some real numbers α_a^2, $\beta_{b,c}^2$ and δ_e^2.

So, replacing A by

$$A - \sum_{\tau=1,2}\sum_{a=1}^{q}\alpha_a^\tau A^{[a,\tau]} - \sum_{\tau=1,2}\sum_{1\le b+c\le r,\, b\neq 0}\beta^\tau_{b,c}A^{[b,c,\tau]} - \sum_{\tau=1,2}\sum_{e=1}^{s}\delta_e^\tau A^{\langle e,\tau\rangle}$$

we can assume that

$$(*)\qquad A\bigg(\Big(\frac{\partial}{\partial x^1}\Big)^C_{j_0^{r,s,q}(x^1,y^1)}\bigg) = A\bigg(\Big(\frac{\partial}{\partial y^1}\Big)^C_{j_0^{r,s,q}(x^1,y^1)}\bigg) = 0.$$

6. Proof of Theorem 1. Because of Lemma 2 and the assumption $(*)$ of Section 5 it is sufficient to verify that

$$A\Big(\frac{d}{dt}_0 (j_0^{r,s,q}(x^1,y^1) + tj_0^{r,s,q}(x^2,0))\Big)$$
$$= A\bigg(\frac{d}{dt}_0 \Big(j_0^{r,s,q}(x^1,y^1) + tj_0^{r,s,q}(0,y^2)\Big)\bigg) = 0.$$

We prove the first equality only. The proof of the second one is similar. Set

$$A\Big(\frac{d}{dt}_0 (j_0^{r,s,q}(x^1,y^1) + tj_0^{r,s,q}(x^2,0))\Big) = \frac{d}{dt}_0 (j_0^{r,s,q}(x^1,y^1) + tj_0^{r,s,q}(\gamma^1,\gamma^2)).$$

We have to show that $j_0^q(\gamma^1) = 0$, $j_0^s(\gamma^2(0,\cdot) = 0$ and $j_0^r(\gamma_2) = 0$.

We prove $j_0^r(\gamma_2) = 0$ only. The proof of the first two equalities is similar.

Given an n-manifold N we have the inclusion $T^{s*}N \subset T^{r,s,q*}(\mathbb{R}^m \times N)$ given by $j_z^s\eta \mapsto j_{(0,z)}^{r,s,q}(x^1,\eta)$ for $\eta : N \to \mathbb{R}$, $z \in N$, $\eta(z) = 0$, where we identify η with $\eta \circ \mathrm{pr}_N$, $\mathrm{pr}_N : \mathbb{R}^m \times N \to N$ being the projection. Then for any $j_z^s\eta \in T^{s*}N$ we have the induced inclusion $T_{j_z^r\eta}T^{s*}N \subset T_{j_{(0,z)}^{r,s,q}(x^1,\eta)}T^{r,s,q*}(\mathbb{R}^m \times N)$.

For m-tuples α with $|\alpha| \le r$ define an $\mathcal{M}f_n$-natural affinor $B_\alpha : TT^{s*}N \to TT^{s*}N$ on $T^{s*}N$ as follows.

Let $w \in T_{j_z^s\eta}T^{s*}N$, where $\eta : N \to \mathbb{R}$, $z \in N$, $\eta(z) = 0$. Then $w \in T_{j_{(0,z)}^{r,s,q}(x^1,\eta)}T^{r,s,q*}(\mathbb{R}^m \times N)$, and we can apply A to w. We have the elements $j_z^{r-|\alpha|}(\eta_\alpha^w) \in J_z^{r-|\alpha|}(N,\mathbb{R})$ (with $\eta_\alpha^w : N \to \mathbb{R}$, $\eta_{(0)}^w(z) = 0$) linearly depending on w by

$$A(w) = \frac{d}{dt}_0 \Big(j_{(0,z)}^{r,s,q}(x^1,\eta) + t j_{(0,z)}^{r,s,q}\Big(\varrho^w, \sum_\alpha x^\alpha \eta_\alpha^w\Big)\Big).$$

We put

$$B_\alpha(w) = \frac{d}{dt}_0 (j_z^s \eta + t j_z^s(\eta^{s-r+|\alpha|}\eta_\alpha^w)) \in T_{j_z^s\eta} T^{s*}N.$$

From $(*)$ we deduce that $B_\alpha((\partial/\partial y^1)^C_{j_0^r(y^1)}) = 0$. Then by the classification result from [6] we obtain $B_\alpha = 0$ for all α as above. Then (in particular) $j_0^r\gamma^2 = 0$. ■

7. The (r,s)-cotangent bundle $T^{r,s*}$. Let r,s,m,n be natural numbers with $s \geq r$. The concept of r-jets can be generalized as follows (see [3]). Let $Y \to M$ be a fibered manifold and N be a manifold. We recall that two maps $f,g : Y \to N$ determine the same (r,s)-jet $j_y^{r,s}f = j_y^{r,s}g$ at $y \in Y_x$, $x \in M$, if $j_y^r f = j_y^r g$ and $j_y^s(f|Y_x) = j_y^s(g|Y_x)$. The space of all (r,s)-jets of Y into N is denoted by $J^{r,s}(Y,N)$.

The vector r-cotangent bundle functor $T^{r*} = J^r(\cdot,\mathbb{R})_0 : \mathcal{M}f_m \to \mathcal{VB}$ can be generalized as follows (see [4], [7]). The space $T^{r,s*} = J^{r,s}(Y,\mathbb{R})_0$, $0 \in \mathbb{R}$, has an induced structure of a vector bundle over Y. Every $\mathcal{FM}_{m,n}$-map $f : Y \to Z$ induces a vector bundle map $T^{r,s*}f : T^{r,s*}Y \to T^{r,s*}Z$ covering f, $T^{r,s*}f(j_y^{r,s}\gamma) = j_{f(y)}^{r,s}(\gamma \circ f^{-1})$ for $\gamma : Y \to \mathbb{R}$ with $\gamma(y) = 0$. The correspondence $T^{r,s*} : \mathcal{FM}_{m,n} \to \mathcal{VB}$ is a vector bundle functor in the sense of [3]. We call it the (r,s)-*cotangent bundle functor.*

8. Natural affinors on $T^{r,s,q*}$. We present some examples of natural affinors on $T^{r,s*}$.

EXAMPLE 5. There is the identity affinor Id on $T^{r,s*}Y$ for any fibered manifold Y from $\mathcal{FM}_{m,n}$.

To present next examples we need some observations.

(a) There is a canonical 1-form θ on $T^{r,s*}Y$ given by

$$\theta_{j_y^{r,s}\gamma} = d_y(\gamma^V),$$

where $y \in Y$, $\gamma : Y \to \mathbb{R}$ is a fibered map with $\gamma(y) = 0$, and $f^V = f \circ \pi : T^{r,s*}Y \to \mathbb{R}$ is the vertical lifting of $f : Y \to \mathbb{R}$ to $T^{r,s*}Y$.

(b) For $e = 1,\dots,s$ there is a canonical vertical vector field $L^{(e)}$ on $T^{r,s*}Y$ given by

$$L^{(e)} j_y^{r,s}\gamma = (j_y^{r,s}\gamma, j_y^{r,s}(\gamma^e)) \in \{j_y^{r,s}\gamma\} \times T_y^{r,s*}Y \cong V_{j_y^{r,s}\gamma} T^{r,s*}Y,$$

where $y \in Y$ and $\gamma : Y \to \mathbb{R}$ is a fibered map with $\gamma(y) = 0$.

(c) If θ is the canonical 1-form on $T^{r,s*}Y$ and L is the canonical vector field on $T^{r,s*}Y$, then $\theta \otimes L$ is the canonical affinor on $T^{r,s*}Y$.

Example 6. For $e = 1, \ldots, s$ there is an $\mathcal{FM}_{m,n}$-natural affinor

$$A^{(e)} = \theta \otimes L^{(e)}$$

on $T^{r,s*}Y$ for any fibered manifold Y in $\mathcal{FM}_{m,n}$.

The second main result in the present paper is the following classification theorem.

Theorem 2. *Let m, n, r, s be integers such that $m \geq 2$, $n \geq 2$, $r \geq 1$ and $s \geq r$. The vector space of all $\mathcal{FM}_{m,n}$-natural affinors on $T^{r,s*}$ is $(s+1)$-dimensional. The natural affinors from Examples 5 and 6 form an $\mathbb{R}$-basis of this vector space.*

The proof of Theorem 2 is quite similar to the one of Theorem 1.

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Reçu par la Rédaction le 9.5.2003 (1443)