Distribution of zeros and shared values of difference operators

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Abstract. We investigate the distribution of zeros and shared values of the difference operator on meromorphic functions. In particular, we show that if \( f \) is a transcendental meromorphic function of finite order with a small number of poles, \( c \) is a non-zero complex constant such that \( \Delta_c^k f \neq 0 \) for \( n \geq 2 \), and \( a \) is a small function with respect to \( f \), then \( f^n \Delta_c^k f \) equals \( a (\neq 0, \infty) \) at infinitely many points. Uniqueness of difference polynomials with the same \( 1 \)-points or fixed points is also proved.

1. Introduction and results. We apply the standard notation of value distribution theory: \( m(r, f) \), \( N(r, f) \), \( T(r, f) \), \( N(r, f) \) and \( S(r, f) \). Let \( f(z) \) be meromorphic in the plane and let \( c \) be a non-zero complex number. Define the differences of \( f(z) \) by

\[
\Delta_c f(z) = f(z+c) - f(z), \quad \Delta_c^{k+1} f(z) = \Delta_c(\Delta_c^k f(z)), \quad k = 1, 2, \ldots
\]

Value distribution of difference operators (1.1) on meromorphic functions has become a subject of great interest recently. Bergweiler and Langley [BL] investigated the distribution of zeros of \( \Delta_c^k f(z) \). Halburd and Korhonen [HK1] established a difference analogue of Nevanlinna’s second main theorem. For other results in this field, the reader is referred to [Z] and [ZK]. An analogue of the logarithmic derivative lemma, developed by Chiang and Feng [CF] and Halburd and Korhonen [HK2] independently, reads as follows.

**Theorem 1.1.** Let \( f \) be a meromorphic function of finite order and let \( c \) be a non-zero complex constant. Then

\[
m\left( r, \frac{f(z+c)}{f(z)} \right) + m\left( r, \frac{f(z)}{f(z+c)} \right) = S(r, f),
\]

where \( S(r, f) = o(T(r, f)) \) as \( r \to \infty \) outside of a possible exceptional set of finite logarithmic measure.

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Noting that
\[
\Delta^k_c f(z) = \sum_{j=0}^{k} C_j^k (-1)^{k-j} f(z + jc),
\]
it is easy to get

(1.2) \[ m \left( r, \frac{\Delta^k_c f}{f} \right) = S(r, f) \]

from Theorem 1.1 (see also [HK1, Lemma 2.3]).

Let \( f \) be a transcendental meromorphic function and let \( n \) be a positive integer. Hayman [H67] conjectured that \( f^n f' \) assumes every non-zero value \( a \in \mathbb{C} \) infinitely often. For results concerning this conjecture, see [BE, H59, M]. Laine and Yang [LY] proved that if \( f(z) \) is a transcendental entire function of finite order and \( n \geq 2 \), then \( f(z)^n f(z + c) \) assumes every non-zero value \( a \in \mathbb{C} \) infinitely often. Liu and Yang [LiY] considered the case of \( f(z)^n \Delta_c f(z) \). They proved

**Theorem 1.2.** Let \( f \) be a transcendental entire function of finite order and let \( c \) be a non-zero complex constant with \( \Delta_c f(z) \not\equiv 0 \). Then for \( n \geq 2 \) and any polynomial \( p(z) \not\equiv 0 \), \( f(z)^n \Delta_c f(z) - p(z) \) has infinitely many zeros.

We intend to study the distribution of zeros of \( f(z)^n \Delta_c f(z) - a(z) \), where \( a(z) (\not\equiv 0, \infty) \) is a small function with respect to \( f(z) \), i.e., \( T(r, a) = S(r, f) \). The following theorem is an extension of Theorem 1.2.

**Theorem 1.3.** Let \( f(z) \) be a transcendental meromorphic function of finite order \( \rho \), and let \( n, k \) be positive integers. Suppose that \( c \) is a non-zero complex number such that \( \Delta^k_c f(z) \not\equiv 0 \) and \( a(z) (\not\equiv 0, \infty) \) is a small function with respect to \( f(z) \). If \( n \geq 2 \) and the exponent of convergence of poles of \( f \) satisfies

\[
\lambda(1/f) := \limsup_{r \to \infty} \frac{\log N(r, f)}{\log r} < \rho,
\]

then \( f(z)^n \Delta^k_c f(z) - a(z) \) has infinitely many zeros, and

\[
N \left( r, \frac{1}{f^n \Delta^k_c f - a} \right) \geq T(r, f) + S(r, f).
\]

The following two counterexamples from [LiY] show that Theorem 1.3 is not true if \( n = 1 \) or the restriction on the order of \( f \) is removed.

**Example 1.4.** Let \( f(z) = z + e^z \), \( c = 2k\pi i \). Then
\[
f(z) \Delta_c f - cz = ce^z \not\equiv 0, \quad \forall z \in \mathbb{C}.
\]

**Example 1.5.** Let \( f(z) = ze^{ez} \), \( e^z = -n \). Then \( f \) is an entire function of infinite order and
finite order. Suppose that \( n \) \( \geq n \) has a finite number of zeros for any \( n \in \mathbb{N} \).

We have the following result for meromorphic functions in general.

**Theorem 1.6.** Let \( f(z) \) be a transcendental meromorphic function of finite order. Suppose that \( n, k, c \) and \( a(z) \) are as in Theorem 1.3. If \( n \geq 3k + 5 \), then \( f(z)^n \Delta_c^k f(z) - a(z) \) has infinitely many zeros, and

\[
N\left( r, \frac{1}{f^n \Delta_c^k f - a} \right) \geq T(r, f) + S(r, f).
\]

We now introduce the definition of sharing a common value: we say that functions \( f \) and \( g \) share a value \( a \) CM (counting multiplicities) if \( f = a \) and \( g = a \) have the same zeros with the same multiplicities (see, e.g., [H64, YY]).

Yang and Hua [YH] studied differential polynomials sharing a common value and obtained the following uniqueness theorem corresponding to Hayman’s conjecture.

**Theorem 1.7.** Let \( f \) and \( g \) be nonconstant entire functions, and let \( n \geq 6 \) be an integer. If \( f^n f' \) and \( g^n g' \) share 1 CM, then either \( f(z) = c_1 e^{cz} \) and \( g(z) = c_2 e^{-cz} \), where \( c_1, c_2 \) and \( c \) are constants satisfying \((c_1 c_2)^n + 1 c^2 = -1\), or \( f = t g \) for a constant \( t \) such that \( t^{n+1} = 1 \).

Fang and Qiu [FQ] obtained the following result concerning fixed points.

**Theorem 1.8.** Let \( f \) and \( g \) be nonconstant entire functions, and let \( n \geq 6 \) be a positive integer. If \( f^n f' \) and \( g^n g' \) share \( z \) CM, then either \( f(z) = c_1 e^{cz^2} \) and \( g(z) = c_2 e^{-cz^2} \), where \( c_1, c_2 \) and \( c \) are constants satisfying \( 4(c_1 c_2)^n + 1 c^2 = -1 \), or \( f = t g \) for a constant \( t \) such that \( t^{n+1} = 1 \).

In the present paper, we get analogous results on difference operators.

**Theorem 1.9.** Let \( f \) and \( g \) be nonconstant entire functions of finite order, and let \( n \geq 5 \) be an integer. Suppose that \( c \) is a non-zero complex constant such that \( \Delta_c f(z) \neq 0 \) and \( \Delta_c g(z) \neq 0 \). If \( f^n \Delta_c f \) and \( g^n \Delta_c g \) share 1 CM, and \( g(z + c) \) and \( g(z) \) share 0 CM, then

1. \( f(z) = c_1 e^{az} \) and \( g(z) = c_2 e^{-az} \), where \( c_1, c_2 \) and \( a \) are constants satisfying \((c_1 c_2)^n + 1 (e^{ac} + e^{-ac} - 2) = -1\); or
2. \( f = t g \), where \( t \) is a constant satisfying \( t^{n+1} = 1 \).
Theorem 1.10. Let $f$ and $g$ be nonconstant entire functions of finite order, and let $n \geq 5$ be an integer. Suppose that $c$ is a non-zero complex constant such that $\Delta_c f(z) \neq 0$ and $\Delta_c g(z) \neq 0$. If $f^n \Delta_c f$ and $g^n \Delta_c g$ share $z$ CM, and $g(z+c)$ and $g(z)$ share 0 CM, then $f = tg$, where $t$ is a constant satisfying $t^{n+1} = 1$.

2. Some preliminary results. In this section, we give some results needed in this paper. Chiang and Feng [CF] estimated the characteristic function and the counting function of $f(z+c)$, and got the following two results.

Lemma 2.1 ([CF] Theorem 2.1). Let $f$ be a meromorphic function of finite order $\rho$ and let $c$ be a non-zero complex constant. Then, for each $\varepsilon > 0$,

$$T(r, f(z+c)) = T(r, f) + O(r^{\rho - 1 + \varepsilon}) + O(\log r).$$

It is evident that $S(r, f(z+c)) = S(r, f)$ from Lemma 2.1.

Recall that $\lambda(1/f)$ denotes the exponent of convergence of poles of a meromorphic function $f$, defined in Theorem 1.3.

Lemma 2.2 ([CF] Theorem 2.2). Let $f$ be a meromorphic function with $\lambda(1/f)$ finite and let $c$ be a non-zero complex constant. Then, for each $\varepsilon > 0$,

$$N(r, f(z+c)) = N(r, f) + O(r^{\lambda(1/f) - 1 + \varepsilon}) + O(\log r).$$

The following result bases on Lemmas 2.1 and 2.2.

Lemma 2.3. Let $f(z)$ be a transcendental meromorphic function of finite order, and let $n$, $k$ be positive integers. Suppose that $c$ is a non-zero complex number such that $\Delta_c^k f(z) \neq 0$ and $a(z)$ ($\neq 0$, $\infty$) is a small function with respect to $f(z)$. Denote $F(z) = f(z)^n \Delta_c^k f(z)$. Then

$$n + 1)T(r, f) \leq T(r, f^{n+1}) = m(r, f^{n+1}) + \gamma(r, f^n)$$

$$\quad \quad \quad \quad \quad \quad \quad \quad \quad + \gamma(r, 1/(F-a)) - N(r, F)$$

$$\quad \quad \quad \quad \quad \quad \quad \quad \quad + N(r, f^n) + N(r, F) + S(r, f).$$

Proof. We deduce from the second main theorem that

$$n + 1)T(r, f) = T(r, f^{n+1}) = m(r, f^{n+1}) + N(r, f^{n+1})$$

$$\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad + N(r, f^{n+1}/F) + m(r, F) + N(r, f^{n+1})$$

$$\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad = m(r, f/\Delta_c^k f) + T(r, F) - N(r, F) + N(r, f^{n+1})$$

$$\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad + \gamma(r, 1/F) + \gamma(r, 1/(F-a)) + N(r, F) - N(r, F) + N(r, f^{n+1}).$$

Noting that

$$\gamma(r, 1/F) = \gamma(r, f/\Delta_c^k f) \leq \gamma(r, 1/F) + \gamma(r, f/\Delta_c^k f),$$

$$\gamma(r, 1/(F-a)) = \gamma(r, 1/(F-a)) = \gamma(r, 1/(F-a)),$$

$$\gamma(r, 1/(F-a)) = \gamma(r, 1/(F-a)) = \gamma(r, 1/(F-a)).$$
we see from (1.2) that
\[
(2.3) \quad m\left(r, \frac{f}{\Delta^k c f}\right) + N\left(r, \frac{1}{f}\right) \leq m\left(r, \frac{f}{\Delta^k c f}\right) + \overline{N}\left(r, \frac{1}{f}\right) + N\left(r, \frac{f}{\Delta^k c f}\right)
= T\left(r, \frac{f}{\Delta^k c f}\right) + \overline{N}\left(r, \frac{1}{f}\right)
= T\left(r, \frac{\Delta^k c f}{f}\right) + \overline{N}\left(r, \frac{1}{f}\right) + O(1)
= N\left(r, \frac{\Delta^k c f}{f}\right) + \overline{N}\left(r, \frac{1}{f}\right) + S(r, f).
\]

Applying (2.3) and (2.2), we obtain the assertion. ☐

When nonconstant meromorphic functions $F$, $G$ share at least one finite value CM, the following lemma plays a key role.

**Lemma 2.4 ([YH, Lemma 3]).** Let $F$ and $G$ be nonconstant meromorphic functions. If $F$ and $G$ share 1 CM, then one of the following three cases holds:

1. $\max\{T(r, F), T(r, G)\} \leq N_2(r, 1/F) + N_2(r, 1/G) + N_2(r, F) + N_2(r, G) + S(r, F) + S(r, G)$;
2. $FG = 1$;
3. $F = G$,

where $N_2(r, 1/F)$ denotes the counting function of zeros of $F$ such that simple zeros are counted once and multiple zeros twice.

### 3. Proofs

**Proof of Theorem 1.3.** For each $\varepsilon > 0$ we have $N(r, f) = O(r^{\lambda(1/f)+\varepsilon})$. Since $f$ is transcendental, it follows from Lemma 2.2 that
\[
(3.1) \quad N(r, f(z + c)) = N(r, f) + O(r^{\lambda(1/f)-1+\varepsilon}) + O(\log r) = O(r^{\lambda(1/f)+\varepsilon}).
\]
Thus $N(r, \Delta^k c f) = O(r^{\lambda(1/f)+\varepsilon})$. Denote $F(z) = f(z)^n \Delta^k c f(z)$. Lemma 2.3 gives
\[
(3.2) \quad (n + 1)T(r, f) \leq \overline{N}(r, 1/f) + N(r, \Delta^k c f/f) + \overline{N}(r, 1/(F - a)) - N(r, F) + N(r, f^{n+1}) + \overline{N}(r, F) + S(r, f)
\leq 2N(r, 1/f) + N(r, \Delta^k c f) + \overline{N}(r, 1/(F - a)) + N(r, f^{n+1}) + S(r, f)
\leq 2T(r, f) + \overline{N}(r, 1/(F - a)) + O(r^{\lambda(1/f)+\varepsilon}) + S(r, f).
\]
Since $\lambda(1/f) < \rho$, we can choose $\varepsilon$ small enough such that $\lambda(1/f) + 2\varepsilon < \rho$. Hence $O(r, \lambda(1/f) + \varepsilon) = S(r, f)$, and (3.2) yields
\[(n - 1)T(r, f) \leq \overline{N}(r, 1/(F - a)) + S(r, f).\]

**Proof of Theorem 1.6.** Define $F(z)$ as before. Using (1.2), we have
\[N(r, f^{n+1}) - N(r, F) = N(r, F f/\Delta_c^k f) - N(r, F) \leq N(r, f/\Delta_c^k f)\]
\[\leq T(r, \Delta_c^k f/f + O(1) \leq N(r, \Delta_c^k f/f + S(r, f)).\]

From this and (2.1), we deduce using Lemma 2.2 that
\[nT(r, f) \leq N(r, \Delta_c^k f/f + \overline{N}(r, 1/(F - a)) - N(r, F)\]
\[+ N(r, f^{n+1}) + \overline{N}(r, F + S(r, f)\]
\[\leq 2N(r, \Delta_c^k f/f + \overline{N}(r, F + \overline{N}(r, 1/(F - a)) + S(r, f)\]
\[\leq 3N(r, \Delta_c^k f/f + \overline{N}(r, F + \overline{N}(r, 1/(F - a)) + S(r, f)\]
\[\leq 3N(r, 1/f) + kN(r, f)) + \overline{N}(r, F + \overline{N}(r, 1/(F - a)) + S(r, f)\]
\[\leq (3k + 4)T(r, f) + \overline{N}(r, 1/(F - a)) + S(r, f),\]
which is
\[(n - 3k - 4)T(r, f) \leq \overline{N}(r, 1/(F - a)) + S(r, f).\]

The assertion follows as $n \geq 3k + 5$. ■

**Proof of Theorem 1.10.** Denote
\[(3.3) \quad F(z) = \frac{f(z)^n\Delta_c f(z)}{z}, \quad G(z) = \frac{g(z)^n\Delta_c g(z)}{z}.\]

Then $F$ and $G$ share 1 CM. Since $f$ is a transcendental entire function, we deduce from the definition of $F$ that $N_2(r, F) = O(\log r) = S(r, f)$ and
\[N_2(r, 1/F) \leq N_2(r, 1/(f^n \Delta_c f)) \leq N_2(r, 1/f^{n+1}) + N_2(r, f/\Delta_c f)\]
\[\leq 2\overline{N}(r, 1/f) + T(r, \Delta_c f/f) + O(1)\]
\[\leq 2\overline{N}(r, 1/f) + N(r, \Delta_c f/f) + S(r, f)\]
\[\leq 2\overline{N}(r, 1/f) + N(r, 1/f) + S(r, f) \leq 3T(r, f) + S(r, f).\]

Thus
\[(3.4) \quad N_2(r, 1/F) + N_2(r, F) \leq 3T(r, f) + S(r, f).\]

Similarly, we get $N_2(r, G) = S(r, g)$ and
\[N_2(r, 1/G) \leq 2\overline{N}(r, 1/g) + N(r, \Delta_c g/g) + S(r, g).\]

Noting that $g(z + c)$ and $g(z)$ share 0 CM, we obtain $N(r, \Delta_c g/g) = 0$ and the last inequality gives
\[N_2(r, 1/G) \leq 2T(r, g) + S(r, g).\]

Thus
\[(3.5) \quad N_2(r, 1/G) + N_2(r, G) \leq 2T(r, g) + S(r, g).\]
Assume that case (1) of Lemma 2.4 holds. Using (3.4) and (3.5), we have
(3.6) \[ \max\{T(r, F), T(r, G)\} \leq 3T(r, f) + 2T(r, g) + S(r, f) + S(r, g). \]
On the other hand, it follows from Theorem 1.1 that
(3.7) \[ (n + 1)T(r, f) = T(r, f^{n+1}) = m(r, f^{n+1}) \leq m(r, f^{n+1}/F) + m(r, F) = m(r, zf/\Delta_c f) + T(r, F) \leq m(r, f/\Delta_c f) + T(r, F) + O(\log r) \leq N(r, \Delta_c f/f) + T(r, F) + S(r, f) \leq T(r, f) + T(r, F) + S(r, f), \]
which means
\[ (3.8) \quad T(r, F) \geq nT(r, f) + S(r, f). \]
By the same reasoning, it follows from \(N(r, \Delta_c g/g) = 0\) that
\[ (3.9) \quad T(r, G) \geq (n + 1)T(r, g) + S(r, g). \]
Combining (3.7), (3.8) with (3.6), we conclude that
(3.10) \[ (n - 3)T(r, f) \leq 2T(r, g) + S(r, f) + S(r, g) \]
and
(3.11) \[ (n - 1)T(r, g) \leq 3T(r, f) + S(r, f) + S(r, g). \]
The last two inequalities yield an immediate contradiction, as \(n \geq 5\). Hence \(F(z) \cdot G(z) \equiv 1\) or \(F(z) \equiv G(z)\) by Lemma 2.4. We discuss the two cases separately.

**Case 1.** Suppose that \(F(z) \cdot G(z) \equiv 1\). Then
(3.12) \[ f^n(z)(f(z + c) - f(z))g^n(z)(g(z + c) - g(z)) = z^2. \]
Notice, that for \(n \geq 5\), zero is a Picard exceptional value of both \(f\) and \(g\) from (3.9). Then \(f(z) = e^{Q(z)}\) and \(g(z) = e^{P(z)}\), where \(Q(z)\) and \(P(z)\) are polynomials. It follows from (3.9) that
\[ (e^{Q(z+c)-Q(z)}-1)(e^{P(z+c)-P(z)}-1) = z^2 e^{-(n+1)[Q(z)+P(z)]}. \]
Denote \(\phi(z) = e^{Q(z+c)-Q(z)}\). Then \(\phi(z) \neq 0, \infty\) for any \(z \in \mathbb{C}\). If \(\phi(z) \not\equiv\) const, then it is a transcendental entire function. We infer from the above equation and the second main theorem that
\[ T(r, \phi) \leq N(r, \phi) + N(r, 1/\phi) + N(r, 1/(\phi - 1)) + S(r, \phi) = N(r, 1/z^2) + S(r, \phi) = O(\log r) + S(r, \phi) = S(r, \phi), \]
which is a contradiction. Therefore, \(\phi(z) \equiv\) const. Similarly, \(e^{P(z+c)-P(z)} \equiv\) const. It follows that \(z^2 e^{-(n+1)[Q(z)+P(z)]} \equiv\) const, which is impossible.

**Case 2.** Suppose that \(F(z) \equiv G(z)\). Then
(3.13) \[ f^n(z)(f(z + c) - f(z)) = g^n(z)(g(z + c) - g(z)). \]
Let $h(z) = f(z)/g(z)$. We deduce that

\[(h^{n+1}(z) - 1)g(z) = (h^n(z)h(z + c) - 1)g(z + c).\]  

If $h(z + c) \equiv h(z)$, then $(h^{n+1} - 1)\Delta_c g = 0$ and $h(z)^{n+1} \equiv 1$. Thus $h(z)$ is a constant, say $t$, satisfying $t^{n+1} = 1$.

Assume now that $h(z + c) \not\equiv h(z)$. Suppose that there exists a point $z_0$ such that $h(z_0)^{n+1} = 1$. Then $h(z_0)^n h(z_0 + c) = 1$ from (3.10) since $g(z)$ and $g(z + c)$ share 0 CM. Hence $h(z_0) = h(z_0 + c)$ and

$$N(r, 1/(h^{n+1} - 1)) \leq N(r, 1/(h(z + c) - h)) \leq 2T(r, h) + S(r, h)$$

by Lemma 2.1. In the above inequality, we apply the second main theorem to $h^{n+1}$, resulting in

$$T(r, h^{n+1}) \leq N(r, h^{n+1}) + N(r, 1/h^{n+1}) + N(r, 1/(h^{n+1} - 1)) + S(r, h) \leq 4T(r, h) + S(r, h).$$

Then

$$(n + 1)T(r, h) \leq 4T(r, h) + S(r, h),$$

which means $h$ is a constant, because $n \geq 5$. Let $h = t$. Making use of (3.10) again, we obtain $t^{n+1} = 1$. □

**Proof of Theorem 1.9.** Denote

$$F(z) = f(z)^n \Delta_c f(z), \quad G(z) = g(z)^n \Delta_c g(z).$$

Then $F$ and $G$ share 1 CM. By the same arguments as in the proof of Theorem 1.10, we have $F(z) \cdot G(z) \equiv 1$ or $F(z) \equiv G(z)$ by Lemma 2.4. If $F(z) \equiv G(z)$, we obtain the second assertion by the same reasoning as in Case 2 in the proof of Theorem 1.10.

It thus remains to consider the case $F(z) \cdot G(z) \equiv 1$. Then

\[(3.11) \quad f^n(z)(f(z + c) - f(z))g^n(z)(g(z + c) - g(z)) = 1.\]

Case 1 in the proof of Theorem 1.10 gives us $f(z) = e^{Q(z)}$ and $g(z) = e^{P(z)}$, where $Q(z)$ and $P(z)$ are polynomials. We conclude from (3.11) that $e^{Q(z+c) - Q(z)} - 1$ is never zero. Denote $H(z) = e^{Q(z+c) - Q(z)}$. Then $H(z) \not\equiv 0, 1, \infty$ for any $z \in \mathbb{C}$. By Picard’s theorem, $H$ is a constant, so $\deg Q = 1$. Similarly, $\deg P = 1$. Assume now that

$$f(z) = c_1 e^{az}, \quad g(z) = c_2 e^{bz},$$

where $a, b, c_1$ and $c_2$ are non-zero constants. Applying (3.11) again, we get $a = -b$ and $(c_1c_2)^{n+1}(e^{az} + e^{-az} - 2) = -1$. The first assertion follows. This completes the proof of Theorem 1.9. □

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