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On an integral-type operator from Privalov spaces to Bloch-type spaces

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Abstract. Let $H(B)$ denote the space of all holomorphic functions on the unit ball B of $\mathbb{C}^n$. Let φ be a holomorphic self-map of B and $g \in H(B)$ such that $g(0) = 0$. We study the integral-type operator

$$C_\varphi^g f(z) = \int_0^1 \Re f(\varphi(tz))g(tz)\,\frac{dt}{t}, \qquad f \in H(B).$$

The boundedness and compactness of C_φ^g from Privalov spaces to Bloch-type spaces and little Bloch-type spaces are studied.

1. Introduction. Let D be the unit disk in the complex plane and B be the unit ball of $\mathbb{C}^n$. Let $z = (z_1, \dots, z_n)$ and $w = (w_1, \dots, w_n)$ be points in $\mathbb{C}^n$. We write

$$\langle z, w\rangle = z_1\bar{w}_1 + \cdots + z_n\bar{w}_n, \qquad |z| = \sqrt{|z_1|^2 + \cdots + |z_n|^2}.$$

Thus $B = \{z \in \mathbb{C}^n : |z| < 1\}$. Let ∂B be the unit sphere in $\mathbb{C}^n$ and $d\sigma$ be the normalized Lebesgue measure on ∂B. We denote by $H(B)$ the class of all holomorphic functions on B. It is a Fréchet space (locally convex, metrizable and complete) with respect to the compact-open topology. By Montel's theorem, bounded sets in $H(B)$ are relatively compact and hence bounded sequences in $H(B)$ admit convergent subsequences. Convergence in this space will be referred to as locally uniform (l.u.) convergence.

For $f \in H(B)$, $z = (z_1, \dots, z_n) \in B$, let $\nabla f(z) = (\partial f/\partial z_1, \dots, \partial f/\partial z_n)$ denote the complex gradient of f. Let $\Re f$ stand for the radial derivative of f, that is,

$$\Re f(z) = \sum_{j=1}^n z_j \frac{\partial f}{\partial z_j}(z), \qquad z = (z_1, \dots, z_n) \in B.$$

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A positive continuous function μ on the interval $[0,1)$ is called *normal* if there exist $\delta \in [0,1)$ and s and t with $0 < s < t$ such that (see, e.g., [16])

$$\frac{\mu(r)}{(1-r)^s} \text{ is decreasing on } [\delta,1) \quad \text{and} \quad \lim_{r\to 1}\frac{\mu(r)}{(1-r)^s}=0;$$

$$\frac{\mu(r)}{(1-r)^t} \text{ is increasing on } [\delta,1) \quad \text{and} \quad \lim_{r\to 1}\frac{\mu(r)}{(1-r)^t}=\infty.$$

Throughout this paper, μ will denote a normal function on $[0,1)$. An $f \in H(B)$ is said to belong to the *Bloch-type space*, denoted by $\mathcal{B}_\mu = \mathcal{B}_\mu(B)$, if

$$b_\mu(f) := \sup_{z\in B} \mu(|z|)|\Re f(z)| < \infty.$$

The Bloch-type space is a Banach space with the norm $\|f\|_{\mathcal{B}_\mu} = |f(0)| + b_\mu(f)$. Let $\mathcal{B}_{\mu,0}$ denote the subspace of $\mathcal{B}_\mu$ consisting of those $f \in \mathcal{B}_\mu$ for which

$$\lim_{|z|\to 1} \mu(|z|)|\Re f(z)| = 0.$$

This space is called the *little Bloch-type space* (see, e.g., [20]). When $\mu(r) = 1-r^2$, we get the classical Bloch space and little Bloch space respectively. For more information on the Bloch space, see for example [24].

Let $1 < p < \infty$ and $f \in H(B)$. We say that f belongs to the *Privalov space*, denoted by $\mathcal{N}^p = \mathcal{N}^p(B)$, if

$$\sup_{0<r<1} \int_{\partial B} [\log^+ |f(r\xi)|]^p \, d\sigma(\xi) < \infty.$$

Here $\log^+ x$ is $\log x$ if $x > 1$ and 0 if $0 \le x \le 1$. By the elementary inequalities $\log^+ x \le \log(1+x) \le \log 2 + \log^+ x$, we see that $f \in \mathcal{N}^p$ if and only if

$$\|f\|^p_{\mathcal{N}^p} = \sup_{0<r<1} \int_{\partial B} [\log(1+|f(r\xi)|)]^p \, d\sigma(\xi) < \infty.$$

From [19], we see that the Privalov space $\mathcal{N}^p$ is a topological vector space with respect to the F-norm $\|\cdot\|_{\mathcal{N}^p}$. Under $\|\cdot\|_{\mathcal{N}^p}$, $\mathcal{N}^p$ is a Fréchet space and the topology of $\mathcal{N}^p$ is stronger than that of locally uniform convergence. This is a consequence of the estimate (see [19])

$$\text{(1)} \qquad \log(1+|f(z)|) \le \frac{(1+|z|)^{n/p}}{(1-|z|)^{n/p}} \|f\|_{\mathcal{N}^p}, \qquad f \in \mathcal{N}^p.$$

If ϕ is an analytic self-map of D, the composition operator induced by ϕ is

$$(C_\phi f)(z) = (f\circ\phi)(z), \qquad f \in H(D).$$

It is of interest to provide function-theoretic characterizations when ϕ induces bounded or compact composition operators on various spaces. The book [3] contains much information on this topic.

Let ϕ be an analytic self-map of D and $h \in H(D)$. In [9], Li and Stević defined and studied the generalized composition operator

$$C_\phi^h f(z) = \int_0^z f'(\phi(\xi))h(\xi)\,d\xi, \qquad f \in H(D),\ z \in D.$$

Composition operators from the Privalov space to the Bloch space and the little Bloch space in the unit disk were studied in [22]. The boundedness and compactness of the generalized composition operator on Zygmund spaces and Bloch-type spaces were investigated in [9]. See also [10, 11, 17] for the study of the operator C_ϕ^h.

Let φ be a holomorphic self-map of B and $g \in H(B)$ such that $g(0) = 0$. For $f \in H(B)$, the integral-type operator

$$C_\varphi^g f(z) = \int_0^1 \Re f(\varphi(tz))g(tz)\,\frac{dt}{t} \tag{2}$$

was recently introduced in [25]. The operator C_φ^g is a generalization of the generalized composition operator on the unit disk. The operator C_φ^g was studied in [14, 18, 25, 26]. It is easy to see that $C_z^g = L_g$, where

$$L_g f(z) = \int_0^1 \Re f(tz)g(tz)\,\frac{dt}{t},$$

which is called the *Riemann–Stieltjes operator* and studied in [1, 2, 5, 6, 7, 8, 12, 13, 23, 25, 26].

In this paper we study the boundedness and compactness of the operator C_φ^g from the Privalov space to the Bloch-type space and the little Bloch-type space in the unit ball. As a consequence, we obtain a characterization of the action of the Riemann–Stieltjes operator L_g from the Privalov space to the Bloch space and the little Bloch space. These results are new even in the unit disk.

Constants are denoted by C in this paper, they are positive and may differ from one occurrence to another.

2. Main results and proofs. In this section we will state our main results and prove them. To carry out the proofs, the following lemmas are needed.

LEMMA 1. *Suppose $f, g \in H(B)$ and $g(0) = 0$. Then*

$$\Re[C_\varphi^g(f)](z) = \Re f(\varphi(z))g(z).$$

Proof. A calculation with (2) gives the result (see, e.g., [4]); we omit the details.

Similarly to the proof of Lemma 4 of [15], we can get the following result. We omit the details.

Lemma 2. *A closed set K in $\mathcal{B}_\mu$ is compact if and only if it is bounded and satisfies*

$$\lim_{|z|\to 1}\sup_{f\in K}\mu(|z|)|\Re f(z)|=0.$$

A subset T of $\mathcal{N}^p_\alpha$ is called *bounded* if it is bounded for the defining F-norm $\|\cdot\|_{\mathcal{N}^p_\alpha}$. Given a Banach space X, we say that a linear map $L:\mathcal{N}^p_\alpha\to X$ is *bounded* if $L(T)\subset X$ is bounded for every bounded subset T of $\mathcal{N}^p_\alpha$. We say that L is *compact* if $L(T)\subset X$ is relatively compact for every bounded subset $T\subset\mathcal{N}^p_\alpha$.

The following criterion for compactness follows from arguments similar, for example, to those outlined in [3, 5, 22].

Lemma 3. *Let $p>1$ and φ be a holomorphic self-map of B, $g\in H(B)$ such that $g(0)=0$. Then $C^g_\varphi:\mathcal{N}^p\to\mathcal{B}_\mu$ is compact if and only if $C^g_\varphi:\mathcal{N}^p\to\mathcal{B}_\mu$ is bounded and for any sequence $(f_k)_{k\in\mathbb{N}}$ which is bounded in $\mathcal{N}^p$ and converges to zero l.u., $\lim_{k\to\infty}\|C^g_\varphi f_k\|_{\mathcal{B}_\mu}=0$.*

We are now in a position to formulate and prove the main results of this paper.

Theorem 1. *Let $p>1$, φ be a holomorphic self-map of B, and $g\in H(B)$ be such that $g(0)=0$. Then the following statements are equivalent.*

(i) $C^g_\varphi:\mathcal{N}^p\to\mathcal{B}_\mu$ *is bounded.*
(ii) $C^g_\varphi:\mathcal{N}^p\to\mathcal{B}_\mu$ *is compact.*
(iii)

$$M_1:=\sup_{z\in B}\mu(|z|)|g(z)|<\infty \tag{3}$$

and for every $c>0$,

$$\lim_{|\varphi(z)|\to 1}\frac{\mu(|z|)|g(z)|}{1-|\varphi(z)|^2}\exp\left[\frac{c}{(1-|\varphi(z)|^2)^{n/p}}\right]=0. \tag{4}$$

Proof. (ii)⇒(i). It is obvious.

(iii)⇒(ii). Assume that the conditions (3) and (4) hold. Combining (3) with (4) we get

$$M_2(c):=\sup_{z\in B}\frac{\mu(|z|)|g(z)|}{1-|\varphi(z)|^2}\exp\left[\frac{c}{(1-|\varphi(z)|^2)^{n/p}}\right]<\infty, \tag{5}$$

for every $c>0$. For arbitrary $f\in\mathcal{N}^p$, by (1) and the Cauchy estimate we have

$$(6)\qquad (1-|z|^2)|\Re f(z)| \le (1-|z|^2)|\nabla f(z)| \le C\int_{\partial B}\left|f\left(z+\frac{1-|z|}{2}\xi\right)\right|d\sigma \le \exp\left[\frac{C\|f\|_{\mathcal{N}^p}}{(1-|z|^2)^{n/p}}\right].$$

Take a bounded set $T\subset\mathcal{N}^p_\alpha$. Then there exists a positive constant M such that $\|f\|_{\mathcal{N}^p}\le M$ for all $f\in T$. By Lemma 1, the fact that $(C^g_\varphi f)(0)=0$ and (5) we have

$$(7)\qquad \begin{aligned}\|C^g_\varphi f\|_{\mathcal{B}_\mu} &= (C^g_\varphi f)(0)+\sup_{z\in B}\mu(|z|)|\Re(C^g_\varphi f)(z)|\\ &= \sup_{z\in B}\mu(|z|)|\Re f(\varphi(z))|\,|g(z)|\\ &\le \sup_{z\in B}\frac{\mu(|z|)|g(z)|}{1-|\varphi(z)|^2}\exp\left[\frac{C\|f\|_{\mathcal{N}^p}}{(1-|\varphi(z)|^2)^{n/p}}\right]\\ &\le \sup_{z\in B}\frac{\mu(|z|)|g(z)|}{1-|\varphi(z)|^2}\exp\left[\frac{CM}{(1-|\varphi(z)|^2)^{n/p}}\right]<\infty\end{aligned}$$

for every $f\in T$. This implies that $C^g_\varphi(T)$ is a bounded subset of $\mathcal{B}_\mu$. Therefore $C^g_\varphi:\mathcal{N}^p\to\mathcal{B}_\mu$ is bounded.

Let $(f_k)_{k\in\mathbb{N}}$ be a sequence in $\mathcal{N}^p$ with $\sup_{k\in\mathbb{N}}\|f_k\|_{\mathcal{N}^p}\le Q$ and $f_k\to 0$ l.u. on B. By means of (4) we arrive at the following: for every $\varepsilon>0$, there is a $\delta\in(0,1)$ such that

$$(8)\qquad \frac{\mu(|z|)|g(z)|}{1-|\varphi(z)|^2}\exp\left[\frac{c}{(1-|\varphi(z)|^2)^{n/p}}\right]<\varepsilon$$

when $\delta<|\varphi(z)|<1$. From (3) and (8), we have

$$\begin{aligned}\|C^g_\varphi f_k\|_{\mathcal{B}_\mu} &= \sup_{z\in B}\mu(|z|)|\Re(C^g_\varphi f_k)(z)|\\ &\le \Big(\sup_{\{z\in B\,:\,|\varphi(z)|\le\delta\}}+\sup_{\{z\in B\,:\,\delta<|\varphi(z)|<1\}}\Big)\mu(|z|)|g(z)|\,|\Re f_k(\varphi(z))|\\ &\le \sup_{\{z\in B\,:\,|\varphi(z)|\le\delta\}}M_1|\Re f_k(\varphi(z))|\\ &\quad+\sup_{\{z\in B\,:\,\delta<|\varphi(z)|<1\}}\frac{\mu(|z|)|g(z)|}{1-|\varphi(z)|^2}\exp\left[\frac{CQ}{(1-|\varphi(z)|^2)^{n/p}}\right]\\ &\le M_1\sup_{\{z\in B\,:\,|\varphi(z)|\le\delta\}}|\Re f_k(\varphi(z))|+\varepsilon.\end{aligned}$$

By the Cauchy's estimate we see that the sequence $|\Re f_k|$ converges to zero l.u. on B and hence

$$\lim_{k\to\infty}\sup_{\{z\in B\,:\,|\varphi(z)|\le\delta\}}|\Re f_k(\varphi(z))|=0.$$

Using this fact and letting $k \to \infty$ in the last inequality, we deduce that $\lim_{k\to\infty} \|C_\varphi^g f_k\|_{\mathcal{B}_\mu} \le \varepsilon$. Since ε is an arbitrary positive number, we have

$$\lim_{k\to\infty} \|C_\varphi^g f_k\|_{\mathcal{B}_\mu} = 0,$$

and the result follows from Lemma 3.

(i)⇒(iii). Suppose that $C_\varphi^g : \mathcal{N}^p \to \mathcal{B}_\mu$ is bounded. Take

$$f_a(z) = \frac{\langle z, a\rangle}{|a|^2}, \qquad |a| \neq 0.$$

Then by the boundedness of the operator $C_\varphi^g : \mathcal{N}^p \to \mathcal{B}_\mu$ we get

$$\sup_{z\in B} \mu(|z|)|g(z)| < \infty. \tag{9}$$

For $w \in B$ and any $c > 0$, set

$$f_w(z) = \exp\bigg[c\bigg\{\frac{1-|\varphi(w)|^2}{(1-\langle z,\varphi(w)\rangle)^2}\bigg\}^{n/p}\bigg].$$

Using the inequality

$$\log^+ x \le \log(1+x) \le \log 2 + \log^+ x,$$

we see that $\|f_w\|_{\mathcal{N}^p} \le C$ (see, e.g., [21]). In addition,

$$\begin{aligned}(10)\quad &\Re f_w(z)\\ &\quad = \frac{2nc}{p}\exp\bigg[c\bigg\{\frac{1-|\varphi(w)|^2}{(1-\langle z,\varphi(w)\rangle)^2}\bigg\}^{n/p}\bigg]\frac{(1-|\varphi(w)|^2)^{n/p}\langle z,\varphi(w)\rangle}{(1-\langle z,\varphi(w)\rangle)^{2n/p+1}},\end{aligned}$$

so that

$$\begin{aligned}(11)\quad C\|C_\varphi^g\|_{\mathcal{N}^p\to\mathcal{B}_\mu} &\ge \|C_\varphi^g f_w\|_{\mathcal{B}_\mu} = \sup_{z\in B}\mu(|z|)|\Re(C_\varphi^g f_w)(z)|\\ &\ge \frac{2nc}{p}\,\frac{\mu(|w|)|g(w)|\,|\varphi(w)|^2}{(1-|\varphi(w)|^2)^{1+n/p}}\exp\bigg[\frac{c}{(1-|\varphi(w)|^2)^{n/p}}\bigg].\end{aligned}$$

This leads to

$$\frac{\mu(|w|)|g(w)|}{(1-|\varphi(w)|^2)}\exp\bigg[\frac{c}{(1-|\varphi(w)|^2)^{n/p}}\bigg] \le C\|C_\varphi^g\|_{\mathcal{N}^p\to\mathcal{B}_\mu}\frac{(1-|\varphi(w)|^2)^{n/p}}{|\varphi(w)|^2},$$

which implies that (4) holds. The proof of Theorem 1 is complete. ∎

THEOREM 2. *Let $p > 1$, φ be a holomorphic self-map of B, and $g \in H(B)$ be such that $g(0) = 0$. Then the following statements are equivalent.*

(i) *$C_\varphi^g : \mathcal{N}^p \to \mathcal{B}_{\mu,0}$ is bounded.*
(ii) *$C_\varphi^g : \mathcal{N}^p \to \mathcal{B}_{\mu,0}$ is compact.*
(iii) *For every $c > 0$,*

$$\lim_{|z|\to 1}\frac{\mu(|z|)|g(z)|}{1-|\varphi(z)|^2}\exp\bigg[\frac{c}{(1-|\varphi(z)|^2)^{n/p}}\bigg] = 0. \tag{12}$$

Proof. (iii)$\Rightarrow$ (ii). Suppose that (12) holds. It is clear that (12) implies (3) and (4). From Theorem 1 we see that $C_\varphi^g : \mathcal{N}^p \to \mathcal{B}_\mu$ is bounded. By (7) we have

$$\mu(|z|)|\Re(C_\varphi^g f)(z)| \le \frac{\mu(|z|)|g(z)|}{1-|\varphi(z)|^2}\exp\left[\frac{C\|f\|_{\mathcal{N}^p}}{(1-|\varphi(z)|^2)^{n/p}}\right], \tag{13}$$

which together with the boundedness of $C_\varphi^g : \mathcal{N}^p \to \mathcal{B}_\mu$ implies that $C_\varphi^g : \mathcal{N}^p \to \mathcal{B}_{\mu,0}$ is bounded. In addition, taking the supremum in (13) over the unit ball of the space $\mathcal{N}^p$, then letting $|z| \to 1$, we obtain

$$\lim_{|z|\to 1} \sup_{\|f\|_{\mathcal{N}^p}\le 1} \mu(|z|)|\Re(C_\varphi^g f)(z)| = 0. \tag{14}$$

From Lemma 2 and (14), we see that $C_\varphi^g : \mathcal{N}^p \to \mathcal{B}_{\mu,0}$ is compact.

(ii)$\Rightarrow$(i). This is obvious.

(i)$\Rightarrow$(iii). Suppose that $C_\varphi^g : \mathcal{N}^p \to \mathcal{B}_{\mu,0}$ is bounded. Take

$$f_a(z) = \frac{\langle z, a\rangle}{|a|^2}, \qquad |a| \ne 0.$$

Then by the boundedness of $C_\varphi^g : \mathcal{N}^p \to \mathcal{B}_{\mu,0}$ we get

$$\lim_{|z|\to 1} \mu(|z|)|g(z)| = 0. \tag{15}$$

Suppose for contradiction that (iii) is not true. Then there are c_1, ε_1 and a sequence $\{z_j\}$ tending to ∂B such that

$$\frac{\mu(|z_j|)|g(z_j)|}{1-|\varphi(z_j)|^2}\exp\left[\frac{c_1}{(1-|\varphi(z_j)|^2)^{n/p}}\right] \ge \varepsilon_1. \tag{16}$$

Since $\lim_{|z|\to 1} \mu(|z|)|g(z)| = 0$, (16) shows that $\{z_j\}$ has a subsequence $\{z_{j_k}\}$ with $|\varphi(z_{j_k})| \to 1$. Again using the boundedness of $C_\varphi^g : \mathcal{N}^p \to \mathcal{B}_\mu$, we have (4), thus

$$\lim_{|\varphi(z_{j_k})|\to 1} \frac{\mu(|z_{j_k}|)|g(z_{j_k})|}{1-|\varphi(z_{j_k})|^2}\exp\left[\frac{c}{(1-|\varphi(z_{j_k})|^2)^{n/p}}\right] = 0, \tag{17}$$

contradicting (16). This completes the proof of the theorem. ∎

Remark 1. For every $c > 0$,

$$\lim_{|z|\to 1} \frac{\mu(|z|)|g(z)|}{1-|\varphi(z)|^2}\exp\left[\frac{c}{(1-|\varphi(z)|^2)^{n/p}}\right] = 0 \tag{18}$$

is equivalent to $\lim_{|z|\to 1} \mu(|z|)|g(z)| = 0$ and

$$\lim_{|\varphi(z)|\to 1} \frac{\mu(|z|)|g(z)|}{1-|\varphi(z)|^2}\exp\left[\frac{c}{(1-|\varphi(z)|^2)^{n/p}}\right] = 0. \tag{19}$$

The above equivalence shows that $C_\varphi^g : \mathcal{N}^p \to \mathcal{B}_{\mu,0}$ is bounded if and only if $C_\varphi^g : \mathcal{N}^p \to \mathcal{B}_\mu$ is bounded and $\lim_{|z|\to 1} \mu(|z|)|g(z)| = 0$.

Remark 2. From Theorems 1 and 2, we see that the following statements are equivalent.

(i) $L_g : \mathcal{N}^p \to \mathcal{B}$ is bounded;
(iii) $L_g : \mathcal{N}^p \to \mathcal{B}$ is compact;
(iii) $L_g : \mathcal{N}^p \to \mathcal{B}_0$ is bounded;
(iv) $L_g : \mathcal{N}^p \to \mathcal{B}_0$ is compact;
(v) $g \equiv 0$.

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