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Existence and uniqueness of positive periodic solutions of delayed Nicholson's blowflies models

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Abstract. This paper is concerned with a class of Nicholson's blowflies models with multiple time-varying delays. By applying the coincidence degree, some criteria are established for the existence and uniqueness of positive periodic solutions of the model. Moreover, a totally new approach to proving the uniqueness of positive periodic solutions is proposed. In particular, an example is employed to illustrate the main results.

1. Introduction. As is well known, Nicholson's blowflies equation was introduced by Nicholson [N] to model laboratory fly populations. Its dynamics was later studied in [GBN] and [NG]. Consequently, the theory of Nicholson's blowflies equation has made a remarkable progress in the past forty years with main results scattered in numerous research papers (for details see [BBT, BIT, CDZ, KLS, YZ, ZWZ]). In particular, there have been extensive studies on the problem of the existence of positive periodic solutions for Nicholson's blowflies equation. We refer the reader to [C, CL, LD, SA] and the references cited therein. In [C], Chen obtained the existence of positive periodic solutions of Nicholson's blowflies model of the form

$$N'(t) = -\delta(t)N(t) + P(t)N(t-\sigma(t))e^{-a(t)N(t-\tau(t))}, \tag{1.1}$$

where $\delta \in C(\mathbb{R},\mathbb{R})$, $P,\sigma,\tau \in C(\mathbb{R},(0,\infty))$, and $a \in C(\mathbb{R},(0,\infty))$ are T-periodic functions with $\int_0^T \delta(t)\,dt > 0$. Let k be a positive integer. When $\tau(t) = \sigma(t) = kT$, S. Saker and S. Agarwal [SA] established the existence of positive periodic solutions for (1.1). In [LD], Li and Du studied the generalized Nicholson's blowflies model

$$N'(t) = -\delta(t)N(t) + \sum_{i=1}^{m} p_i(t)N(t-\tau_i(t))e^{-\gamma_i(t)N(t-\tau_i(t))}, \tag{1.2}$$

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where $\delta, p_i, \gamma_i \in C(\mathbb{R}^+, (0,\infty))$ and $\tau_i \in C(\mathbb{R}^+, \mathbb{R}^+)$ are T-periodic functions for $i = 1, \dots, m$. They obtained a sufficient and necessary condition for the existence of positive periodic solutions for (1.2). Recently, Chen and Liu [CL] established some criteria for the solutions of this model to converge locally exponentially to a positive almost periodic solution. However, the existence and uniqueness of positive periodic solutions of Nicholson's blowflies model is difficult to establish. Moreover, to the best of our knowledge, few authors have considered conditions for the existence and uniqueness of positive periodic solutions of Nicholson's blowflies equation in terms of its coefficients.

The main purpose of this paper is to give conditions for the existence and uniqueness of positive periodic solutions for Nicholson's blowflies model (1.2).

Throughout this paper, given a bounded continuous function g defined on $\mathbb{R}$, let

$$g^- = \inf_{t\in\mathbb{R}} g(t), \qquad g^+ = \sup_{t\in\mathbb{R}} g(t). \tag{1.3}$$

Then, we denote

$$A = 2\int_0^T \delta(t)\,dt, \qquad B = \int_0^T \sum_{i=1}^m p_i(t)\,dt, \tag{1.4}$$

$$\gamma^- = \min_{i\in\{1,\dots,m\}} \gamma_i^-, \qquad \gamma^+ = \max_{i\in\{1,\dots,m\}} \gamma_i^+. \tag{1.5}$$

For convenience, let $\kappa \in (0,1)$ be the unique real number such that

$$\frac{1-\kappa}{e^\kappa} = \frac{1}{e^2}. \tag{1.6}$$

One can easily show that

$$\sup_{x\ge\kappa} \left|\frac{1-x}{e^x}\right| = \frac{1}{e^2}. \tag{1.7}$$

The paper is organized as follows. In Section 2, we shall derive new sufficient conditions for the existence and uniqueness of positive periodic solution of model (1.2). In Section 3, we shall give an example and a remark to illustrate our results.

2. Existence and uniqueness of positive periodic solution. We will need the continuation theorem of coincidence degree theory formulated in [GM].

LEMMA 2.1 (Continuation Theorem). *Let X and Z be Banach spaces. Consider an operator equation $Lx = \lambda Nx$, where $L : \operatorname{Dom} L \subset X \to Z$ is a Fredholm operator of index zero and $\lambda \in [0,1]$ is a parameter. Let P and Q*

denote two projectors such that

$$P : X \to \operatorname{Ker} L \quad \textit{and} \quad Q : Z \to Z/\operatorname{Im} L.$$

Assume that $N : \overline{\Omega} \to Z$ *is L-compact on* $\overline{\Omega}$, *where* Ω *is open bounded in* X. *Furthermore, suppose that*

(1) $Lx \neq \lambda Nx$ *for all* $x \in \partial\Omega \cap D(L), \lambda \in (0,1)$;
(2) $Nx \notin \operatorname{Im} L$ *for all* $x \in \partial\Omega \cap \operatorname{Ker} L$;
(3) *the Brouwer degree* $\deg\{QN, \Omega \cap \operatorname{Ker} L, 0\}$ *is not zero.*

Then $Lx = Nx$ *has a solution in* $\overline{\Omega}$.

THEOREM 2.1. *Suppose that*

$$\ln \frac{2B}{A} > A. \tag{2.1}$$

Then (1.2) *has a positive* T*-periodic solution.*

Proof. Set $N(t) = e^{x(t)}$. Then (1.2) can be rewritten as

$$\begin{aligned} x'(t) = &-\delta(t) + \sum_{i=1}^{m} p_i(t) e^{x(t-\tau_i(t)) - x(t) - \gamma_i(t) e^{x(t-\tau_i(t))}} \\ := &\ \Delta(x,t). \end{aligned} \tag{2.2}$$

Thus, to prove Theorem 2.1, it suffices to show that (2.2) has a T-periodic solution. Let $X = Z = \{x \in C(\mathbb{R},\mathbb{R}) : x(t+T) = x(t) \text{ for all } t \in \mathbb{R}\}$ be equipped with the norm $\|x\| = \max_{t\in[0,T]} |x(t)|$. For any $x \in X$, it is easy to see that $\Delta(x,\cdot) \in C(\mathbb{R},\mathbb{R})$ is T-periodic. Let

$$\begin{aligned} &L : D(L) = \{x \in X : x \in C^1(\mathbb{R},\mathbb{R})\} \ni x \mapsto x' \in Z, \\ &P : X \ni x \mapsto \frac{1}{T}\int_0^T x(s)\, ds \in X, \\ &Q : Z \ni z \mapsto \frac{1}{T}\int_0^T z(s)\, ds \in Z, \\ &N : X \ni x \mapsto \Delta(x,\cdot) \in Z. \end{aligned}$$

Clearly,

$$\operatorname{Im} L = \Big\{x \in Z : \int_0^T x(s)\, ds = 0\Big\} = \operatorname{Ker} Q, \qquad \operatorname{Ker} L = \mathbb{R} = \operatorname{Im} P.$$

It follows that L is a Fredholm operator with index zero. Set $L_P = L|_{D(L)\cap \operatorname{Ker} P}$. Then L_P has continuous inverse L_P^{-1} defined by

$$L_P^{-1} : \operatorname{Im} L \to D(L) \cap \operatorname{Ker} P,\ L_P^{-1} y(t) = -\frac{1}{T}\int_0^T\int_0^t y(s)\, ds\, dt + \int_0^t y(s)\, ds. \tag{2.3}$$

To apply Lemma 2.1, we first show that N is L-compact on $\overline{\Omega}$, where Ω is a bounded open subset of X. From (2.3), it follows that

$$QNx = \frac{1}{T}\int_0^T Nx(t)\,dt = \frac{1}{T}\int_0^T \Big[-\delta(t) + \sum_{i=1}^m p_i(t)e^{x(t-\tau_i(t))-x(t)-\gamma_i(t)e^{x(t-\tau_i(t))}}\Big]\,dt, \tag{2.4}$$

$$L_P^{-1}(I-Q)Nx = \int_0^t Nx(s)\,ds - \frac{t}{T}\int_0^T Nx(s)\,ds - \frac{1}{T}\int_0^T\int_0^t Nx(s)\,ds\,dt + \frac{1}{T}\int_0^T\int_0^t QNx(s)\,ds\,dt. \tag{2.5}$$

Obviously, QN and $L_P^{-1}(I-Q)N$ are continuous. It is not difficult to show that $L_P^{-1}(I-Q)N(\overline{\Omega})$ is compact for any open bounded set $\Omega \subset X$ by using the Arzelà–Ascoli theorem. Moreover, $QN(\overline{\Omega})$ is clearly bounded. Thus, N is L-compact on $\overline{\Omega}$ with any open bounded set $\Omega \subset X$.

Considering the operator equation $Lx = \lambda Nx$, $\lambda \in (0,1)$, we have

$$x'(t) = \lambda \Delta(x,t). \tag{2.6}$$

Assume that $x \in X$ is a solution of (2.6) for some $\lambda \in (0,1)$. Then

$$\int_0^T \Big|\sum_{i=1}^m p_i(t)e^{x(t-\tau_i(t))-x(t)-\gamma_i(t)e^{x(t-\tau_i(t))}}\Big|\,dt = \int_0^T\sum_{i=1}^m p_i(t)e^{x(t-\tau_i(t))-x(t)-\gamma_i(t)e^{x(t-\tau_i(t))}}\,dt = \int_0^T \delta(t)\,dt. \tag{2.7}$$

It follows from (2.6) and (2.7) that

$$\int_0^T |x'(t)|\,dt \le \lambda\int_0^T \Big|\sum_{i=1}^m p_i(t)e^{x(t-\tau_i(t))-x(t)-\gamma_i(t)e^{x(t-\tau_i(t))}}\Big|\,dt + \lambda\int_0^T |\delta(t)|\,dt < 2\int_0^T \delta(t)\,dt = A. \tag{2.8}$$

Since $x \in X$, there exist $\xi, \eta \in [0,T]$ such that

$$x(\xi) = \min_{t\in[0,T]} x(t), \qquad x(\eta) = \max_{t\in[0,T]} x(t). \tag{2.9}$$

It follows from (2.7) and (2.8) that

$$\frac{A}{2} = \int_0^T \delta(t)\,dt = \int_0^T \sum_{i=1}^m p_i(t) e^{x(t-\tau_i(t))-x(t)-\gamma_i(t)e^{x(t-\tau_i(t))}}\,dt$$
$$\geq e^{x(\xi)-x(\eta)-\gamma^+ e^{x(\eta)}} \sum_{i=1}^m \int_0^T p_i(t)\,dt = Be^{x(\xi)-x(\eta)-\gamma^+ e^{x(\eta)}},$$

and

$$\frac{A}{2} = \int_0^T \delta(t)\,dt = \int_0^T \sum_{i=1}^m p_i(t) e^{x(t-\tau_i(t))-x(t)-\gamma_i(t)e^{x(t-\tau_i(t))}}\,dt$$
$$\leq e^{x(\eta)-x(\xi)-\gamma^- e^{x(\xi)}} \sum_{i=1}^m \int_0^T p_i(t)\,dt = Be^{x(\eta)-x(\xi)-\gamma^- e^{x(\xi)}},$$

which implies that

$$x(\xi) \leq \ln\frac{A}{2B} + x(\eta) + \gamma^+ e^{x(\eta)},$$

and

$$x(\eta) \geq \ln\frac{A}{2B} + x(\xi) + \gamma^- e^{x(\xi)}.$$

Using (2.8) yields

$$x(t) \leq x(\xi) + \int_0^T |x'(t)|\,dt \leq \ln\frac{A}{2B} + x(\eta) + \gamma^+ e^{x(\eta)} + A,$$
$$x(t) \geq x(\eta) - \int_0^T |x'(t)|\,dt \geq \ln\frac{A}{2B} + x(\xi) + \gamma^- e^{x(\xi)} - A.$$

In particular,

$$x(\eta) \leq x(\xi) + \int_0^T |x'(t)|\,dt \leq \ln\frac{A}{2B} + x(\eta) + \gamma^+ e^{x(\eta)} + A,$$
$$x(\xi) \geq x(\eta) - \int_0^T |x'(t)|\,dt \geq \ln\frac{A}{2B} + x(\xi) + \gamma^- e^{x(\xi)} - A.$$

It follows that

$$x(\eta) \geq \ln\left(\frac{1}{\gamma^+}\left(\ln\frac{2B}{A} - A\right)\right), \qquad x(\xi) \leq \ln\left(\frac{1}{\gamma^-}\left(\ln\frac{2B}{A} + A\right)\right).$$

Again from (2.8), we have

$$x(t) \geq x(\eta) - \int_0^T |x'(t)|\,dt \geq \ln\left(\frac{1}{\gamma^+}\left(\ln\frac{2B}{A} - A\right)\right) - A := H_1, \tag{2.10}$$

$$x(t) \leq x(\xi) + \int_0^T |x'(t)|\,dt \leq \ln\left(\frac{1}{\gamma^-}\left(\ln\frac{2B}{A} + A\right)\right) + A := H_2. \tag{2.11}$$

Let $H > \max\{|H_1|, |H_2|\}$, and define $\Omega = \{x \in X : \|x\| < H\}$. Then (2.10) and (2.11) imply that there is no $\lambda \in (0,1)$ and $x \in \partial\Omega$ such that $Lx = \lambda Nx$.

If $x \in \partial\Omega \cap \operatorname{Ker} L = \partial\Omega \cap \mathbb{R}$, then $x = \pm H$. We have

$$QN(-H) > 0 \quad \text{and} \quad QN(H) < 0. \tag{2.12}$$

Indeed, if $QN(-H) \leq 0$, it follows from (2.4) that

$$\begin{aligned}\frac{A}{2} = \int_0^T \delta(t)\,dt &\geq \sum_{i=1}^m \int_0^T p_i(t) e^{-\gamma_i(t)e^{-H}}\,dt \\ &\geq e^{-\gamma^+ e^{-H}} \sum_{i=1}^m \int_0^T p_i(t)\,dt = Be^{-\gamma^+ e^{-H}},\end{aligned}$$

which implies

$$-H \geq \ln\left(\frac{1}{\gamma^+}\ln\frac{2B}{A}\right) > \ln\left(\frac{1}{\gamma^+}\left(\ln\frac{2B}{A} - A\right)\right) - A = H_1.$$

This is a contradiction and implies that $QN(-H) > 0$. Similarly, if $QN(H) \geq 0$, it follows from (2.4) that

$$\begin{aligned}\frac{A}{2} = \int_0^T \delta(t)\,dt &\leq \sum_{i=1}^m \int_0^T p_i(t) e^{-\gamma_i(t)e^{H}}\,dt \\ &\leq e^{-\gamma^- e^{H}} \sum_{i=1}^m \int_0^T p_i(t)\,dt = Be^{-\gamma^- e^{H}}.\end{aligned}$$

Consequently,

$$H \leq \ln\left(\frac{1}{\gamma^-}\ln\frac{2B}{A}\right) < \ln\left(\frac{1}{\gamma^+}\left(\ln\frac{2B}{A} + A\right)\right) + A = H_2,$$

a contradiction to the choice of H. Thus, $QN(H) < 0$.

Furthermore, define a continuous function $H(x,\mu)$ by setting

$$H(x,\mu) = -(1-\mu)x + \mu\frac{1}{T}\int_0^T \left[-\delta(t) + \sum_{i=1}^m p_i(t) e^{-\gamma_i(t)e^{x}}\right] dt.$$

It follows from (2.12) that $xH(x,\mu) \neq 0$ for all $x \in \partial\Omega \cap \operatorname{Ker} L$. Hence, using the homotopy invariance theorem, we obtain

$$\begin{aligned}
&\deg\{QN, \Omega \cap \operatorname{Ker} L, 0\} \\
&\quad = \deg\left\{\frac{1}{T}\int_0^T \left[-\delta(t) + \sum_{i=1}^m p_i(t)e^{-\gamma_i(t)e^x}\right] dt, \Omega \cap \operatorname{Ker} L, 0\right\} \\
&\quad = \deg\{-x, \Omega \cap \operatorname{Ker} L, 0\} \neq 0.
\end{aligned}$$

In view of all the discussion above, we conclude from Lemma 2.1 that Theorem 2.1 is proved.

COROLLARY 2.1. *Let* (2.1) *hold, and*

$$\ln\left(\frac{1}{\gamma^+}\left(\ln\frac{2B}{A} - A\right)\right) - A := H_1 \geq \ln\kappa. \tag{2.13}$$

If $N(t)$ is a positive T-periodic solution of (1.2), *then*

$$N(t) \geq \kappa \quad \text{for all } t \in \mathbb{R}. \tag{2.14}$$

Proof. Let $N(t)$ be a positive T-periodic solution of (1.2), and let $x(t) = \ln N(t)$. Then $x(t)$ is a T-periodic solution of (2.2). Applying techniques similar to the proof of Theorem 2.1, we can obtain

$$x(t) \geq \ln\left(\frac{1}{\gamma^+}\left(\ln\frac{2B}{A} - A\right)\right) - A := H_1 \geq \ln\kappa \quad \text{for all } t \in \mathbb{R},$$

which implies that

$$N(t) = e^{x(t)} \geq \kappa \quad \text{for all } t \in \mathbb{R}.$$

This completes the proof.

THEOREM 2.2. *Let* (2.1) *and* (2.13) *hold. Moreover, assume that*

$$\gamma^- = \min_{i \in \{1,\dots,m\}} \gamma_i^- \geq 1, \qquad \sum_{i=1}^m \frac{p_i^+}{\delta^-} < e^2. \tag{2.15}$$

Then equation (1.2) *has a unique positive T-periodic solution.*

Proof. Assume that $N_1(t)$ and $N_2(t)$ are two positive T-periodic solutions of (1.2). Set $y(t) = N_1(t) - N_2(t)$, where $t \in \mathbb{R}$. Then

$$\begin{aligned}
y'(t) = -\delta(t)y(t) + \sum_{i=1}^m p_i(t)[&N_1(t-\tau_i(t))e^{-\gamma_i(t)N_1(t-\tau_i(t))} \\
&- N_2(t-\tau_i(t))e^{-\gamma_i(t)N_2(t-\tau_i(t))}].
\end{aligned} \tag{2.16}$$

Define a continuous function $\Gamma(u)$ by setting

$$\Gamma(u) = -(\delta^- - u) + \sum_{i=1}^{m} p_i^+ \frac{1}{e^2} e^{ur}, \qquad u \in [0,1],\ r = \max_{i\in\{1,\dots,m\}} \tau_i^+. \tag{2.17}$$

Then, from (2.15), we have

$$\Gamma(0) = -\delta^- + \sum_{j=i}^{m} p_i^+ \frac{1}{e^2} = -\delta^- \frac{1}{e^2}\left(e^2 - \sum_{j=i}^{m} \frac{p_i^+}{\delta^-}\right) < 0,$$

which implies that there exist two constants $\eta > 0$ and $\lambda \in (0,1]$ such that

$$\Gamma(\lambda) = -(\delta^- - \lambda) + \sum_{i=1}^{m} p_i^+ \frac{1}{e^2} e^{\lambda r} < -\eta < 0. \tag{2.18}$$

We consider the Lyapunov functional

$$V(t) = |y(t)|e^{\lambda t}. \tag{2.19}$$

Calculating the upper right derivative of $V(t)$ along the solution $y(t)$ of (2.16), we have

$$\begin{aligned} D^+(V(t)) &\le -\delta(t)|y(t)|e^{\lambda t} + \sum_{i=1}^{m} p_i(t)|N_1(t-\tau_i(t))e^{-\gamma_i(t)N_1(t-\tau_i(t))} \\ &\qquad - N_2(t-\tau_i(t))e^{-\gamma_i(t)N_2(t-\tau_i(t))}|e^{\lambda t} + \lambda|y(t)|e^{\lambda t} \\ &= \Big[(\lambda - \delta(t))|y(t)| + \sum_{i=1}^{m} p_i(t)|N_1(t-\tau_i(t))e^{-\gamma_i(t)N_1(t-\tau_i(t))} \\ &\qquad - N_2(t-\tau_i(t))e^{-\gamma_i(t)N_2(t-\tau_i(t))}|\Big]e^{\lambda t} \end{aligned} \tag{2.20}$$

for all $t \in \mathbb{R}$. For any fixed $t_0 \in \mathbb{R}$, we claim that

$$\begin{aligned} V(t) &= |y(t)|e^{\lambda t} \\ &< e^{\lambda t_0}(\max_{t\in[0,T]} |N_1(t) - N_2(t)| + 1) =: M \qquad \text{for all } t > t_0. \end{aligned} \tag{2.21}$$

Otherwise, there exists a constant $t_* > t_0$ such that

$$V(t_*) = M \quad \text{and} \quad V(t) < M \quad \text{for all } t < t_*, \tag{2.22}$$

which implies that

$$V(t_*) - M = 0 \quad \text{and} \quad V(t) - M < 0 \quad \text{for all } t < t_*. \tag{2.23}$$

By (2.15) and Corollary 2.1, we get

$$\gamma^- N_i(t) \ge N_i(t) \ge \kappa \qquad \text{for all } t \in \mathbb{R},\ i = 1,2. \tag{2.24}$$

From (1.7), (2.20), (2.23), (2.24) and the inequality

$$
(2.25)\qquad |se^{-s} - te^{-t}| = \left|\frac{1-(s+\theta(t-s))}{e^{s+\theta(t-s)}}\right| |s-t| \\
\le \frac{1}{e^2}|s-t| \quad \text{where } s,t \in [\kappa,\infty), 0<\theta<1,
$$

we obtain

$$
\begin{aligned}
(2.26)\quad 0 &\le D^+(V(t_*) - M) = D^+(V(t_*)) \\
&\le \Big[(\lambda - \delta(t_*))|y(t_*)| \\
&\quad + \sum_{i=1}^{m} p_i(t_*)|N_1(t_* - \tau_i(t_*))e^{-\gamma_i(t_*)N_1(t_*-\tau_i(t_*))} \\
&\qquad - N_2(t_* - \tau_i(t_*))e^{-\gamma_i(t_*)N_2(t_*-\tau_i(t_*))}|\Big]e^{\lambda t_*} \\
&= \Big[(\lambda - \delta(t_*))|y(t_*)| \\
&\quad + \sum_{i=1}^{m} \frac{p_i(t_*)}{\gamma_i(t_*)}|\gamma_i(t_*)N_1(t_* - \tau_i(t_*))e^{-\gamma_i(t_*)N_1(t_*-\tau_i(t_*))} \\
&\qquad - \gamma_i(t_*)N_2(t_* - \tau_i(t_*))e^{-\gamma_i(t_*)N_2(t_*-\tau_i(t_*))}|\Big]e^{\lambda t_*} \\
&\le (\lambda - \delta(t_*))|y(t_*)|e^{\lambda t_*} \\
&\quad + \sum_{i=1}^{m} p_i(t_*)\frac{1}{e^2}|y(t_* - \tau_i(t_*))|e^{\lambda(t_*-\tau_i(t_*))}e^{\lambda\tau_i(t_*)} \\
&\le \Big[(\lambda - \delta^-) + \sum_{i=1}^{m} p_i^+\frac{1}{e^2}e^{\lambda r}\Big]M.
\end{aligned}
$$

Thus,

$$
0 \le (\lambda - \delta^-) + \sum_{i=1}^{m} p_i^+\frac{1}{e^2}e^{\lambda r},
$$

which contradicts (2.18). Hence, (2.21) holds. It follows that

$$
(2.27)\qquad |y(t)| < Me^{-\lambda t} \quad \text{for all } t > t_0.
$$

In view of (2.27) and the periodicity of $y(t)$, we have

$$
y(t) = N_1(t) - N_2(t) = 0 \quad \text{for all } t \in \mathbb{R}.
$$

This completes the proof.

3. An example. In this section we present an example to illustrate our results.

Example 3.1. Consider the Nicholson's blowflies model with multiple time-varying delays:

$$
\begin{aligned}
(3.1)\quad N'(t) = &-\frac{1}{1000000}(1.000001 + 0.000001\cos t)N(t) + \frac{e^{1.9999}}{4000000} \\
&\cdot (0.999999 + 0.000001\sin t)N(t-1-10^{-4}|\sin t|) \\
&\cdot e^{-N(t-1-10^{-4}|\sin t|)} + \frac{e^{1.9999}}{4000000}(0.999999 \\
&+ 0.000001\cos t)N(t-1-10^{-4}|\cos t|)e^{-N(t-1-10^{-4}|\cos t|)}.
\end{aligned}
$$

Obviously,

$$
\begin{aligned}
\delta(t) &= \frac{1}{1000000}(1.000001 + 0.000001\cos t), \\
p_1(t) &= \frac{e^{1.9999}}{4000000}(0.999999 + 0.000001\sin t), \\
p_2(t) &= \frac{e^{1.9999}}{4000000}(0.999999 + 0.000001\cos t), \\
\tau_1(t) &= 1 + 0.00001|\sin t|, \qquad \tau_2(t) = 1 + 0.00001|\cos t|, \\
\gamma_1(t) &= \gamma_2(t) = 1, \gamma_1^- = \gamma_2^- = \gamma_1^+ = \gamma_2^+ = 1, \\
A &= 2\int_0^{2\pi} \delta(t)\,dt = 4.000004\pi \cdot 10^{-6}, \\
B &= \int_0^{2\pi} \sum_{i=1}^{2} p_i(t)\,dt = 1.999998\pi e^{1.9999} \cdot 10^{-6}.
\end{aligned}
$$

Then

$$
\ln\frac{2B}{A} - A \approx 1.9999 > 0, \quad \ln\left(\frac{1}{\gamma^+}\left(\ln\frac{2B}{A} - A\right)\right) - A \approx 0.6931 > 0 > \ln\kappa,
$$

$$
\sum_{i=1}^{2} \frac{p_i^+}{\delta^-} = \frac{1}{2}e^{1.9999} < e^2.
$$

This implies that Nicholson's blowflies model (3.1) satisfies (2.1), (2.13) and (2.15). Hence, from Theorem 2.2, equation (3.1) has a unique positive 2π-periodic solution.

Remark 3.1. Since the delays of Nicholson's blowflies model (3.1) are time-varying, all the results in [SA] are invalid for equation (3.1). In [C, CL, LD], the authors only studied the local existence of positive periodic solution of Nicholson's blowflies model. Therefore, the results in [C, CL, LD] also cannot be applied to equation (3.1) to obtain the existence and uniqueness of positive 2π-periodic solutions. Moreover, in the present paper, we propose a totally new approach to proving the uniqueness of positive periodic solution of Nicholson's blowflies model. This implies that the results of this paper are essentially new.

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