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Bifurcation theorems for nonlinear problems with lack of compactness

by Francesca Faraci (Catania) and Roberto Livrea (Messina)

Abstract. We deal with a bifurcation result for the Dirichlet problem

$$\begin{cases} -\Delta_p u = \dfrac{\mu}{|x|^p}\,|u|^{p-2}u + \lambda f(x,u) & \text{a.e. in } \Omega, \\ u_{|\partial\Omega} = 0. \end{cases}$$

Starting from a weak lower semicontinuity result by E. Montefusco, which allows us to apply a general variational principle by B. Ricceri, we prove that, for μ close to zero, there exists a positive number λ_μ^* such that for every $\lambda \in]0, \lambda_\mu^*[$ the above problem admits a nonzero weak solution u_λ in $W_0^{1,p}(\Omega)$ satisfying $\lim_{\lambda\to 0^+} \|u_\lambda\| = 0$.

1. Introduction. In the present paper we are interested in the existence of solutions for the Dirichlet problem

$$(P_{\lambda,\mu}) \qquad \begin{cases} -\Delta_p u = \dfrac{\mu}{|x|^p}\,|u|^{p-2}u + \lambda f(x,u) & \text{a.e. in } \Omega, \\ u_{|\partial\Omega} = 0, \end{cases}$$

where Ω is a bounded open subset of $\mathbb{R}^n$ $(n \geq 2)$ containing the origin, $1 < p < N$, $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is a Carathéodory function with $f(x,0) = 0$, and λ, μ are two parameters respectively positive and nonnegative.

The presence of the term $\mu/|x|^p$ does not allow us to apply the classical variational approach. The Hardy inequality ensures that $W_0^{1,p}(\Omega)$ is continuously, but not compactly embedded in $L^p(\Omega)$ with respect to the weight $|x|^{-p}$. Because of this lack of compactness we are not able to obtain the weak lower semicontinuity of the energy functional via the classical De Giorgi theorem.

The problem

$$(P_\mu) \qquad \begin{cases} -\Delta_p u = \dfrac{\mu}{|x|^p}\,|u|^{p-2}u + f(x) & \text{a.e. in } \Omega, \\ u_{|\partial\Omega} = 0, \end{cases}$$

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is studied in [3], where $f \in W^{-1,p'}(\Omega)$ and $\mu < H$, H being the best constant in the Hardy inequality. In particular, starting from the coercivity and homogeneity of the energy functional, the authors of [3] are able to prove the required compactness property by finding a minimizing sequence converging to a global minimum.

The authors of [1] are interested in minima of the nondifferentiable functional

$$J(u) = \int_\Omega j(x, \nabla u)\, dx - a \int_\Omega \frac{|u|^2}{|x|^2}\, dx - \int_\Omega f(x)u(x)\, dx,$$

where $f \in L^2(\Omega)$ and $j(x,\xi)$ is a convex function with respect to ξ, satisfying

$$\alpha|\xi|^2 \le j(x,\xi) \le \beta|\xi|^2$$

for every $\xi \in \mathbb{R}^n$, a.e. in Ω. With no information about the weak lower semicontinuity of the functional, they state the existence of a global minimum using a truncation approach.

We refer moreover to the recent papers [2] and [4] for a complete survey of the topic. The authors of [4] deal with the problem

$$(\widetilde{P}) \qquad \begin{cases} -\Delta_p u = \dfrac{\mu}{|x|^s}\, |u|^{q-2}u + \lambda|u|^{r-2}u & \text{a.e. in } \Omega, \\ u_{|\partial\Omega} = 0, \end{cases}$$

where $0 \le s \le p < n$, $q \le p^\star(s) = (p-s)/(n-p)p$. In particular in the case $s = q = p$, $p < r < p^\star$ they obtain infinitely many solutions, at least one of them being positive, for any $\lambda > 0$ and $0 < \mu < H$. We notice that the main assumption in that paper is a bound on r, which is incompatible with our hypothesis on the nonlinearity at zero (see condition (3)).

Here we propose a novel approach to the subject. In particular, combining a weak lower semicontinuity result by E. Montefusco [5] with a recent variational principle by B. Ricceri [6] we establish a bifurcation theorem for problem $(P_{\lambda,\mu})$ just assuming a suitable behaviour of $f(x,t)$ at zero. Moreover, we prove that if λ is sufficiently small, then the energy functional related to the problem is negative and decreasing on the solutions.

Finally, using the same technique, we obtain an analogous result for the problem

$$(P^*_{\lambda,\mu}) \quad \begin{cases} -\Delta_p u = \mu\Big(\displaystyle\int_\Omega |u|^{p^*}\Big)^{p/p^*-1} |u|^{p^*-2}u + \lambda f(x,u) & \text{a.e. in } \Omega, \\ u_{|\partial\Omega} = 0, \end{cases}$$

where n, p and f are as in problem $(P_{\lambda,\mu})$, Ω is a bounded open subset of $\mathbb{R}^n$ and $p^* = np/(n-p)$.

2. Preliminaries. Assume the following growth condition on f: there exist two positive constants a, q with

$$q < \frac{n(p-1)+p}{n-p}$$

and a nonnegative constant b such that

$$|f(x,\xi)| \le a|\xi|^q + b \tag{1}$$

for every $\xi \in \mathbb{R}$ and a.e. in Ω.

Denote by X the space $W_0^{1,p}(\Omega)$ endowed with the norm

$$\|u\| = \Big(\int_\Omega |\nabla u(x)|^p\,dx\Big)^{1/p}.$$

Let us recall the Hardy inequality

$$\int_\Omega \frac{|u(x)|^p}{|x|^p}\,dx \le \frac{1}{H}\int_\Omega |\nabla u(x)|^p\,dx, \tag{2}$$

where Ω is an open set in $\mathbb{R}^n$ containing the origin and H is the best constant in the inclusion of $W_0^{1,p}(\Omega)$ in $L^p(\Omega)$ with weight $|x|^{-p}$. In particular, when Ω is a ball, $H = ((n-p)/p)^p$ (see [3]).

For each $u \in X$ and $\mu \in \mathbb{R}$ put

$$\mathcal{H}_\mu(u) = \frac{1}{p}\int_\Omega |\nabla u(x)|^p\,dx - \frac{\mu}{p}\int_\Omega \frac{|u(x)|^p}{|x|^p}\,dx,$$

$$\Phi(u) = -\int_\Omega \Big(\int_0^{u(x)} f(x,\xi)\,d\xi\Big)\,dx.$$

In [5] it is shown that $\mathcal{H}_\mu$ is a well defined and continuously Gateaux differentiable functional in X. Moreover, if $\mu \in [0,H[$, then $\mathcal{H}_\mu$ is weakly lower semicontinuous and coercive.

Standard arguments show that Φ is a well defined and continuously Gateaux differentiable functional whose Gateaux derivative is a compact operator from X to X^*.

A weak solution of problem $(P_{\lambda,\mu})$ is any $u \in X$ such that

$$\int_\Omega |\nabla u(x)|^{p-2}\nabla u(x)\cdot\nabla v(x)\,dx - \mu\int_\Omega \frac{|u(x)|^{p-2}}{|x|^p}u(x)v(x)\,dx - \lambda\int_\Omega f(x,u(x))v(x)\,dx = 0 \quad \text{for all } v \in X.$$

For each $\lambda,\mu \in \mathbb{R}$, consider the functional $J_{\lambda,\mu} : X \to \mathbb{R}$ defined by

$$J_{\lambda,\mu}(u) = \mathcal{H}_\mu(u) + \lambda\Phi(u)$$

and observe that it is the energy functional related to $(P_{\lambda,\mu})$.

Finally, for the reader's convenience, we recall the main tool that we will use. It is due to B. Ricceri and it can be stated as follows:

THEOREM A ([6, Theorem 2.5]). *Let X be a reflexive real Banach space, and let $\Phi, \Psi : X \to \mathbb{R}$ be two sequentially weakly lower semicontinuous functionals. Assume also that Ψ is (strongly) continuous and $\lim_{\|x\|\to+\infty} \Psi(x) = +\infty$. For each $\varrho > \inf_X \Psi$, put*

$$\varphi(\varrho) = \inf_{x\in\Psi^{-1}(]-\infty,\varrho[)} \frac{\Phi(x) - \inf_{\mathrm{cl}_w \Psi^{-1}(]-\infty,\varrho[)} \Phi}{\varrho - \Psi(x)},$$

where cl_w is the closure in the weak topology. Then, for each $\varrho > \inf_X \Psi$ and each $\lambda > \varphi(\varrho)$, the restriction of the functional $\Phi + \lambda\Psi$ to $\Psi^{-1}(]-\infty, \varrho[)$ has a global minimum.

3. Main result

THEOREM 3.1. *Let $f : \Omega \times \mathbb{R} \to \mathbb{R}$ be a Carathéodory function with $f(x, 0) = 0$, satisfying condition (1). Assume that there are a nonempty open set $D \subseteq \Omega$ and a set $B \subseteq D$ of positive measure such that*

$$\limsup_{\xi\to0^+} \frac{\inf_{x\in B} \int_0^\xi f(x,t)\,dt}{|\xi|^p} = +\infty, \qquad (3)$$
$$\liminf_{\xi\to0^+} \frac{\inf_{x\in D} \int_0^\xi f(x,t)\,dt}{|\xi|^p} > -\infty.$$

Then for every $\mu \in [0, H[$ there exists a positive number λ^*_μ such that for every $\lambda \in]0, \lambda^*_\mu[$ problem $(P_{\lambda,\mu})$ admits a nonzero weak solution u_λ in $W^{1,p}_0(\Omega)$. Moreover,

$$\lim_{\lambda\to0^+} \|u_\lambda\| = 0$$

and the function $\lambda \mapsto J_{\lambda,\mu}(u_\lambda)$ is negative and decreasing in $]0, \lambda^*_\mu[$.

Proof. Fix $\mu \in [0, H[$. We want to apply Theorem A, where $X = W^{1,p}_0(\Omega)$, $\Psi = \mathcal{H}_\mu$ and Φ is the functional introduced in Section 2.

Since $\mu \in [0, H[$, we have already observed in Section 2 that Φ and Ψ are two sequentially weakly lower semicontinuous and continuously Gateaux differentiable functionals. Moreover, Ψ is coercive and clearly $\inf_{u\in X} \Psi(u) = 0$.

Let $\overline{\varrho} > 0$ be such that $\varphi(\overline{\varrho}) > 0$ and put $\lambda^*_\mu = 1/\varphi(\overline{\varrho})$. Thanks to Theorem A, for every $\lambda \in]0, \lambda^*_\mu[$ there exists $u_\lambda \in \Psi^{-1}(]-\infty, \overline{\varrho}[)$ such that

$$\Phi'(u_\lambda) + \frac{1}{\lambda}\Psi'(u_\lambda) = 0 \qquad (4)$$

and, in particular, u_λ is a global minimum of the restriction of $\Phi + \frac{1}{\lambda}\Psi$ to $\Psi^{-1}(]-\infty, \overline{\varrho}[)$.

Fix $\lambda \in]0, \lambda^*_\mu[$; we will prove that $u_\lambda \neq 0$. To this end, let us prove that

(5) $$\liminf_{\|u\|\to 0^+} \frac{\Phi(u)}{\Psi(u)} = -\infty.$$

Thanks to (3) we can fix a sequence $\{\xi_k\}$ in $\mathbb{R}^+$ converging to zero and two constants δ and Γ with $\delta > 0$ such that

$$\lim_{k\to+\infty} \frac{\inf_{x\in B} \int_0^{\xi_k} f(x,t)\,dt}{|\xi_k|^p} = +\infty$$

and

$$\inf_{x\in D} \int_0^{\xi} f(x,t)\,dt \geq \Gamma|\xi|^p$$

for all $\xi \in [0,\delta]$. Next, fix a set $C \subset B$ of positive measure and a function $v \in X$ such that $v(x) \in [0,1]$ for all $x \in \Omega$, $v(x) = 1$ for all $x \in C$, and $v(x) = 0$ for all $x \in \Omega \setminus D$. Let $Q > 0$, put

$$|||v|||^p = \int_\Omega \frac{|v(x)|^p}{|x|^p}\,dx$$

and consider a positive number T with

$$Q < \frac{T\,\mathrm{meas}(C) + \Gamma \int_{D\setminus C} |v(x)|^p\,dx}{\frac{1}{p}\|v\|^p - \frac{\mu}{p}|||v|||^p}.$$

Then there is $\nu \in \mathbb{N}$ such that $\xi_k < \delta$ and

$$\inf_{x\in B} \int_0^{\xi_k} f(x,t)\,dt \geq T|\xi_k|^p$$

for all $k > \nu$. Now, for each $k > \nu$, one has

(6) $$-\frac{\Phi(\xi_k v)}{\Psi(\xi_k v)} = \frac{\int_C (\int_0^{\xi_k} f(x,t)\,dt)\,dx + \int_{D\setminus C}(\int_0^{\xi_k v(x)} f(x,t)\,dt)\,dx}{\frac{1}{p}\|\xi_k v\|^p - \frac{\mu}{p}|||\xi_k v|||^p}$$
$$\geq \frac{T\,\mathrm{meas}(C) + \Gamma \int_{D\setminus C} |v(x)|^p\,dx}{\frac{1}{p}\|v\|^p - \frac{\mu}{p}|||v|||^p} > Q.$$

From (6), clearly (5) follows. Hence, there is a sequence $\{w_k\}$ in X converging to zero such that for k large enough we have $w_k \in \Psi^{-1}(]-\infty, \overline{\varrho}[)$, and

$$\Phi(w_k) + \frac{1}{\lambda}\Psi(w_k) < 0.$$

Since u_λ is a global minimum of the restriction of $\Phi + \frac{1}{\lambda}\Psi$ to $\Psi^{-1}(]-\infty, \overline{\varrho}[)$, we can conclude that

$$\Phi(u_\lambda) + \frac{1}{\lambda}\Psi(u_\lambda) < 0 = \Phi(0) + \frac{1}{\lambda}\Psi(0) \tag{7}$$

so that $u_\lambda \neq 0$.

Observing that the weak solutions of problem $(P_{\lambda,\mu})$ are exactly the critical points of the functional $J_{\lambda,\mu}$ and that

$$J_{\lambda,\mu} = \lambda\left(\Phi + \frac{1}{\lambda}\Psi\right), \tag{8}$$

one finds that the first part of our theorem is completely proved.

Since Ψ is coercive and $u_\lambda \in \Psi^{-1}(]-\infty, \overline{\varrho}[)$ for every $\lambda \in]0, \lambda^*_\mu[$, there exists a positive number L such that

$$\|u_\lambda\| \leq L$$

for every $\lambda \in]0, \lambda^*_\mu[$. Therefore, since Φ' is a compact operator, there exists a positive number M such that

$$\left|\int_\Omega f(x, u_\lambda(x))u_\lambda(x)\,dx\right| \leq \|\Phi'(u_\lambda)\|_{X^*}\|u_\lambda\| \leq M \cdot L^2$$

for every $\lambda \in]0, \lambda^*_\mu[$.

By (4), $J'_\lambda(u_\lambda) = 0$ for every $\lambda \in]0, \lambda^*_\mu[$ and in particular $(J'_\lambda(u_\lambda))(u_\lambda) = 0$, that is,

$$p \cdot \Psi(u_\lambda) = \|u_\lambda\|^p - \mu|||u_\lambda|||^p = \lambda\int_\Omega f(x, u_\lambda(x))u_\lambda(x)\,dx \tag{9}$$

for every $\lambda \in]0, \lambda^*_\mu[$. Hence

$$\lim_{\lambda\to 0^+}\Psi(u_\lambda) = 0.$$

Finally, putting together (2) and (9) yields

$$\frac{1}{p}\|u_\lambda\|^p \leq \Psi(u_\lambda) + \frac{\mu}{p \cdot H}\|u_\lambda\|^p$$

for every $\lambda \in]0, \lambda^*_\mu[$, hence

$$\|u_\lambda\|^p \leq \frac{p \cdot H}{H - \mu}\Psi(u_\lambda)$$

for every $\lambda \in]0, \lambda^*_\mu[$ and

$$\lim_{\lambda\to 0^+}\|u_\lambda\| = 0.$$

From (7) and (8) it follows that the function $\lambda \mapsto J_{\lambda,\mu}(u_\lambda)$ is negative in $]0, \lambda^*_\mu[$. Finally, if we fix $\lambda_1, \lambda_2 \in]0, \lambda^*_\mu[$ with $\lambda_1 < \lambda_2$ and put

$$m_{\lambda_1} = \Phi(u_{\lambda_1}) + \frac{1}{\lambda_1}\Psi(u_{\lambda_1}) = \inf_{v\in\Psi^{-1}(]-\infty,\overline{\varrho}[)}\left(\Phi(v) + \frac{1}{\lambda_1}\Psi(v)\right),$$

$$m_{\lambda_2} = \Phi(u_{\lambda_2}) + \frac{1}{\lambda_2}\Psi(u_{\lambda_2}) = \inf_{v\in\Psi^{-1}(]-\infty,\overline{\varrho}[)}\left(\Phi(v) + \frac{1}{\lambda_2}\Psi(v)\right),$$

then

$$J_{\lambda_1,\mu}(u_\lambda) = \lambda_1 m_{\lambda_1} > \lambda_2 m_{\lambda_1} \geq \lambda_2 m_{\lambda_2} = J_{\lambda_2,\mu}(u_{\lambda_2}).$$

Hence, $\lambda \mapsto J_{\lambda,\mu}(u_\lambda)$ is decreasing in $]0, \lambda^*_\mu[$ and the proof is complete. ∎

REMARK 1. Observe that Theorem 3.1 is a bifurcation result. In fact, since $f(x,0) = 0$ it follows that 0 is a solution of $(P_{\lambda,\mu})$ for every λ, μ. Hence, $\lambda = 0$ is a bifurcation point for problem $(P_{\lambda,\mu})$, in the sense that $(0,0)$ belongs to the closure in $W^{1,p}_0(\Omega) \times \mathbb{R}$ of the set

$$\{(u,\lambda) \in W^{1,p}_0(\Omega) \times]0,+\infty[: u \text{ is a weak solution of } (P_{\lambda,\mu}),\ u \neq 0\}.$$

Anyway, also when $f(x,0) \neq 0$ and f, in addition to assumption (3), satisfies the growth condition

$$|f(x,\xi)| \leq a(1 + |\xi|^q)$$

for every $\xi \in \mathbb{R}$ and a.e. in Ω, where $a > 0$ and $q < \frac{n(p-1)+p}{n-p}$, the statements of Theorem 3.1 are still true.

Here is an example of application of Theorem 3.1.

EXAMPLE 1. Let Ω be a bounded open subset of $\mathbb{R}^n$ with $n \geq 2$, $1 < p < n$ and $\alpha, \beta : \Omega \to \mathbb{R}$ two continuous and bounded functions. Assume that $\sup_\Omega \alpha > 0$ and that β is a positive function with $\inf_\Omega \beta > 0$. Then for each $\mu \in [0, H[$ there exists a positive number λ^*_μ such that the problem

$$(\widetilde{P}_{\lambda,\mu}) \qquad \begin{cases} -\Delta_p u = \dfrac{\mu}{|x|^p}\,|u|^{p-2}u + \lambda[\alpha(x)|u|^{r-2}u + \beta(x)|u|^{s-2}u] & \text{a.e. in } \Omega, \\ u_{|\partial\Omega} = 0, \end{cases}$$

with $1 < r < p$ and $p < s < p^*$ admits a nonzero weak solution u_λ in $W^{1,p}_0(\Omega)$. Moreover, $\lim_{\lambda\to 0^+} \|u_\lambda\| = 0$ and the energy functional related to problem $(\widetilde{P}_{\lambda,\mu})$ is negative and decreasing in $]0, \lambda^*_\mu[$.

To prove this, we can apply Theorem 3.1 with

$$f(x,\xi) = \alpha(x)|\xi|^{r-2}\xi + \beta(x)|\xi|^{s-2}\xi$$

for every $(x,\xi) \in \overline{\Omega} \times \mathbb{R}$.

It is easy to verify that condition (1) holds. Denote by B_ϱ the open ball centred at x_0 with radius ϱ, and let ϱ be such that $B_\varrho \subseteq \Omega$ and $\min_{B_\varrho} \alpha > 0$. If we put $D = B = B_\varrho$, a simple computation shows that

$$\lim_{\xi\to 0^+} \frac{\inf_{x\in B} \int_0^\xi f(x,t)\,dt}{|\xi|^p} \geq \frac{\min_B \alpha}{r} \lim_{\xi\to 0^+} \frac{1}{|\xi|^{p-r}} = +\infty.$$

Hence all the assumptions of Theorem 3.1 are verified and the conclusion follows.

REMARK 2. We point out that the energy functional $J_{\lambda,\mu}$ related to problem $(\widetilde{P}_{\lambda,\mu})$ is not coercive. In particular, it is unbounded from below.

In fact, if we fix $v \in W_0^{1,p}(\Omega)$ and $\tau \in \mathbb{R}$, then

$$J_{\lambda,\mu}(\tau v) = \frac{\tau^p}{p}\|v\|^p - \frac{\mu\tau^p}{p}|||v|||^p - \lambda\Big[\frac{\tau^r}{r}\Big(\int_\Omega \alpha(x)\,dx\Big)\|v\|_r^r + \frac{\tau^s}{s}\Big(\int_\Omega \beta(x)\,dx\Big)\|v\|_s^s\Big].$$

So, as $s > p > r$, it follows that $\lim_{\tau\to+\infty} J_{\lambda,\mu}(\tau v) = -\infty$.

Let now Ω be a bounded open subset of $\mathbb{R}^n$ and consider, for each $\lambda, \mu \in \mathbb{R}$, the functional $I_{\lambda,\mu} : X \to \mathbb{R}$ defined by putting

$$I_{\lambda,\mu}(u) = \frac{1}{p}\int_\Omega |\nabla u(x)|^p\,dx - \frac{\mu}{p}\Big(\int_\Omega |u(x)|^{p^*}\,dx\Big)^{p/p^*} + \lambda\Phi(u)$$

for each $u \in X$.

Taking into account the Sobolev inequality

$$\Big(\int_\Omega |u(x)|^{p^*}\,dx\Big)^{1/p^*} \le \frac{1}{S^{1/p}}\Big(\int_\Omega |\nabla u(x)|^p\,dx\Big)^{1/p}, \tag{10}$$

where $u \in X$ and S is the best constant in the Sobolev inclusion (see [7]), and observing that $I_{\lambda,\mu}$ is the energy functional related to problem $(P^*_{\lambda,\mu})$ introduced in the introduction, in analogy to Theorem 3.1, we can state the following

THEOREM 3.2. *Let $f : \Omega\times\mathbb{R} \to \mathbb{R}$ be a Carathéodory function satisfying the assumptions of Theorem* 3.1. *Then for every $\mu \in [0, S[$ there exists a positive number ν^*_μ such that for every $\lambda \in]0, \nu^*_\mu[$ problem $(P^*_{\lambda,\mu})$ admits a nonzero weak solution u_λ in $W_0^{1,p}(\Omega)$. Moreover,*

$$\lim_{\lambda\to 0^+}\|u_\lambda\| = 0$$

and the function $\lambda \mapsto I_{\lambda,\mu}(u_\lambda)$ is negative and decreasing in $]0, \nu^*_\mu[$.

References

[1] D. Arcoya and L. Boccardo, *Some remarks on critical point theory for nondifferentiable functionals*, NoDEA Nonlinear Differential Equations Appl. 6 (1999), 79–100.

[2] I. Ekeland and N. Ghoussoub, *Selected new aspects of the calculus of variations in the large*, Bull. Amer. Math. Soc. 39 (2002), 207–265.

[3] J. P. García Azorero and I. Peral, *Hardy inequalities and some critical elliptic and parabolic problems*, J. Differential Equations 144 (1998), 441–476.

[4] N. Ghoussoub and C. Yuan, *Multiple solutions for quasi-linear PDEs involving the critical Sobolev and Hardy exponents*, Trans. Amer. Math. Soc. 352 (2000), 5703–5743.

[5] E. Montefusco, *Lower semicontinuity of functionals via the concentration-compactness principle*, J. Math. Anal. Appl. 263 (2001), 264–276.

[6] B. Ricceri, *A general variational principle and some of its applications*, J. Comput. Appl. Math. 113 (2000), 401–410.

[7] G. Talenti, *Best constant in Sobolev inequality*, Ann. Mat. Pura Appl. 110 (1976), 353–372.

Dipartimento di Matematica
e Informatica
Università di Catania
Viale A. Doria 6
95125 Catania, Italy
E-mail: ffaraci@dmi.unict.it

Dipartimento di Matematica
Università di Messina
Salita Sperone 31
98166 Sant'Agata, Messina, Italy
E-mail: livrea@dipmat.unime.it

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