The pluricomplex Green function
on some regular pseudoconvex domains

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Abstract. Let $D$ be a smooth bounded pseudoconvex domain in $\mathbb{C}^n$ of finite type. We prove an estimate on the pluricomplex Green function $\mathcal{G}_D(z, w)$ of $D$ that gives quantitative information on how fast the Green function vanishes if the pole $w$ approaches the boundary. Also the Hölder continuity of the Green function is discussed.

1. Introduction. Let $D$ be a bounded pseudoconvex domain in $\mathbb{C}^n$ with a smooth boundary. We will investigate the behavior of the pluricomplex Green function $\mathcal{G}_D(\cdot, w)$, $w \in D$, of $D$ when $w$ tends to the boundary.

This function is defined by

$$\mathcal{G}_D(\cdot, w) := \sup \{ u(z) \mid u \in P(w; D) \},$$

where $P(w; D)$ denotes the class of all negative plurisubharmonic functions on $D$ such that $u - \log |\cdot - w|$ is bounded from above near $w$. It has been introduced by Klimek [Kli], and later in hyperconvex domains in general complex manifolds by Demailly [Dem]. In both papers fundamental properties of $\mathcal{G}_D$ were proved (in particular its relationship to the Monge–Ampère operator was clarified in [Dem]).

The fact that $\mathcal{G}_D$ has a logarithmic pole at $w$ makes it an important tool in applications of real methods in complex analysis, in particular those that are based upon the $L^2$-theory for the $\bar{\partial}$-operator with plurisubharmonic weight functions (see [Hör], [OhTă]). We need to know, however, how $\mathcal{G}_D(\cdot, w)$ behaves when $w$ tends to the boundary. First results in this context were obtained in [CCW], [He], and [DiHe] (for quantitative results in special cases see [Car], [Che]).

For a domain $D \subseteq \mathbb{C}^n$ we denote by $\delta_D$ the boundary-distance function. Our main result is

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Theorem 1.1. Let $D \subset \subset \mathbb{C}^n$ be a smooth bounded domain, and let $w_0 \in \partial D$. Assume that there exist an open neighborhood $U_1 \ni w_0$, constants $C > 0$ and $0 < \varepsilon \leq 1/2$, and a $C^2$-smooth plurisubharmonic function $\Phi : D \cap U_1 \to (-1, 0)$ such that:

(a) The function $z \mapsto \Phi(z) - C^{-1}|z|^2$ is plurisubharmonic.
(b) One has $\Phi(z) \geq -C\delta_D(z)^{2\varepsilon}$ for all $z \in D \cap U_1$.

Then there exists a constant $C > 0$, and a neighborhood $U_2 \subset \subset U_1$ of $w_0$, such that

\begin{equation}
|G_D(z, w)| \leq C M(z, w)^{1-1/n} \log \left(1 + C \left(\frac{\delta_D(w)^\varepsilon}{|z - w|}\right)^{1/n}\right)
\end{equation}

and

\begin{equation}
|G_D(z, w)| \leq C M(z, w)^{1-1/n} \log \left(1 + C \left(\frac{\delta_D(z)^\varepsilon \delta_D(w)^\varepsilon}{|z - w|^2}\right)^{1/n}\right),
\end{equation}

where

\[ M(z, w) := \log \log \left(1 + C \frac{\delta_D(w)^\varepsilon}{|z - w|}\right) + 1 + C \log \frac{R_D}{|z - w|} \]

for all $z, w \in D \cap U_2$. Here we denote by $R_D$ the diameter of $D$.

In [Car] the case of strongly pseudoconvex domains was treated, where one can find a plurisubharmonic function $\psi$ with properties (a) and (b) for $\varepsilon = 1/2$. For a convex domain of finite type an estimate for $G_D$ was established in [Che] that implies (1.2) without the factor $M(z, w)$.

The methods from [Car], [Che] do not carry over to our case since the Green function is not symmetric in general; whether or not holomorphic peak functions are available under the above comparatively weak hypotheses is also an open question.

As a corollary we obtain from Theorem 1.1

Theorem 1.2. Let $D \subset \subset \mathbb{C}^n$ be a smooth bounded domain, and let $w_0 \in \partial D$ be such that there exist an open neighborhood $U_1 \ni w_0$, constants $C > 0$ and $0 < \varepsilon \leq 1/2$, and a $C^2$-smooth plurisubharmonic function $\Phi : D \cap U_1 \to (-1, 0)$ with properties (a) and (b). Then we can choose an open neighborhood $U_2 \subset \subset U_1$ of $w_0$ such that for any $w \in D \cap U_2$ the sublevel set $\{G_D(\cdot, w) < -1\}$ is contained in a ball about $w$ of radius \(\leq C'\delta_D(w)^\varepsilon \log^n \frac{1}{\delta_D(w)}\). In particular, the Bergman metric $B_D$ of $D$ grows at least like

\[ B_D(w; X) \geq C_2 \frac{|X|}{\delta_D(w)^\varepsilon \log^n \frac{1}{\delta_D(w)}} \]

for all $w \in D \cap U_2$ and $X \in \mathbb{C}^n$. 
The first assertion is clear. The second, concerning the growth order of the Bergman metric, follows from Proposition 4.1 from [DiHe].

This improves the estimate of Theorem 1.13 from [DiHe] insofar as the points $w$ are not restricted to nontangential approach to the boundary point $w^0$ and the growth is up to a log-term exactly of order $\varepsilon$ and not only $\varepsilon - t$ (with arbitrary $0 < t \ll 1$).

In [Blo2] and [NPT] the question of Hölder continuity of the Green function was treated for a special class of domains. With the methods applied in those papers we will show (1.1).

As a by-product we further obtain

**Theorem 1.3.** Suppose $D$ is as in Theorem 1.1 Then there is a constant $C > 0$ such that

$$|G_D(z'', w) - G_D(z', w)| \leq C \frac{|z' - z''|^{\varepsilon^2/3n}}{M(z', z'', w)^{(1+\varepsilon)/n}} \log \frac{R_D}{M(z', z'', w)}$$

for any $z', z'' \in D \setminus \{w\}$, where

$$M(z', z'', w) = \min\{|z' - w|, |z'' - w|\}.$$

Later we will consider pseudoconvex domains that are uniformly extendable in a pseudoconvex way of some finite order $N \geq 2$. They belong to the class of pseudoconvex domains to which the above results apply.

The notion of pseudoconvex extendability is explained in the following

**Definition 1.4 (cf. [DiHe Def. 1.10]).** Let $D \subset\subset \mathbb{C}^n$ be pseudoconvex and smoothly bounded. We call $D$ uniformly extendable of order $N$ in a pseudoconvex way near a point $w^0 \in \partial D$ if there exist an open neighborhood $U' \ni w^0$, a constant $C_1 > 0$ and a $C^2$-smooth function $\psi : U' \times U' \to \mathbb{R}$ such that:

(i) The open set $\{\psi(q, \cdot) < 0\} \cap U'$ is pseudoconvex and the surface $\{\psi(q, \cdot) = 0\} \cap U'$ is smooth and passes through $q$ when $q \in \partial D \cap U'$.

(ii) For $x \in U'$, $q \in U' \cap \partial D$ we have the estimate

$$C_1(r(x) - |x - q|) \leq \psi(q, x) \leq r(x) - \frac{1}{C_1} |x - q|^N.$$

In [DiFo] it was shown that real-analytically bounded pseudoconvex domains have this property. This result was extended later in [Cho] to the larger class of smooth bounded pseudoconvex domains that are of finite type in the sense of [DA]. We will prove

**Lemma 1.5.** Assume that the domain $D \subset\subset \mathbb{C}^n$ is uniformly extendable of order $N$ in a pseudoconvex way near a point $w^0 \in \partial D$. Then there exist an open neighborhood $U_1$ of $w^0$, a continuous plurisubharmonic function $\Phi : D \cap U_1 \to \mathbb{R}$ and constants $C_1, c_1 > 0$ such that:
(i) On \( D \cap U_1 \) we have \(-C_1 \delta_D^{2/N} \leq \Phi < 0\).
(ii) The function \( z \mapsto \Phi(z) - c_1 |z|^2 \) is plurisubharmonic on \( D \cap U_1 \).

Hence Theorems 1.1 and 1.2 apply with \( \varepsilon = 1/N \).

In conjunction with a result of [Cho] one obtains the following:

**Lemma 1.6.** Let \( D \subset \subset \mathbb{C}^n \) be a smooth bounded domain and \( w_0 \in \partial D \) a point such that there exist an open neighborhood \( U_1 \ni w_0 \), constants \( C > 0 \) and \( 0 < \varepsilon \leq 1/2 \), and a family \((\lambda_\delta)_0 < \delta < \delta_0\) of plurisubharmonic functions on \( U_1 \) satisfying:

(a) For all \( \delta \in (0, \delta_0) \) one has \( 0 \leq \lambda_\delta \leq 1 \).
(b) On the strip \( S_\delta := \{ z \in D \cap U_1 | \delta_D(z) < \delta \} \) the function \( z \mapsto \lambda_\delta(z) - \delta^{-2\varepsilon}|z|^2 \) is still plurisubharmonic.
(c) For any derivative \( D\lambda_\delta \) of \( \lambda_\delta \) of order \( k \leq 2 \) one has \( |D\lambda_\delta| \leq C\delta^{-k} \).

Then \( D \) is uniformly extendable of order \( 1/\varepsilon \) near \( w_0 \). In particular Theorems 1.1 and 1.2 apply.

**2. Estimating the Green function in terms of the boundary distance of its first argument.** Our plan is to estimate the Green function \( \mathcal{G}_D(P,Q) \) in terms of \( \delta_D(P) \) and \( |P-Q| \), and then to compare \( \mathcal{G}_D(P,Q) \) with \( \mathcal{G}_D(Q,P) \).

**Proposition 2.1.** Let \( D \subset \subset \mathbb{C}^n \) satisfy the hypotheses of Theorem 1.1 near \( w^0 \in \partial D \). Then there exist a constant \( C_5 > 0 \) and a radius \( R_1 > 0 \) such that for any \( P \in D \cap B(w^0, R_1) \) and \( Q \in D \) one has

\[
|\mathcal{G}_D(P,Q)| \leq \frac{1}{2} \log \left( 1 + C_5 \frac{\delta_D(P)^{2\varepsilon}}{|P-Q|^2} \right).
\]

**Proof.** Let us take radii \( R_1 < \tilde{R}_1 < R_2 \) such that \( B(w_0, 2R_2) \subset U_1 \) and \( R_2 \geq 5\tilde{R}_1 \). Furthermore, we may assume that each \( x \in B(w^0, 3R_2/2) \) has an orthogonal projection \( x^* \in \partial D \cap B(w^0, 3R_2) \). We may certainly suppose that \( \delta_D(P) < R_1/2 \). Let us consider two cases:

**Case I:** \( |Q - w^0| < 3R_1 \). On \( U_1 \) the function \( \Phi_1(z) := \Phi(z) - c_1 |z - Q|^2 - c_1 |z - P^*|^2 \), where \( c_1 < \frac{1}{2C} \), is negative and plurisubharmonic. Also we have

\[
\frac{-\Phi_1(z)}{c_1 |z - Q|^2} \geq 1 + \frac{|z - P^*|^2}{|z - Q|^2}.
\]

But for \( z \in \partial B(w^0, R_2) \) we have, for \( P \in B(w^0, R_1) \),

\[
|z - P^*| \geq |z - w^0| - |w^0 - P| - |P - P^*| \geq R_2 - \frac{3}{2} R_1 \geq \frac{7}{10} R_2
\]

and

\[
|z - Q| \leq |z - w^0| + |w^0 - Q| \leq R_2 + 3R_1 \leq \frac{8}{5} R_2.
\]
Therefore,

\[-\Phi_1(z) \geq \frac{1}{c_1 |z - Q|^2} \geq 1 + \left( \frac{7}{16} \right)^2\]

and hence

\[\frac{1}{2} \log \frac{c_1 |z - Q|^2}{-\Phi_1(z)} \leq -c_3 := -\frac{1}{2} \log \left( 1 + \left( \frac{7}{16} \right)^2 \right).

The function

\[
\Phi_2(z) := \begin{cases} 
\max \left\{ \frac{1}{2} \log \frac{c_1 |z - Q|^2}{-\Phi_1(z)}, -c_3 \right\} & \text{if } z \in B(w^0, R_2) \cap D, \\
-c_3 & \text{if } z \in D \setminus (B(w^0, R_2) \cap D),
\end{cases}
\]

now becomes plurisubharmonic on \( D \) and thus it is a good candidate for \( \mathcal{G}_D(P, Q) \). Since \( |P - w^0| < R_1 < R_2 \), we obtain

\[|\mathcal{G}_D(P, Q)| \leq -\Phi_2(P) \leq \frac{1}{2} \log \frac{-\Phi_1(P)}{c_1 |P - Q|^2} \]
\[= \frac{1}{2} \log \frac{-\Phi(P) + c_1 |P - Q|^2 + c_1 |P - P^*|^2}{c_1 |P - Q|^2} \]
\[\leq \frac{1}{2} \log \left( 1 + c_2 \frac{\delta_D(P)^{2\varepsilon}}{|P - Q|^2} \right)\]

with some positive constant \( c_2 \), as desired.

**Case II: \( |Q - w^0| \geq 3R_1 \).** Now we put

\[\Phi_3(z) := \Phi(z) - c_1 |z - P^*|^2\]

in \( D \cap U_1 \). This function is plurisubharmonic and if \( |z - P^*| = R_1/2 \) we obtain \( \Phi_3(z) \leq -c_1 R_1^2/4 \). Thus the function

\[\Phi_4(z) = \begin{cases} 
\max \{ \Phi_3(z), -c_1 R_1^2/4 \} & \text{if } z \in B(P^*, R_1/2) \cap D, \\
-c_1 R_1^2/4 & \text{if } z \in D \setminus (B(P^*, R_1/2) \cap D),
\end{cases}\]

becomes well-defined and plurisubharmonic on \( D \). Next we define an appropriate candidate for \( \mathcal{G}_D(\cdot, Q) \). Let

\[\Phi_5(z) = \begin{cases} 
\max \left\{ C_7 \Phi_4(z), \log \frac{|z - Q|}{R_D} \right\} & \text{if } z \in D \setminus (B(Q, R_1/2) \cap D), \\
\log \frac{|z - Q|}{R_D} & \text{if } z \in B(Q, R_1/2) \cap D,
\end{cases}\]

where \( C_7 > 0 \) is chosen so large that

\[C_7 \Phi_4(z) \leq \log \frac{R_1}{2R_D} \quad \text{for } z \in D \cap \partial B(Q, R_1/2).\]
Note that this is possible, since for such points \( z \) one has
\[
|z - P^*| \geq |Q - P^*| - |z - Q| \geq |Q - w^0| - |w^0 - P^*| - |z - Q|
\]
\[
\geq |Q - w^0| - |w^0 - P - \delta_D(P) - |z - Q| \geq R_1,
\]
hence \( \Phi_4(z) = -c_1 R_1^2 / 4 \).

We find that
\[
|G_D(P, Q)| \leq -\Phi_5(P) \leq -c_7 \Phi(P) = -c_7 C_7|P - P^*|^2
\]
\[
\leq C_8 \delta_D(P)^{2\epsilon},
\]
because in our situation we have
\[
|P - Q| \geq |Q - w_0| - |P - w_0| \geq 2R_1.
\]

This implies also
\[
\frac{\delta_D(P)^{\epsilon}}{|P - Q|} \leq \frac{R_2}{2R_1}.
\]

The function \( V(t) := \frac{1}{t} \log(1 + t) \) is decreasing on \((0, \infty)\). This yields, if we choose \( C_5 \) so large that
\[
\frac{2R_2}{R_2^{2+2\epsilon}} \log \left( 1 + C_5^2 \frac{R_2^{2\epsilon}}{4R_1^2} \right) \geq C_8,
\]
the estimate
\[
\frac{1}{2} \log \left( 1 + C_5^2 \frac{\delta_D(P)^{2\epsilon}}{|P - Q|^2} \right) = \frac{1}{2} C_5^2 \frac{\delta_D(P)^{2\epsilon}}{|P - Q|^2} V \left( C_5^2 \frac{\delta_D(P)^{2\epsilon}}{|P - Q|^2} \right)
\]
\[
\geq \frac{1}{2} C_5^2 \frac{\delta_D(P)^{2\epsilon}}{|P - Q|^2} V \left( C_5^2 \frac{R_2^{2\epsilon}}{4R_1^2} \right)
\]
\[
\geq \frac{1}{2R_2^2} C_5^2 V \left( C_5^2 \frac{R_2^{2\epsilon}}{4R_1^2} \right) \delta_D(P)^{2\epsilon}
\]
\[
= \frac{2R_2^2}{R_2^{2+2\epsilon}} \log \left( 1 + C_5^2 \frac{R_2^{2\epsilon}}{4R_1^2} \right) \delta_D(P)^{2\epsilon}
\]
\[
\geq C_8 \delta_D(P)^{2\epsilon} \geq |G_D(P, Q)|.
\]

The proposition is proved. \( \blacksquare \)

3. A first Hölder estimate for the Green function. We adopt the methods from \cite{Blo2} and \cite{NPT}. Let \( D \) be a bounded hyperconvex domain in \( \mathbb{C}^n \). Let \( w \in D \) be fixed and \( r = \frac{2}{3}\delta_D(w) \). We consider for \( r > \eta > 0 \) the family
\[
\mathcal{P}_\eta := \{ v \mid v \text{ plurisubharmonic on } D, v \leq 0 \text{ on } D, v \leq \log(\eta/r) \text{ on } \overline{B}(w, \eta) \}
\]
and its upper envelope
\[
u^n(z) = \sup \{ v(z) \mid v \in \mathcal{P}_\eta \}.
\]

Then (for details see \cite{Kli2}, Sec. 4.5) we have:
(a) \( u^n \) is continuous, and \( u^n(z) \to 0 \) as \( z \) tends to a boundary point of \( D \).

(b) The function \( u^n \) is maximal plurisubharmonic outside \( \overline{B}(w, \eta) \).

(c) We have \( u^n = \log(\eta/r) \) on \( \overline{B}(w, \eta) \) and \( u^n(z) \leq \log(\max\{|z-w|, \eta\}/r) \) on \( D \).

(d) For \( \eta_1 \leq \eta_2 \) we have \( u^{\eta_1} \leq u^{\eta_2} \). Further \( \lim_{\eta \searrow 0} u^n(z) = G_D(z, w) \) on \( D \).

We recall from \([\text{NPT}]\) the following construction, associated with two given different points \( P, w \in D \). Let \( D \) denote the unit disc in the plane. We choose a holomorphic function \( F_{P,w} \) on \( D \) with values in \( D \) such that \( F_{P,w}(w) = 0 \) and \( \arctgh|F_{P,w}(P)| \) is equal to the Carathéodory distance \( c_D(P, w) \) between \( P \) and \( w \). Then we put, for \( h \in \mathbb{C}^n, \ z \in D \),

\[
H_{P,w,h}(z) := z + \frac{F_{P,w}(z)}{F_{P,w}(P)} h.
\]

Since \( tgh c_D(P, Q) \geq |P - Q|/R_D \) for \( P, Q \in D \), we obtain

\[
|H_{P,w,h}(z) - z| \leq R_D |F_{P,w}(z)| \frac{|h|}{|P - w|}
\]

for \( z \in D \). If \( |z - w| < \delta_D(w) \), then we get

\[
|F_{P,w}(z)| \leq tgh c_D(z, w) \leq \frac{|z - w|}{\delta_D(w)},
\]

and from (3.1) we see that

\[
|H_{P,w,h}(z) - z| \leq R_D \frac{|z - w|}{|P - w|} \frac{|h|}{\delta_D(w)}.
\]

We will make use of this later.

**Lemma 3.1.** Let \( D \) be a pseudoconvex domain as in Theorem 1.1. Then, with some constant \( C_1 > 0 \), for \( z', z'' \in D \setminus \{w\} \) we have

\[
|G_D(z', w) - G_D(z'', w)| \leq \log \left( 1 + C_1 \frac{R_D^\varepsilon}{\delta_D(w)} \frac{|z' - z''|^\varepsilon}{M(z', z'', w)^\varepsilon} \right)
\]

provided that

\[
|z' - z''| \leq \frac{\delta_D(w)}{8R_D} M(z', z'', w),
\]

where we write \( M(z', z'', w) = \min\{|z' - w|, |z'' - w|\} \).

**Proof.** We follow an idea from \([\text{NPT}]\). Let \( h := z'' - z' \), and consider the domain

\[
D_1 := \{ z \in D \mid H_{z',w,h}(z) \in D \}.
\]

Then \( \overline{B}(w, \eta) \subset D_1 \) for small enough \( \eta \), since \( w \in D_1 \).
If $z \in D$ and $H_{z',w,h}(z) \in \partial D$, we have
\begin{equation}
\delta_D(z) \leq |H_{z',w,h}(z) - z| \leq \frac{R_D}{|z' - w|}|h| \tag{3.4}
\end{equation}
and
\begin{equation}
|z - w| \geq |H_{z',w,h}(z) - w| - |H_{z',w,h}(z) - z| \\
\geq \delta_D(w) - \frac{R_D}{|z' - w|}|h| \geq \frac{1}{2}\delta_D(w), \tag{3.5}
\end{equation}
using (3.3). This implies (together with Prop. 2.1)
\[ u_\eta(z) \geq G_{\hat{D}}(z,w) \geq -\log \left( 1 + C_5 \frac{\delta_D(z)^c}{|z - w|} \right) \]
\[ \geq -\log \left( 1 + C_5 \frac{R_D^c}{|z - w|} \frac{|h|^c}{|z' - w|^c} \right) \text{ by (3.4)} \]
\[ \geq -\log \left( 1 + 2C_5 \frac{R_D^c}{\delta_D(w)} \frac{|h|^c}{|z' - w|^c} \right) \text{ by (3.5)}. \]
For $z \in \partial D$ we even have $u_\eta(z) = 0$. In each case we see that the last displayed estimate holds for any $z \in \partial D_1$. In particular,
\[ u_\eta(H_{z',w,h}(z)) \leq 0 \leq u_\eta(z) + \log \left( 1 + 2C_5 \frac{R_D^c}{\delta_D(w)} \frac{|h|^c}{|z' - w|^c} \right) \]
on $\partial D_1$. We want to prove this estimate also for $z \in \partial B(w,\eta)$.

For this purpose we take an arbitrary $z \in D$ with $|z - w| = \eta$. Then, by (3.2),
\[ |H_{z',w,h}(z) - w| \leq |z - w| + \frac{R_D}{\delta_D(w)} \frac{|z - w|}{|z' - w|}|h| \]
\[ = \left( 1 + \frac{R_D}{\delta_D(w)} |h| \frac{|z' - w|}{|z' - w|^c} \right) \eta. \]
This gives
\[ u_\eta(H_{z',w,h}(z)) \leq \log \frac{\max\{|H_{z',w,h}(z) - w|, \eta\}}{r} \]
\[ \leq \log(\eta/r) + \log \left( 1 + \frac{R_D}{\delta_D(w)} \frac{|h|}{|z' - w|} \right) \]
\[ \leq u_\eta(z) + \log \left( 1 + 2C_5 \frac{R_D^c}{\delta_D(w)} \frac{|h|^c}{|z' - w|^c} \right), \text{ by (3.3)}. \]
Since $u_\eta$ is maximal on $D_1 \setminus \overline{B}(w,\eta)$, the above estimate holds even on $D_1 \setminus \overline{B}(w,\eta)$, since it holds on $\partial(D_1 \setminus \overline{B}(w,\eta))$. We choose $z = z'$ and get, because $H_{z',w,h}(z') = z''$,
\[ u_\eta(z'') \leq u_\eta(z') + \log \left( 1 + 2C_5 \frac{R_D^c}{\delta_D(w)} \frac{|h|^c}{M(z',z'',w)^c} \right). \]
Letting $\eta$ tend to zero and recalling the definition of $h$, we obtain the desired estimate
\[ G_D(z'', w) - G_D(z', w) \leq \log \left( 1 + 2C_5 \frac{R_D^e}{\delta_D(w)} \frac{|z' - z''|^e}{M(z', z'', w)^e} \right). \]
Interchanging the roles of $z'$ and $z''$ we can complete the proof. ■

4. Proof of Theorem 1.1

4.1. Proof of estimate (1.1). We must consider two cases.

Case 1: $\delta_D(w)^e \leq |z - w|$. The starting point is the following estimate that was obtained in \[\text{[He]}\] (based upon an inequality of \[\text{[Bło1]}\]):
\[ (4.6) \int_D |\mathcal{G}_D(\cdot, w)| d\mu_{z, \eta} \leq (2\pi)^n (n!)^{1/n} \eta^{(n-1)/n} |\mathcal{G}_D(w, z)|^{1/n}, \]
where $d\mu_{z, \eta}$ denotes for any $\eta > 0$ the measure
\[ d\mu_{z, \eta} := (dd^c \max\{\mathcal{G}_D(\cdot, z), -\eta\})^n. \]
This measure is supported on the set \( \{\mathcal{G}_D(\cdot, z) = -\eta\} \subset B(z, R_De^{-\eta}) \), and its total mass is $(2\pi)^n$ (see \[\text{[He]}\]).

We want to apply Lemma 3.1 for $z' = z$. For this we must choose $\eta > 1$ such that
\[ (4.7) R_De^{-\eta} \leq \frac{\delta_D(w)}{8R_D} \min\{|z - w|, |z'' - w|\} \]
for $|z'' - z| < R_De^{-\eta}$. Now we note that
\[ |z'' - w| \geq |z - w| - |z'' - z| \geq |z - w| - R_De^{-\eta} \geq \frac{1}{2} |z - w|, \]
if only $\eta \geq \log \frac{2R_D}{|z - w|}$. We must choose
\[ (4.8) \eta \geq \log \frac{16R_D}{|z - w|\delta_D(w)} \]
in order to arrange for (4.7). Lemma 3.1 and (4.6) yield
\[ (n!)^{1/n} \eta^{(n-1)/n} |\mathcal{G}_D(w, z)|^{1/n} \]
\[ \geq (2\pi)^{-n} \int_D |\mathcal{G}_D(z'', w)| d\mu_{z, \eta}(z'') \geq (2\pi)^{-n} \int_D |\mathcal{G}_D(z, w)| d\mu_{z, \eta}(z'') \]
\[ - (2\pi)^{-n} \int_D |\mathcal{G}_D(z'', w) - \mathcal{G}_D(z, w)| d\mu_{z, \eta}(z'') \]
\[ = |\mathcal{G}_D(z, w)| - (2\pi)^{-n} \int_D |\mathcal{G}_D(z'', w) - \mathcal{G}_D(z, w)| d\mu_{z, \eta}(z'') \]
\[ \geq |\mathcal{G}_D(z, w)| - (2\pi)^{-n} \int_D \log \left( 1 + 2C_5 \frac{R_{De}^e}{\delta_D(w)} \frac{e^{-\eta}}{|z - w|^e} \right) d\mu_{z, \eta}(z'') \]
\[ = |\mathcal{G}_D(z, w)| - \log(1 + M_\eta e^{-\eta}) \]
with the abbreviation
\[
M_\eta := (2C_5)^\varepsilon \frac{R_D^{2+1}}{\delta_D(w)|z-w|^\varepsilon}.
\]

We now choose
\[
(4.10) \quad \eta := \left(\frac{1}{n+\varepsilon}\right)^{\frac{1}{\varepsilon}} \log \frac{1}{|G_D(w,z)|} + \frac{1+\varepsilon^2}{\varepsilon^2} \log \frac{2C_5}{R_D^{1-\varepsilon}} + \frac{1+\varepsilon^2}{\varepsilon^2} \log \frac{R_D}{|z-w|}.
\]

Next we show that
\[
(4.11) \quad \eta > \frac{1}{\varepsilon} \log M_\eta + \frac{1}{n\varepsilon} \log \frac{1}{|G_D(w,z)|}.
\]

By Proposition 2.1 we have
\[
|G_D(w,z)| \leq C_5 \delta_D(w)^\varepsilon.
\]

This leads to
\[
\eta - \frac{1}{\varepsilon} \log M_\eta - \frac{1}{n\varepsilon} \log \frac{1}{|G_D(w,z)|} = \frac{1}{\varepsilon^2} \log \frac{1}{|G_D(w,z)|} + \frac{1+\varepsilon^2}{\varepsilon^2} \log \frac{R_D}{|z-w|} + \frac{1+\varepsilon^2}{\varepsilon^2} \log \frac{2C_5}{R_D^{1-\varepsilon}}
\]
\[
- \log(2C_5) - \frac{1+\varepsilon^2}{\varepsilon} \log R_D + \frac{1}{\varepsilon} \log \delta_D(w) + \log |z-w|
\]
\[
\geq - \frac{1}{\varepsilon^2} \log C_5 - \frac{1}{\varepsilon} \log \delta_D(w) + \frac{1}{\varepsilon^2} \log |z-w| + \frac{1+\varepsilon^2}{\varepsilon^2} \log \frac{R_D}{|z-w|}
\]
\[
+ \frac{1+\varepsilon^2}{\varepsilon^2} \log \frac{2C_5}{R_D^{1-\varepsilon}} - \log(2C_5) - \frac{1+\varepsilon^2}{\varepsilon} \log R_D + \frac{1}{\varepsilon} \log \delta_D(w) + \log |z-w|
\]
\[
= \frac{1}{\varepsilon^2} \log 2 > 0,
\]

which yields (4.11). Further we get (4.8) from
\[
\eta - \log \frac{16R_D}{|z-w|\delta_D(w)}
\]
\[
\geq \frac{1}{\varepsilon} \log M_\eta - \frac{1}{n\varepsilon} \log C_5 - \log(16R_D) + \log \delta_D(w) + \log |z-w|
\]
\[
\geq - \log(8R_D^{1-\varepsilon}C_5^{1/n\varepsilon - 1}) + \left(\frac{1}{\varepsilon} - 1\right) \log \frac{1}{\delta_D(w)} > 0
\]

using (4.11) and $|G_D(w,z)| \leq C_5 \delta_D(w)^\varepsilon/|z-w| \leq C_5$ (recall that we suppose $\delta_D(w)^\varepsilon/|z-w| \leq 1$).
Finally,

\[
\begin{align*}
&\log \frac{M_\eta}{\exp(\eta^{1/n}|G_D(w,z)|^{1/n}) - 1} \\
&\leq \log \frac{M_\eta}{\eta^{1/n}|G_D(w,z)|^{1/n}} = \log M_\eta - \left(1 - \frac{1}{n}\right) \log \eta + \frac{1}{n} \log \frac{1}{|G_D(w,z)|} \\
&\leq \varepsilon \eta - \left(1 - \frac{1}{n}\right) \log \eta \leq \varepsilon \eta, \quad \text{by (4.11)},
\end{align*}
\]

and

\[
\log(1 + M_\eta e^{-\varepsilon \eta}) \leq \eta^{1/n}|G_D(w,z)|^{1/n}.
\]

Plugging these into (4.9) we find by means of (2.1), applied to |G_D(w,z)|,

\[
|G_D(z,w)| \leq (1 + (n!)^{1/n})^{\varepsilon_\eta} \eta^{(n-1)/n}|G_D(w,z)|^{1/n}
\]

\[
\leq \frac{2}{\varepsilon_\eta} (1 + (n!)^{1/n}) M(z,w)|G_D(w,z)|^{1/n},
\]

from which the claim follows.

**Case 2:** \( \delta_D(w)^\varepsilon > |z - w| \). With a constant \( \widehat{M}_\eta > 1 \) to be chosen later, we consider the function

\[
v(x) := \widehat{M}_\eta \log \left(\frac{1}{2} \frac{|x - w|}{\delta_D(w)^\varepsilon}\right)
\]

on the domain

\[
\Omega_r := D \cap B(w, \delta_D(w)^\varepsilon) \setminus B(w, r),
\]

where the radius \( r > 0 \) is less than \( |z - w| \) and satisfies

\[
\widehat{M}_\eta \log \frac{r}{\delta_D(w)^\varepsilon} \leq \log \frac{r}{R_D},
\]

which is equivalent to

\[
\log r < \frac{\varepsilon \widehat{M}_\eta \log \delta_D(w) - \log R_D}{\widehat{M}_\eta - 1}.
\]

Then \( z \in \Omega_r \), and \( w \notin \Omega_r \).

On \( \Omega_r \) we have \( v \leq -\widehat{M}_\eta \log 2 < 0 \).

Next let us consider \( v \) on \( \partial \Omega_r \). For \( x \in D \cap \partial B(w, r) \) we can estimate

\[
v(x) = \widehat{M}_\eta \log \frac{r}{\delta_D(w)^\varepsilon} \leq \log \frac{r}{R_D} < G_D(x,w).
\]

For \( x \in \partial D \) we obtain

\[
v(x) \leq -\widehat{M}_\eta \log 2 < 0 = G_D(x,w).
\]
Finally, let \( x \in D \cap \partial B(w, \delta_D(w)^\varepsilon) \). Then, by Case 1, because \(|x - w| = \delta_D(w)^\varepsilon\),
\[
|J_D(x, w)| \leq 2\varepsilon^{-2}(1 + (n!)^{1/n})M(x, w)^{1-1/n}|J_D(w, x)|^{1/n}.
\]
But
\[
|J_D(w, x)| \log^{n-1} \left( \frac{1}{|J_D(w, x)|} \right) \leq C_6
\]
and
\[
\log \frac{R_D}{|x - w|} = \log \frac{R_D}{\delta_D(w)^\varepsilon} \leq \log \frac{R_D}{|z - w|}.
\]
(We are considering the case \( \delta_D(w)^\varepsilon > |z - w| \).) This proves
\[
|J_D(x, w)| \leq C_7 + C_8 \left( \log \frac{R_D}{|z - w|} \right)^{1-1/n}
\]
and, since \( v(x) = -\hat{M}_\eta \log 2 \), we get
\[
|J_D(x, w)| \leq \frac{1}{\hat{M}_\eta} \left( C_7^\prime + C_8^\prime \left( \log \frac{R_D}{|z - w|} \right)^{1-1/n} \right) |v(x)|.
\]
Let
\[
\hat{M}_\eta := C_9 M(z, w)^{1-1/n},
\]
where the constant \( C_9 \) can be chosen independently of \( z, w \) in such a way that \(|J_D(x, w)| \leq |v(x)|\). Hence, by the maximality of \( J_D(\cdot, w) \) we get \( v \leq J_D(\cdot, w) \) on \( \Omega_r \). This implies
\[
|J_D(z, w)| \leq |v(z)| \leq \hat{M}_\eta \log \left( \frac{2 \delta_D(w)^\varepsilon}{|z - w|} \right) \leq n\hat{M}_\eta \log \left( 1 + 2 \left( \frac{\delta_D(w)^\varepsilon}{|z - w|} \right)^{1/n} \right),
\]
from which the desired estimate on \( J_D(z, w) \) will follow. ■

4.2. Proof of estimate (1.2). Our aim is the proof of
\[
|J_D(z, w)| \leq CM(z, w)^{1-1/n} \log \left( 1 + C \left( \frac{\delta_D(z)^\varepsilon \delta_D(w)^\varepsilon}{|z - w|^2} \right)^{1/n} \right).
\]
We fix distinct \( z, w \in D \cap U_1 \). Without loss of generality we may assume that they are close to the boundary so that the orthogonal projections \( z^*, w^* \) to the boundary are well-defined.

Let \( c > 0 \) denote a small constant such that
\[
4c^{1/\varepsilon} R_D^{1/\varepsilon - 1} < 1.
\]
If $\delta_D(z)^e \geq c|z - w|$, then (1.1) yields
$$|\mathcal{G}_D(z, w)| \leq nCM(z, w)^{1-1/n} \log \left( 1 + C \left( \frac{\delta_D(w)^e}{|z - w|} \right)^{1/n} \right) \leq nCM(z, w)^{1-1/n} \log \left( 1 + Cc^{-1/n} \left( \frac{\delta_D(z)^e \delta_D(w)^e}{|z - w|^2} \right)^{1/n} \right).$$

So we suppose that $\delta_D(z)^e \leq c|z - w|$. Now we define
$$V := D \cap B(z^*, 2c^{1/e} R_D^{1/e-1}|z - w|)$$
and note that
$$|z - z^*| = \delta_D(z) \leq (c|z - w|)^{1/e} \leq c^{1/e} R_D^{1/e-1}|z - w|,$$
hence $z \in V$. At the same time we have
$$|w - z^*| \geq |z - w| - |z - z^*| = |z - w| - \delta_D(z) \geq |z - w| - (c|z - w|)^{1/e} > 2c^{1/e} R_D^{1/e-1}|z - w|$$
by the choice of $c$. Hence $w \notin V$, and $\mathcal{G}_D(\cdot, w)$ is a maximal plurisubharmonic function on $V$. We define on $V$ a plurisubharmonic comparison function $v_2$. For this we use
$$\psi(x) := \Phi(x) - \gamma|x - z^*|^2,$$
which is negative and for small enough $\gamma > 0$ also plurisubharmonic. Then, for any $x \in V$,
$$|x - w| \geq |z - w| - |x - z| \geq |z - w| - \delta_D(z) - |x - z^*| \geq (1 - 3c^{1/e} R_D^{1/e-1})|z - w| \geq \frac{1}{4}|z - w|.$$
By (1.1) we have the estimate
$$\mathcal{G}_D(x, w) \geq -CM(x, w)^{1-1/n} \log \left( 1 + \tilde{C} \left( \frac{\delta_D(w)^e}{|x - w|} \right)^{1/n} \right) \geq -C'M(z, w)^{1-1/n} \log(1 + CM_1^{1/n}) \geq -C'M(z, w)^{1-1/n} \log \left( 1 + CM_1^{1/n} \left( \frac{-\psi(x)}{\gamma|x - z^*|^2} \right)^{1/2n} \right)$$
with some constant $C'$ and $M_1 := 4\delta_D(w)^e/|z - w|$. Our plurisubharmonic comparison function $v_2$ is now defined by
$$v_2(x) := -C'M(z, w)^{1-1/n} \log \left( 1 + C \left( \frac{\delta_D(w)^e}{\gamma_1|z - w|^2} \right)^{1/n} (-\psi(x))^{1/2n} \right)$$
with a constant $\gamma_1$ that will be chosen in a moment.

It is easily verified that $v_2$ is plurisubharmonic on $V$. We compare $v_2$ and $\mathcal{G}_D(\cdot, w)$ on $\partial V$. On $V \cap \partial D$ certainly $v_2 \leq 0 = \mathcal{G}_D(\cdot, w)$. 

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For \( x \in D \cap \partial B(z^*, 2c^{1/\varepsilon} R_D^{1/\varepsilon-1}|z - w|) \) we have
\[
\gamma_1 |z - w| = \gamma_1 \frac{|x - z^*|}{2c^{1/\varepsilon} R_D^{1/\varepsilon-1}} = \frac{1}{4} \sqrt{\gamma} |x - z^*|
\]
for \( \gamma_1 = \frac{\sqrt{\gamma}}{2} c^{1/\varepsilon} R_D^{1/\varepsilon-1} \), and therefore
\[
v_2(x) = -C'' M(z, w)^{-1/n} \log \left(1 + C M_1^{1/n}(-\psi(x))^{1/2n}\right)
\leq -C'' M(z, w)^{-1/n} \log \left(1 + C M_1^{1/n}(-\psi(x))^{1/2n}\right)^{1/2n} \leq \mathcal{G}_D(x, w).
\]
Hence \( v_2 \leq \mathcal{G}_D(\cdot, w) \) on \( \partial V \) and, by the comparison principle, \( v_2 \leq \mathcal{G}_D(\cdot, w) \) on \( V \). But this gives
\[
|\mathcal{G}_D(z, w)| \leq |v_2(z)|
\]
\[
= C' M(z, w)^{1-1/n} \log \left(1 + C \left(\frac{\delta_D(w)^{\varepsilon}}{\gamma_1 |z - w|^2}\right)^{1/n} (-\psi(z))^{1/2n}\right)
\leq C' M(z, w)^{1-1/n} \log \left(1 + \widehat{C} \left(\frac{\delta_D(z)^{\varepsilon} \delta_D(w)^{\varepsilon}}{|z - w|^2}\right)^{1/n}\right)
\]
with some new constant \( \widehat{C} \). Note that
\[
|\psi(z)| = |\Phi(z)| + \delta_D(z) \leq (C + 1) \delta_D(z)^{2\varepsilon}.
\]
This finishes the proof of Theorem 1.1.

**4.3. Proof of Theorem 1.3.** We let \( h := z' - z'' \) and consider two cases.

**Case 1:** \( |h|^{2\varepsilon/3} \leq \delta_D(w) \) and \( |h| \leq (\delta_D(w)/8R_D) M(z', z'', w) \). Then, by Lemma 3.1 we have
\[
|\mathcal{G}_D(z', w) - \mathcal{G}_D(z'', w)| \leq \log \left(1 + C' \frac{|h|^{\varepsilon}}{\delta_D(w) M(z', z'', w)^{\varepsilon}}\right)
\leq \log \left(1 + C' \frac{|h|^{\varepsilon/3}}{M(z', z'', w)^{\varepsilon}}\right),
\]
which proves the claimed estimate.

**Case 2:** \( |h|^{2\varepsilon/3} \geq \delta_D(w) \) or \( |h| \geq (\delta_D(w)/8R_D) M(z', z'', w) \). Now we simply estimate
\[
|\mathcal{G}_D(z', w) - \mathcal{G}_D(z'', w)| \leq |\mathcal{G}_D(z', w)| + |\mathcal{G}_D(z'', w)|
\]

With some new constant \( \hat{C} \). Note that
\[
|\psi(z)| = |\Phi(z)| + \delta_D(z) \leq (C + 1) \delta_D(z)^{2\varepsilon}.
\]
This finishes the proof of Theorem 1.1.
and want to apply (1.1) to the right-hand side. For this we note that
\[ M(z', w) \leq \left| \log \log \left( 1 + C \left( \frac{\delta_D(w)^e}{|z' - w|} \right) \right) \right| + \log \frac{R_D}{M(z', z'', w)} \]
\[ \leq C' \log \frac{R_D}{M(z', z'', w)}, \]
and \( M(z'', w) \leq C' \log \frac{R_D}{M(z', z'', w)}. \) This results in
\[ |G_D(z', w)| \leq \hat{C} M(z', w)^{1-1/n} \log \left( 1 + C \left( \frac{\delta_D(w)^e}{|z' - w|} \right)^{1/n} \right) \]
\[ \leq \hat{C} M(z', w)^{1-1/n} \log \left( 1 + \hat{C} \frac{|h|^{2\varepsilon/n}}{M(z', z'', w)^{1+\varepsilon/n}} \right) \]
and
\[ |G_D(z'', w)| \leq \hat{C} M(z'', w)^{1-1/n} \log \left( 1 + \hat{C} \frac{|h|^{2\varepsilon/n}}{M(z', z'', w)^{1+\varepsilon/n}} \right), \]
which in conjunction with the estimates on \( M(z', w) \) and \( M(z'', w) \) gives the desired Hölder estimate for \( |G_D(z', w)|. \)

5. The case of pseudoconvex extendable domains. Proofs of Lemmas 1.5 and 1.6

5.1. Proof of Lemma 1.5. We assume that \( N > 2 \), otherwise the assertion is well-known. As in the definition of pseudoconvex extendability, let \( \psi \in \mathcal{C}^2(U' \times U') \) be an extending function of order \( N \), defined on a neighborhood \( U' \) of \( w^0 \). Then there exists a constant \( C_2 > 0 \) such that its Levi form \( \mathcal{L}_{\psi(q, \cdot)} \) satisfies (for all \( q \in \partial D \cap U' \))
\[ \mathcal{L}_{\psi(q, \cdot)}(z; X) \geq -C_2 \left( |\psi(q, z)| |X|^2 + ||\partial\psi(q, \cdot), X|| |X| \right). \]

For any constant \( A > 0 \) and any \( q \in \partial D \cap U' \), the function
\[ \sigma(q, z) := \psi(q, z)e^{-A|z-q|^2} \]
also extends in a pseudoconvex way on \( \partial D \) near \( w^0 \), more explicitly
\[ -C_3(-r(z) + |z - q|) \leq \sigma(q, z) \leq e^{-Ar^2} r(z) - c_2 |z - q|^N, \]
where \( R' \) is the diameter of \( U' \) and \( c_2 > 0 \) is a small constant.

We choose open neighborhoods \( U_1 \subset U_2 \subset U' \) of \( w^0 \) such that, given \( z \in U_1 \), its orthogonal projection \( z^* \) onto \( \partial D \) lies inside \( U_2 \). By making \( A \) very large and then shrinking \( U_1 \) we can arrange that for any \( q \in \partial D \cap U_2 \), the function \(-(-\sigma(q, z))^{2/N} \) is plurisubharmonic on \( D \cap U_1 \). Now we put, for \( z \in D \cap U_1 \),
\[ \Phi'(z) := \sup_{q \in \partial D \cap U_2} \left( -(-\sigma(q, z))^{2/N} + \frac{1}{4} c_2^{2/N} |z - q|^2 \right). \]
Our claim is that \( \Phi' \) satisfies the estimate
\[
-C_1' \delta_D^{2/N} \leq \Phi' \leq -C_2' \delta_D^{2/N}
\]
with suitable constants \( C_1', C_2' > 0 \).

For this we observe that for any \( t, s \geq 0 \),
\[
(t^{2/N} + s^{2/N})^{N/2} \leq 2^N (t + s).
\]
This implies
\[
((-e^{-AR^2} r(z))^{2/N} + c_2^{2/N} |z - q|^2)^{N/2} \leq 2^N (-e^{-AR^2} r(z) + c_2 |z - q|^N)
\]
\[
\leq -2^N \sigma(q, z)
\]
by (5.2), or
\[
(-e^{-AR^2} r(z))^{2/N} + c_2^{2/N} |z - q|^2 \leq 4(-\sigma(q, z))^{2/N}.
\]
This gives
\[
\Phi'(z) \leq -\frac{1}{4} e^{-2AR^2/N} (-r(z))^{2/N}.
\]
The lower estimate is easier to show. Let \( z \in D \cap U_1 \); then \( z^* \in \partial D \cap U_2 \), and we find that
\[
\Phi'(z) \geq -(-\sigma(z^*, z))^{2/N} + \frac{1}{4} c_2^{2/N} |z - z^*|^2
\]
\[
\geq -C_3(-r(z) + |z - z^*|))^{2/N} \geq -C_4 \delta_D(z)^{2/N}.
\]

The upper semicontinuous regularization \( \Phi \) of \( \Phi' \) is plurisubharmonic and satisfies property (i). But also property (ii) holds, since the function \( \Phi''(z) := \Phi'(z) - \frac{1}{5} c_2^{2/N} |z|^2 \) is the supremum of a family of plurisubharmonic functions, and furthermore \( z \mapsto \Phi(z) - \frac{1}{5} c_2^{2/N} |z|^2 \) equals the upper semicontinuous regularization of \( \Phi'' \) and hence is also plurisubharmonic. ■

5.2. Proof of Lemma 1.6

We only need to recall Cho’s proof. We give a sketch of this proof and then state where to modify it.

Let \( \phi \in \mathcal{C}_0^\infty (B(0, 2) \setminus B(0, 1/4)) \) be a function such that \( \phi(z) = 1 \) for \( 1/2 < |z| < 1 \). Also let \( \psi \in \mathcal{C}_0^\infty (\mathbb{C}^n) \) be a smooth function such that \( \psi(z) = 1 \) for \( |z| \geq 2 \), and \( \psi(z) = 0 \) if \( |z| < 1 \).

For some large integer \( \mathcal{N} \) we put \( \phi_{\mathcal{N}}(z) = \psi(2^{\mathcal{N}} e z) \) and \( \phi_k(z) = \phi(2^k e z) \) for \( k > \mathcal{N} \). Let \( \zeta \in \partial D \). Then we consider, with a suitable small number \( \alpha > 0 \), the function
\[
E_{\zeta}(z) := \sum_{k=\mathcal{N}}^{\infty} 2^{-2k} \phi_k(z - \zeta)(\lambda_{2^{-k} \alpha}(z) - 2).
\]
The only difference between this definition for \( E_{\zeta} \) and that of Cho’s proof is the factor \( 2^{-2k} \) in front of \( \phi_k(z - \zeta)(\lambda_{2^{-k} \alpha}(z) - 2) \). In Cho’s proof the factor was \( 2^{-4k} \).
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There exists $L \in \mathbb{N}$ such that for any $\zeta$ and $z$ there are at most $L$ integers $k$ such that $z \in \text{supp} \phi_k(\cdot - \zeta)$. Again we have

$$|DE_\zeta(z)| \leq La^{-\ell}2^{\ell-2}k$$

for any $\ell$th order derivative $DE_\zeta$ of $E_\zeta$ and $z \in \text{supp} \phi_k(\cdot - \zeta)$. This shows that $E_\zeta$ is of class $C^2$. The rest of the proof of the pseudoconvexity of the surface $\{E_\zeta = 0\}$ is completely analogous to that in [Cho]. Because of the factor $2^{-2k}$ instead of $2^{-4k}$, now the function $E_\zeta$ extends in a pseudoconvex way to order $\leq 1/\varepsilon$ instead of $2/\varepsilon$. ■

References

[Hör] L. Hörmander, $L^2$ estimates and existence theorems for the $\bar{\partial}$-operator, Acta Math. 113 (1965), 89–152.
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