

Exponential convergence of shunting inhibitory cellular neural networks with continuously distributed delays

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Abstract. We study delay shunting inhibitory cellular neural networks without almost periodic coefficients. Some sufficient conditions are established to ensure that all solutions of the networks converge exponentially to an almost periodic function. This complements previously known results.

1. Introduction. Consider the shunting inhibitory cellular neural networks (SICNNs) with continuously distributed delays given by

$$(1.1) \quad x'_{ij}(t) = -a_{ij}(t)x_{ij}(t) - \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl}(t) \int_0^{\infty} K_{ij}(u)f(x_{kl}(t-u)) du x_{ij}(t) \\ + L_{ij}(t), \quad i = 1, \dots, m, j = 1, \dots, n,$$

where C_{ij} denotes the cell at the (i, j) position of the lattice, the r -neighborhood $N_r(i, j)$ of C_{ij} is

$$N_r(i, j) = \{C_{kl} : \max(|k - i|, |l - j|) \leq r, 1 \leq k \leq m, 1 \leq l \leq n\},$$

x_{ij} is the activity of the cell C_{ij} , $L_{ij}(t)$ is the external input to C_{ij} , $a_{ij}(t) > 0$ represents the passive decay rate of the cell activity at time t , $C_{ij}^{kl}(t) \geq 0$ is the connection or coupling strength of postsynaptic activity of the cell transmitted to the cell C_{ij} at time t , and the activity function $f(\cdot)$ is a continuous function representing the output or firing rate of the cell C_{kl} .

Since Bouzerdoum and Pinter [1–3] described SICNNs as new cellular neural networks (CNNs), SICNNs have been extensively applied in psychophysics, speech, perception, robotics, adaptive pattern recognition, vision, and image processing. Hence, they have been the object of intensive analysis by numerous authors in recent years. In particular, there have been extensive results on the problem of the existence and stability of almost pe-

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riodic solutions for system (1.1) in the literature. We refer the reader to [4, 5, 9, 10, 11, 13] and the references cited therein. Suppose that the following condition is satisfied:

$$(H_0) \quad a_{ij}, C_{ij}^{kl}, L_{ij} : \mathbb{R} \rightarrow \mathbb{R} \text{ are almost periodic functions, where } i = 1, \dots, m, j = 1, \dots, n.$$

Most authors cited above then deduced that all solutions of system (1.1) converge exponentially to an almost periodic function. However, to the best of our knowledge, few authors have considered the convergence behavior for all solutions of system (1.1) without the assumption (H_0) . Thus, it is worthwhile to investigate the convergence behavior for all solutions of system (1.1) in this case.

The main purpose of this paper is to study system (1.1) with almost periodic coefficients being perturbed by exponentially small perturbations as $t \rightarrow +\infty$. By applying techniques similar to those in [12], we derive some sufficient conditions ensuring that all solutions of system (1.1) converge exponentially to an almost periodic function. Moreover, an example is provided to illustrate the effectiveness of our results.

Consider the following delayed shunting inhibitory cellular neural networks:

$$(1.2) \quad x'_{ij}(t) = -a_{ij}^*(t)x_{ij}(t) - \sum_{C_{kl} \in N_r(i,j)} \tilde{C}_{ij}^{kl}(t) \int_0^\infty K_{ij}(u)f(x_{kl}(t-u)) du x_{ij}(t) + L_{ij}^*(t), \quad i = 1, \dots, m, j = 1, \dots, n.$$

Throughout this paper, for $i = 1, \dots, m, j = 1, \dots, n$, it will be assumed that $a_{ij}^*, \tilde{C}_{ij}^{kl}, L_{ij}^* : \mathbb{R} \rightarrow \mathbb{R}$ are almost periodic functions such that

$$(1.3) \quad \begin{cases} 0 < \underline{a}_{ij} := \inf_{t \in \mathbb{R}} a_{ij}^*(t) < +\infty, & \overline{C}_{ij}^{kl} := \sup_{t \in \mathbb{R}} \tilde{C}_{ij}^{kl}(t) < +\infty, \\ L_{ij}^+ := \sup_{t \in \mathbb{R}} |L_{ij}^*(t)| < +\infty. \end{cases}$$

Throughout, we set

$$\{x_{ij}(t)\} = (x_{11}(t), \dots, x_{1n}(t), \dots, x_{i1}(t), \dots, x_{in}(t), \dots, x_{m1}(t), \dots, x_{mn}(t)).$$

For $x(t) = \{x_{ij}(t)\} \in \mathbb{R}^{m \times n}$, we define the norm $\|x(t)\| = \max_{(i,j)} \{|x_{ij}(t)|\}$. If $g_1, g_2 : \mathbb{R} \rightarrow \mathbb{R}$ are continuous, and there exist constants $\zeta, \delta > 0$ such that

$$|g_1(t)/g_2(t)| \leq \zeta \quad \text{for all } t \geq \delta,$$

we write $g_1(t) = O(g_2(t))$.

We also assume that the following conditions hold:

$$(H_1) \quad \text{For } i \in \{1, \dots, m\}, j \in \{1, \dots, n\}, \text{ the delay kernels } K_{ij} : [0, \infty) \rightarrow \mathbb{R} \text{ are continuous, integrable and there exist constants } k_{ij} \geq 0 \text{ and}$$

$\lambda > 0$ such that

$$\int_0^\infty |K_{ij}(s)| ds \leq k_{ij}, \quad \int_0^\infty |K_{ij}(u)|e^{\lambda u} du < +\infty.$$

(H₂) There exist constants M_f and μ_f such that

$$|f(u) - f(v)| \leq \mu_f |u - v|, \quad |f(u)| \leq M_f \quad \text{for all } u, v \in \mathbb{R}.$$

(H₃) The quantities defined by

$$L = \max_{(i,j)} \frac{L_{ij}^+}{\underline{a}_{ij}}, \quad \delta = \max_{(i,j)} \frac{\sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} M_f k_{ij}}{\underline{a}_{ij}}$$

satisfy $0 \leq L < +\infty, 0 < \delta < 1$, and there exist constants $\eta, \lambda > 0$ such that for all i, j as above,

$$(1.4) \quad (\lambda - \underline{a}_{ij}) + \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \left(M_f k_{ij} + \mu_f \int_0^\infty |K_{ij}(u)|e^{\lambda u} du \frac{L}{1-\delta} \right) < -\eta < 0.$$

(H₄) For i, j as above, $a_{ij}, C_{ij}^{kl}, L_{ij} : \mathbb{R} \rightarrow \mathbb{R}$ are continuous functions, and

$$a_{ij}(t) - a_{ij}^*(t) = O(e^{-\lambda t}), \quad L_{ij}(t) - L_{ij}^*(t) = O(e^{-\lambda t}), \\ C_{ij}^{kl}(t) - \tilde{C}_{ij}^{kl}(t) = O(e^{-\lambda t}).$$

The initial conditions associated with system (1.1) are of the form

$$(1.5) \quad x_{ij}(s) = \varphi_{ij}(s), \quad s \in (-\infty, 0], \quad i = 1, \dots, m, \quad j = 1, \dots, n,$$

where $\varphi_{ij}(\cdot)$ denotes a real-valued bounded continuous function defined on $(-\infty, 0]$.

DEFINITION 1 (see [6, 7]). Let $u : \mathbb{R} \rightarrow \mathbb{R}^{m \times n}$ be continuous. Then u is said to be *almost periodic* on \mathbb{R} if, for any $\varepsilon > 0$, the set $T(u, \varepsilon) = \{ \delta : \|u(t + \delta) - u(t)\| < \varepsilon, \forall t \in \mathbb{R} \}$ is relatively dense, i.e., for any $\varepsilon > 0$, it is possible to find a real number $l = l(\varepsilon) > 0$ such that for any interval with length $l(\varepsilon)$, there exists a number $\delta = \delta(\varepsilon)$ in this interval such that $\|u(t + \delta) - u(t)\| < \varepsilon$ for all $t \in \mathbb{R}$.

Since $a_{ij}^*, \tilde{C}_{ij}^{kl}$ and L_{ij}^* are almost periodic functions, by using a similar argument to that in the proof of Theorem 2.1 in [13], we can easily show the following lemma.

LEMMA 1.1. *Let (H₁)–(H₃) hold. Then system (1.2) has exactly one almost periodic solution $Z^* = \{x_{ij}^*(t)\} = (x_{11}^*(t), \dots, x_{mn}^*(t))$, and $\sup_{t \in \mathbb{R}} |x_{ij}^*(t)| \leq L/(1 - \delta), i = 1, \dots, m, j = 1, \dots, n$.*

The remaining part of this paper is organized as follows. In Section 2, we present some new sufficient conditions to ensure that all solutions of system

(1.1) converge exponentially to an almost periodic function. In Section 3, we give some examples and remarks to illustrate our results.

2. Main results

THEOREM 2.1. *Let (H_1) – (H_4) hold. Suppose that $Z^*(t) = \{x_{ij}^*(t)\} = (x_{11}^*(t), \dots, x_{mn}^*(t))$ is the almost periodic solution of system (1.2). Then, for every solution $Z(t) = \{x_{ij}(t)\} = (x_{11}(t), \dots, x_{mn}(t))$ of system (1.1) with any initial value $\varphi = \{\varphi_{ij}(t)\} = (\varphi_{11}(t), \dots, \varphi_{mn}(t))$, we have*

$$|x_{ij}(t) - x_{ij}^*(t)| = O(e^{-\lambda t}) \quad \text{for all } i, j.$$

Proof. Set

$$\begin{aligned} \epsilon_{ij}(t) = & -[a_{ij}(t) - a_{ij}^*(t)]x_{ij}^*(t) - \sum_{C_{kl} \in N_r(i,j)} [C_{ij}^{kl}(t) - \tilde{C}_{ij}^{kl}(t)] \\ & \cdot \int_0^\infty K_{ij}(u)f(x_{kl}^*(t-u)) du x_{ij}^*(t) + [L_{ij}(t) - L_{ij}^*(t)]. \end{aligned}$$

Since $Z^*(t) = (x_{11}^*(t), \dots, x_{mn}^*(t))$ is an almost periodic function, by (H_3) and (H_4) , we can choose constants $F, T > 0$ such that

$$(2.1) \quad |\epsilon_{ij}(t)| < \frac{1}{2}Fe^{-\lambda t} \quad \text{for all } t \geq T,$$

and

$$\begin{aligned} (2.2) \quad & [\lambda - a_{ij}(t)] + \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl}(t) \left[M_f k_{ij} + \mu_f \int_0^\infty |K_{ij}(u)|e^{\lambda u} du \frac{L}{1-\delta} \right] \\ & < [\lambda - a_{ij}^*(t)] + \sum_{C_{kl} \in N_r(i,j)} \tilde{C}_{ij}^{kl}(t) \left[M_f k_{ij} + \mu_f \int_0^\infty |K_{ij}(u)|e^{\lambda u} du \frac{L}{1-\delta} \right] + \frac{1}{2}\eta \\ & < [\lambda - \underline{a}_{ij}] + \sum_{C_{kl} \in N_r(i,j)} \bar{C}_{ij}^{kl} \left[M_f k_{ij} + \mu_f \int_0^\infty |K_{ij}(u)|e^{\lambda u} du \frac{L}{1-\delta} \right] + \frac{1}{2}\eta \\ & < -\frac{1}{2}\eta \end{aligned}$$

for all $t \geq T$ and all i, j , where T is a sufficiently large constant.

Let $Z(t) = \{x_{ij}(t)\} = (x_{11}(t), \dots, x_{mn}(t))$ be a solution of system (1.1) with any initial value $\varphi = \{\varphi_{ij}(t)\} = (\varphi_{11}(t), \dots, \varphi_{mn}(t))$, and define

$$y(t) = \{y_{ij}(t)\} = (y_{11}(t), \dots, y_{mn}(t)) = Z(t) - Z^*(t).$$

Then

$$\begin{aligned}
 (2.3) \quad & y'_{ij}(t) \\
 &= -a_{ij}(t)y_{ij}(t) - \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl}(t) \left[\int_0^\infty K_{ij}(u)f(x_{kl}(t-u)) du x_{ij}(t) \right. \\
 &\quad \left. - \int_0^\infty K_{ij}(u)f(x_{kl}^*(t-u)) du x_{ij}^*(t) \right] + \epsilon_{ij}(t)
 \end{aligned}$$

for all i, j . Let $(ij)_t$ be an index such that

$$(2.4) \quad |y_{(ij)_t}(t)| = \|y(t)\|.$$

Calculating the upper right derivative of $e^{\lambda s}|y_{(ij)_s}(s)|$, in view of (H_1) and (H_2) , for all $t \geq T$, we have

$$\begin{aligned}
 (2.5) \quad & D^+(e^{\lambda s}|y_{(ij)_s}(s)|)|_{s=t} \\
 &= \lambda e^{\lambda t}|y_{(ij)_t}(t)| + e^{\lambda t} \text{sign}(y_{(ij)_t}(t)) \left\{ -a_{(ij)_t}(t)y_{(ij)_t}(t) \right. \\
 &\quad - \sum_{C_{kl} \in N_r(i,j)_t} C_{(ij)_t}^{kl}(t) \left[\int_0^\infty K_{(ij)_t}(u)f(x_{kl}(t-u)) du x_{(ij)_t}(t) \right. \\
 &\quad \left. - \int_0^\infty K_{(ij)_t}(u)f(x_{kl}^*(t-u)) du x_{(ij)_t}^*(t) \right] + \epsilon_{(ij)_t}(t) \left. \right\} \\
 &\leq e^{\lambda t} \left\{ [\lambda - a_{(ij)_t}(t)]|y_{(ij)_t}(t)| \right. \\
 &\quad + \sum_{C_{kl} \in N_r(i,j)_t} C_{(ij)_t}^{kl}(t) \left[\int_0^\infty K_{(ij)_t}(u)f(x_{kl}(t-u)) du x_{(ij)_t}(t) \right. \\
 &\quad - \int_0^\infty K_{(ij)_t}(u)f(x_{kl}(t-u)) du x_{(ij)_t}^*(t) \left. \right] \\
 &\quad + \left| \int_0^\infty K_{(ij)_t}(u)f(x_{kl}(t-u)) du x_{(ij)_t}^*(t) \right. \\
 &\quad \left. - \int_0^\infty K_{(ij)_t}(u)f(x_{kl}^*(t-u)) du x_{(ij)_t}^*(t) \right] + |\epsilon_{(ij)_t}(t)| \left. \right\} \\
 &< e^{\lambda t} \left\{ [\lambda - a_{(ij)_t}(t)]|y_{(ij)_t}(t)| + \sum_{C_{kl} \in N_r(i,j)_t} C_{(ij)_t}^{kl}(t) \left[M_f k_{(ij)_t} |y_{(ij)_t}(t)| \right. \right. \\
 &\quad \left. \left. + \mu_f \int_0^\infty |K_{(ij)_t}(u)| |y_{kl}(t-u)| du \frac{L}{1-\delta} \right] \right\} + \frac{1}{2}F.
 \end{aligned}$$

Let

$$(2.6) \quad M(t) = \max_{s \leq t} e^{\lambda s} \|y(s)\|.$$

It is obvious that $e^{\lambda t} \|y(t)\| \leq M(t)$, and $M(t)$ is non-decreasing.

Now, we consider two cases.

CASE (i). Suppose that

$$(2.7) \quad M(t) > e^{\lambda t} \|y(t)\| \quad \text{for all } t \geq T.$$

Then we claim that

$$(2.8) \quad M(t) \equiv M(T) \text{ is a constant for all } t \geq T.$$

Assume, by way of contradiction, that (2.8) does not hold. Then there exists $t_1 > T$ such that $M(t_1) > M(T)$. Since

$$e^{\lambda t} \|y(t)\| \leq M(T) \quad \text{for all } t \leq T,$$

there must exist $\beta \in (T, t_1)$ such that

$$e^{\lambda \beta} \|y(\beta)\| = M(t_1) \geq M(\beta),$$

which contradicts (2.7). This contradiction implies that (2.8) holds. It follows that

$$(2.9) \quad e^{\lambda t} \|y(t)\| < M(t) = M(T) \quad \text{for all } t \geq T.$$

CASE (ii). Suppose that there is a point $t_0 \geq T$ such that $M(t_0) = e^{\lambda t_0} \|y(t_0)\|$. Then, in view of (2.2) and (2.5), we get

$$\begin{aligned} (2.10) \quad & D^+(e^{\lambda s} |y_{(ij)_s}(s)|) |_{s=t_0} \\ & < e^{\lambda t_0} \left\{ [\lambda - a_{(ij)_{t_0}}(t_0)] |y_{(ij)_{t_0}}(t_0)| + \sum_{C_{kl} \in N_r(i,j)_{t_0}} C_{(ij)_{t_0}}^{kl}(t_0) \left[M_f k_{(ij)_{t_0}} |y_{(ij)_{t_0}}(t_0)| \right. \right. \\ & \quad \left. \left. + \mu_f \int_0^\infty |K_{(ij)_{t_0}}(u)| |y_{kl}(t_0 - u)| du \frac{L}{1 - \delta} \right] \right\} + \frac{1}{2} F \\ & = [\lambda - a_{(ij)_{t_0}}(t_0)] |y_{(ij)_{t_0}}(t_0)| e^{\lambda t_0} + \sum_{C_{kl} \in N_r(i,j)_{t_0}} C_{(ij)_{t_0}}^{kl}(t_0) \left[M_f k_{(ij)_{t_0}} |y_{(ij)_{t_0}}(t_0)| e^{\lambda t_0} \right. \\ & \quad \left. + \mu_f \int_0^\infty |K_{(ij)_{t_0}}(u)| e^{\lambda u} |y_{kl}(t_0 - u)| e^{\lambda(t_0 - u)} du \frac{L}{1 - \delta} \right] + \frac{1}{2} F \end{aligned}$$

$$\begin{aligned} &\leq \left\{ [\lambda - a_{(ij)_{t_0}}(t_0)] + \sum_{C_{kl} \in N_r(i,j)_{t_0}} C_{(ij)_{t_0}}^{kl}(t) \left[M_f k_{(ij)_{t_0}} \right. \right. \\ &\quad \left. \left. + \mu_f \int_0^\infty |K_{(ij)_{t_0}}(u)| e^{\lambda u} du \frac{L}{1-\delta} \right] \right\} M(t_0) + \frac{1}{2} F \\ &\leq -\frac{1}{2} \eta M(t_0) + \frac{1}{2} F. \end{aligned}$$

In addition, if $M(t_0) \geq F/\eta$, then $M(t)$ is strictly decreasing in a small neighborhood $(t_0, t_0 + \delta_0)$. This contradicts that $M(t)$ is non-decreasing. Hence,

$$(2.11) \quad e^{\lambda t_0} \|y(t_0)\| = M(t_0) < F/\eta.$$

For all $t > t_0$, by the same approach used in the proof of (2.11), we have

$$(2.12) \quad e^{\lambda t} \|y(t)\| < F/\eta \quad \text{if } M(t) = e^{\lambda t} \|y(t)\|.$$

On the other hand, if $M(t) > e^{\lambda t} \|y(t)\|$ for $t > t_0$, we can choose $t_0 \leq t_3 < t$ such that

$$M(t_3) = e^{\lambda t_3} \|y(t_3)\| < F/\eta \quad \text{and} \quad M(s) > e^{\lambda s} \|y(s)\| \quad \text{for all } s \in (t_3, t].$$

Using a similar argument to the proof of Case (i), we can show that

$$(2.13) \quad M(s) \equiv M(t_3) \text{ is a constant for all } s \in (t_3, t],$$

which implies that

$$e^{\lambda t} \|y(t)\| < M(t) = M(t_3) < F/\eta.$$

In summary, there must exist $N > 0$ with $e^{\lambda t} \|y(t)\| < \max\{M(T), F/\eta\}$ for all $t > N$. This completes the proof of Theorem 2.1.

3. An example. In this section, we give an example to illustrate the results obtained in the previous sections.

EXAMPLE 3.1. Consider the following SICNNs with continuously distributed delays:

$$(3.1) \quad \frac{dx_{ij}}{dt} = -a_{ij}(t)x_{ij} - \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl}(t) \int_0^\infty K_{ij}(u)f(x_{kl}(t-u)) du x_{ij} + L_{ij}(t),$$

$$i, j = 1, 2, 3,$$

where

$$(3.2) \quad \begin{bmatrix} a_{11}(t) & a_{12}(t) & a_{13}(t) \\ a_{21}(t) & a_{22}(t) & a_{23}(t) \\ a_{31}(t) & a_{32}(t) & a_{33}(t) \end{bmatrix} = \begin{bmatrix} 1 + e^{-t} & 1 + e^{-t} & 3 + e^{-t} \\ 3 + e^{-t} & 1 + e^{-t} & 3 + e^{-t} \\ 3 + e^{-t} & 1 + e^{-t} & 3 + e^{-t} \end{bmatrix},$$

$$(3.3) \quad \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} = \begin{bmatrix} 0.1 + e^{-2t} & 0.2 + e^{-2t} & 0.1 + e^{-2t} \\ 0.2 + e^{-2t} & 0 + e^{-2t} & 0.2 + e^{-2t} \\ 0.1 + e^{-2t} & 0.2 + e^{-2t} & 0.1 + e^{-2t} \end{bmatrix}.$$

and

$$(3.4) \quad \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} = \begin{bmatrix} 0.7 + 0.24 \sin^2 \sqrt{2}t + e^{-2t} & 0.41 + 0.5 \cos^2 t + e^{-2t} & 0.94 + 2 \sin^2 t + e^{-2t} \\ 0.91 + 2 \cos^2 t + e^{-2t} & 0.67 + 0.2 \sin^2 t + e^{-2t} & 0.91 + 2 \sin^2 t + e^{-2t} \\ 2.54 + 0.4 \cos^4 t + e^{-2t} & 0.5 + 0.41 \sin^2 t + e^{-2t} & 0.94 + 2 \cos^2 t + e^{-2t} \end{bmatrix}.$$

Let

$$(3.5) \quad x'_{ij}(t) = -a_{ij}^* x_{ij}(t) - \sum_{C_{kl} \in N_r(i,j)} \tilde{C}_{ij}^{kl} \int_0^\infty K_{ij}(u) f(x_{kl}(t-u)) du x_{ij}(t) + L_{ij}^*(t),$$

$i, j = 1, 2, 3,$

where

$$(3.6) \quad \begin{bmatrix} a_{11}^* & a_{12}^* & a_{13}^* \\ a_{21}^* & a_{22}^* & a_{23}^* \\ a_{31}^* & a_{32}^* & a_{33}^* \end{bmatrix} = \begin{bmatrix} 1 & 1 & 3 \\ 3 & 1 & 3 \\ 3 & 1 & 3 \end{bmatrix},$$

$$\begin{bmatrix} \tilde{c}_{11} & \tilde{c}_{12} & \tilde{c}_{13} \\ \tilde{c}_{21} & \tilde{c}_{22} & \tilde{c}_{23} \\ \tilde{c}_{31} & \tilde{c}_{32} & \tilde{c}_{33} \end{bmatrix} = \begin{bmatrix} 0.1 & 0.2 & 0.1 \\ 0.2 & 0 & 0.2 \\ 0.1 & 0.2 & 0.1 \end{bmatrix},$$

$$(3.7) \quad \begin{bmatrix} L_{11}^* & L_{12}^* & L_{13}^* \\ L_{21}^* & L_{22}^* & L_{23}^* \\ L_{31}^* & L_{32}^* & L_{33}^* \end{bmatrix} = \begin{bmatrix} 0.7 + 0.24 \sin^2 \sqrt{2}t & 0.41 + 0.5 \cos^2 t & 0.94 + 2 \sin^2 t \\ 0.91 + 2 \cos^2 t & 0.67 + 0.2 \sin^2 t & 0.91 + 2 \sin^2 t \\ 2.54 + 0.4 \cos^4 t & 0.5 + 0.41 \sin^2 t & 0.94 + 2 \cos^2 t \end{bmatrix}.$$

Set $r = 1$, $K_{ij}(u) = e^{-2u} \sin u$, $i, j = 1, 2, 3$, and $f(x) = \frac{1}{20}(|x - 1| - |x + 1|)$. Then it is straightforward to check that the SICNNs (3.1) and (3.5) satisfy the conditions (H_1) – (H_4) . Hence, from Lemma 1.1 and Theorem 2.1, system

(3.5) has exactly one almost periodic solution. Moreover, all solutions of system (3.1) converge exponentially to the almost periodic solution of system (3.5).

REMARK 3.1. System (3.1) is a very simple form of delayed shunting inhibitory neural networks without almost periodic coefficients. Therefore, the results in [2–5, 8–13] and the references therein cannot be applied to prove that all solutions of system (3.1) converge exponentially to an almost periodic function. This implies that the results of this paper are essentially new.

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References

- [1] A. Bouzerdoum and R. B. Pinter, *Shunting inhibitory cellular neural networks: derivation and stability analysis*, IEEE Trans. Circuits Systems I Fund. Theory Appl. 40 (1993), 215–221.
- [2] —, —, *Analysis and analog implementation of directionally sensitive shunting inhibitory Cellular Neural Networks*, in: Visual Information Processing: From Neurons to Chips, SPIE 1473, 1991, 29–38.
- [3] —, —, *Nonlinear lateral inhibition applied to motion detection in the fly visual system*, in: Nonlinear Vision, R. B. Pinter and B. Nabet (eds.), CRC Press, Boca Raton, FL, 1992, 423–450.
- [4] M. Cai and W. Xiong, *Almost periodic solutions for shunting inhibitory cellular neural networks without global Lipschitz and bounded activation functions*, Phys. Lett. A 362 (2007), 417–423.
- [5] A. Chen, J. Cao and L. Huang, *Almost periodic solution of shunting inhibitory CNNs with delays*, *ibid.* 298 (2002), 161–170.
- [6] A. M. Fink, *Almost Periodic Differential Equations*, Lecture Notes in Math. 377, Springer, Berlin, 1974.
- [7] C. Y. He, *Almost Periodic Differential Equations*, Higher Education Publ. House, Beijing, 1992 (in Chinese).
- [8] Y. Li, C. Liu and L. Zhu, *Global exponential stability of periodic solution of shunting inhibitory CNNs with delays*, Phys. Lett. A 337 (2005), 46–54.
- [9] B. Liu and L. Huang, *Existence and stability of almost periodic solutions for shunting inhibitory cellular neural networks with continuously distributed delays*, *ibid.* 349 (2006), 177–186.
- [10] Y. Liu, Z. You and L. Cao, *On the almost periodic solution of generalized shunting inhibitory cellular neural networks with continuously distributed delays*, *ibid.* 360 (2006), 122–130.
- [11] —, —, —, *Almost periodic solution of shunting inhibitory cellular neural networks with time varying and continuously distributed delays*, *ibid.* 364 (2007), 17–28.

- [12] W. Lu and T. Chen, *Global exponential stability of almost periodic solutions for a large class of delayed dynamical systems*, Sci. China Ser. A Math. 8 (2005), 1015–1026.
- [13] Q. Zhou, B. Xiao, Y. Yu and L. Peng, *Existence and exponential stability of almost periodic solutions for shunting inhibitory cellular neural networks with continuously distributed delays*, Chaos Solitons Fractals 34 (2007), 860–866.

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