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On natural vector bundle morphisms $T_A \circ \otimes_s^q \to \otimes_s^q \circ T_A$ over id_{T_A}

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Abstract. Some properties and applications of natural vector bundle morphisms $T_A \circ \otimes_s^q \to \otimes_s^q \circ T_A$ over id_{T_A} are presented.

1. Introduction. Let $T_A : \mathcal{M}f \to \mathcal{FM}$ be a Weil functor. For any vector bundle (E, M, π) with the standard fibre V, the bundle $(T_A E, T_A M, T_A \pi)$ is a well-defined vector bundle since T_A is product-preserving. One can consider the following vector bundles:

$$\otimes_s^q \pi : \otimes_s^q E \to M,$$
$$T_A(\otimes_s^q \pi) : T_A(\otimes_s^q E) \to T_A M,$$
$$\otimes_s^q (T_A \pi) : \otimes_s^q T_A E \to T_A M.$$

Let $S : M \to \otimes_s^q E$ be a tensor field of type (q, s) and $\Phi : T_A(\otimes_s^q E) \to \otimes_s^q T_A E$ a vector bundle morphism over $\mathrm{id}_{T_A M}$; then $\widetilde{S} := \Phi \circ T_A S$ is a tensor field of the same type on $(T_A E, T_A M, T_A \pi)$. When Φ comes from a natural vector bundle morphism $T_A \circ \otimes_s^q \to \otimes_s^q \circ T_A$ over $\mathrm{id}_{T_A M}$ and $E = TM$ the tangent bundle of a manifold M, one can define some interesting natural operators $\otimes_s^q \circ T \rightsquigarrow (\otimes_s^q \circ T) T_A$ (see [4]) by using the canonical flow natural equivalence $\kappa : T_A \circ T \to T \circ T_A$ (see [7] for the case $q = 1$).

The main result of this paper is Proposition 3.1 that reduces the research of natural vector bundle morphisms $T_A \circ \otimes_s^q \to \otimes_s^q \circ T_A$ over id_{T_A} to that of equivariant linear maps $T_A(\otimes_s^q V) \to \otimes_s^q (T_A V)$.

2. Weil functor

2.1. *Weil algebra*

Definition 2.1. A *Weil algebra* is a finite-dimensional quotient of the algebra of germs $\mathcal{E}_p = C_0^\infty(\mathbb{R}^p, \mathbb{R})$ $(p \in \mathbb{N}^*)$.

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We denote by $\mathcal{M}_p$ the ideal of germs vanishing at 0; $\mathcal{M}_p$ is the maximal ideal of the local algebra $\mathcal{E}_p$.

EXAMPLE 2.1. (1) $\mathbb{R}$ is a Weil algebra since it is canonically isomorphic to the quotient $\mathcal{E}_p/\mathcal{M}_p$.

(2) $J_0^r(\mathbb{R}^p, \mathbb{R}) = \mathcal{E}_p/\mathcal{M}_p^{r+1}$ is a Weil algebra.

2.2. *Covariant description of a Weil functor* $T_A : \mathcal{M}f \to \mathcal{FM}$. We write $\mathcal{M}f$ for the category of finite-dimensional differential manifolds and mappings of class C^∞; furthermore, $\mathcal{FM}$ is the category of fibred manifolds and fibred manifold morphisms.

Let $A = \mathcal{E}_p/I$ be a Weil algebra and consider a manifold M. In the set of $\varphi \in C^\infty(\mathbb{R}^p, M)$ such that $\varphi(0) = x$, one defines an equivalence relation $\mathcal{R}_x$ by: $\varphi \mathcal{R}_x \psi$ if and only if $[h]_x \circ [\psi]_0 - [h]_x \circ [\varphi]_0 \in I$ for any $[h]_x \in C_x^\infty(M, \mathbb{R})$.

The equivalence class of φ is denoted by $j_A\varphi$ and is called the A-*velocity* of φ at 0; the class $j_A\varphi$ depends only on the germ of φ at 0. The quotient set is denoted by $(T_AM)_x$ and the disjoint union of $(T_AM)_x$, $x \in M$, by T_AM.

The mapping $\pi_{A,M} : T_AM \to M$, $j_A\varphi \mapsto \varphi(0)$, defines a bundle structure on T_AM and for any differentiable mapping $f : M \to N$, one defines a bundle morphism $T_Af : T_AM \to T_AN$ (over f) by $T_Af(j_A\varphi) = j_A(f \circ \varphi)$.

The correspondence $T_A : \mathcal{M}f \to \mathcal{FM}$ is a product-preserving bundle functor ([4]).

EXAMPLE 2.2. If $A = J_0^r(\mathbb{R}^p, \mathbb{R})$, then T_A is equivalent to the functor T_p^r of (p, r)-velocities, and if $A = \mathcal{E}_1/\mathcal{M}_1^2$, then $T_A = T$, the tangent functor.

2.3. *The canonical flow natural equivalence* $\kappa : T_A \circ T \to T \circ T_A$. Let T_A, T_B be two Weil functors with $A = \mathcal{E}_p/I$, $B = \mathcal{E}_q/J$; let M be a manifold. For any $\zeta = j_A\varphi \in T_AT_BM$, there is a differentiable mapping $\Phi : \mathbb{R}^p \times \mathbb{R}^q \to M$ such that $\varphi(z) = j_B\Phi_z$ in a neighbourhood of $0 \in \mathbb{R}^p$ (see [4]). By this result, one can define a natural equivalence

$$\kappa : T_A \circ T_B \to T_B \circ T_A$$

as follows:

$$\kappa_M(\zeta) = j_B\eta,$$

where $\eta : \mathbb{R}^q \to T_AM$, $t \mapsto j_A\Phi^t$. In particular, for $T_B = T$, we obtain the canonical flow natural equivalence $\kappa : T_A \circ T \to T \circ T_A$ associated to the bundle functor T_A.

3. Natural vector bundle morphisms $T_A \circ \otimes_s^q \to \otimes_s^q \circ T_A$ over id_{T_A}. In this section, $A = \mathcal{E}_p/I$ is a Weil algebra with $\mathcal{M}_p \supset I \supset \mathcal{M}_p^{r+1}$, r minimal; $(q, s) \in \mathbb{N}^2$; V is a finite-dimensional real vector space.

3.1. *Preliminaries.* We write $\mathcal{VB}$ for the subcategory of $\mathcal{FM}$ of vector bundles and vector bundle morphisms; $\mathcal{D}$ is the subcategory of $\mathcal{VB}$ of vector bundles with the standard fibre V and morphisms of vector bundles which are isomorphisms on fibres.

Let us consider the following vector spaces:

$$T_A(\otimes_s^q V) := T_A((\otimes^s V^*) \otimes (\otimes^q V)),$$
$$\otimes_s^q(T_A(V)) := (\otimes^s (T_A V)^*) \otimes (\otimes^q T_A V).$$

If φ is a linear automorphism of V, one can consider the following linear automorphisms:

$$T_A(\otimes_s^q \varphi) := T_A(\otimes^s({}^t\varphi^{-1}) \otimes (\otimes^q \varphi)),$$
$$\otimes_s^q(T_A\varphi) := \otimes^s ({}^t(T_A\varphi)^{-1}) \otimes (\otimes^q T_A\varphi)$$

respectively on $T_A(\otimes_s^q V)$ and $\otimes_s^q(T_A(V))$.

Finally, consider the functors $T_A \circ \otimes_s^q : \mathcal{D} \to \mathcal{VB}$ and $\otimes_s^q \circ T_A : \mathcal{D} \to \mathcal{VB}$ defined as follows:

$$\begin{cases} T_A \circ \otimes_s^q((E, M, \pi)) = (T_A(\otimes_s^q E), T_A M, T_A(\otimes_s^q \pi)), \\ T_A \circ \otimes_s^q((\bar{f}, f)) = (T_A\bar{f}, T_A(\otimes_s^q f)), \end{cases}$$

and

$$\begin{cases} \otimes_s^q \circ T_A((E, M, \pi)) = (\otimes_s^q(T_A E), T_A M, \otimes_s^q(T_A\pi)), \\ \otimes_s^q \circ T_A((\bar{f}, f)) = (T_A\bar{f}, \otimes_s^q(T_A f)). \end{cases}$$

3.2. *Natural vector bundle morphisms* $T_A \circ \otimes_s^q \to \otimes_s^q \circ T_A$ *over* id_{T_A}. Let us consider the representation $\rho_{q,s,V} : GL(V) \to GL(\otimes_s^q V)$ given by $\rho_{q,s,V}(u) = \otimes_s^q(u)$. Let $\lambda_V : GL(V) \times V \to V$, $(u, x) \mapsto u(x)$, denote the canonical linear action; the map $T_A\lambda_V : T_A GL(V) \times T_A V \to T_A V$ is also a linear action, so there is a unique representation $j_V : T_A GL(V) \to GL(T_A V)$ defined by $j_V(j_A\varphi)(j_A\eta) = T_A\lambda_V(j_A\varphi, j_A\eta)$. The representation $j_{\otimes_s^q V} \circ T_A\rho_{q,s,V} : T_A GL(V) \to GL(T_A(\otimes_s^q V))$ will be denoted $(\rho_{q,s,V})_1$; $(\rho_{q,s,V})_1$ induces a left action of $T_A GL(V)$ on $T_A(\otimes_s^q V)$ defined by $\widetilde{g} \cdot T = (\rho_{q,s,V})_1(\widetilde{g})(T)$. The representations $\rho_{q,s,T_A V}$ and j_V also induce a left action of $T_A GL(V)$ on $\otimes_s^q T_A V$ defined by $\widetilde{g} \cdot \widetilde{T} = \rho_{q,s,T_A V}(j_V(\widetilde{g}))(\widetilde{T})$. The particular case $T_A = J_p^1$ (the bundle functor of $(p,1)$-velocities) is treated in [1].

Definition 3.1. A linear map $\overline{\tau} : T_A(\otimes_s^q V) \to \otimes_s^q(T_A V)$ is said to be *equivariant* if it is $T_A GL(V)$-equivariant with respect to the previous actions, i.e.

$$\rho_{q,s,T_A V}(j_V(\widetilde{g})) \circ \overline{\tau} = \overline{\tau} \circ (\rho_{q,s,V})_1(\widetilde{g})$$

for all $\widetilde{g} \in T_A GL(V)$.

Definition 3.2. A *natural vector bundle morphism* $\tau : T_A \circ \otimes_s^q \to \otimes_s^q \circ T_A$ *over* id_{T_A} is a system of base-preserving vector bundle morphisms, τ_E :

$T_A(\otimes^q_s E) \to \otimes^q_s(T_A E)$, for every $\mathcal{D}$-object E, such that $\otimes^q_s(T_A f) \circ \tau_E = \tau_F \circ T_A(\otimes^q_s f)$ for each $\mathcal{D}$-morphism $f : E \to F$.

PROPOSITION 3.1. *There is a bijective correspondence between the set of all natural vector bundle morphisms $\tau : T_A \circ \otimes^q_s \to \otimes^q_s \circ T_A$ over id_{T_A} and the set of all equivariant linear maps $T_A(\otimes^q_s V) \to \otimes^q_s(T_A V)$.*

Proof. Let $\varphi : \pi^{-1}(U) \to U \times V$ be a local trivialisation of a vector bundle (E, M, π) and put

(1) $$\tau_E \mid_{[T_A(\otimes^q_s \pi)]^{-1}(T_A U)} = (\otimes^q_s T_A \varphi^{-1}) \circ (\mathrm{id}_{T_A U} \times \overline{\tau}) \circ T_A(\otimes^q_s \varphi).$$

1° *The right hand side of* (1) *does not depend on* φ: Indeed, let $\varphi_1 : \pi^{-1}(U) \to U \times V$ be another local trivialisation such that $(\varphi_1 \circ \varphi^{-1})(x,t) = (x, a(x) \cdot t)$; one has

$$\begin{cases} (\otimes^q_s \varphi_1) \circ (\otimes^q_s \varphi^{-1})(x, T) = (x, \rho_{q,s,V}(a(x)) \cdot T), \\ (T_A\varphi_1 \circ T_A \varphi^{-1})(\widetilde{x}, \widetilde{t}) = (\widetilde{x}, j_V(T_A a(\widetilde{x})) \cdot \widetilde{t}), \\ T_A(\otimes^q_s \varphi_1) \circ T_A(\otimes^q_s \varphi^{-1})(\widetilde{x}, \widetilde{T}) = (\widetilde{x}, (\rho_{q,s,V})_1(T_A a(\widetilde{x})) \cdot \widetilde{T}), \\ \otimes^q_s(T_A\varphi_1) \circ \otimes^q_s(T_A\varphi)^{-1}(\widetilde{x}, T_1) = (\widetilde{x}, \rho_{q,s,T_A V}(j_V(T_A a(\widetilde{x})))) \cdot T_1). \end{cases}$$

2° τ *is a natural vector bundle morphism*: Indeed, let $f : E \to E'$ be a $\mathcal{D}$-morphism over $\bar{f}$, $\varphi : \pi^{-1}(U) \to U \times V$ a local trivialisation of E, and $\varphi' : (\pi')^{-1}(U') \to U' \times V$ a local trivialisation of E' such that $\bar{f}(U) \subset U'$. Let us put $(\varphi' \circ f \circ \varphi^{-1})(x,t) = (\bar{f}(x), b(x) \cdot t)$. For any $(\widetilde{x}, \widetilde{T}) \in T_A(\otimes^q_s \varphi) \circ (T_A(\otimes^q_s \pi))^{-1}(T_A U)$,

$$(\otimes^q_s(T_A\varphi') \circ \otimes^q_s(T_A f) \circ \tau_E \circ T_A(\otimes^q_s \varphi^{-1})(\widetilde{x}, \widetilde{T}) \\ = (T_A \bar{f}(\widetilde{x}), \rho_{q,s,T_A V}(j_V(T_A b(\widetilde{x})))) \cdot \overline{\tau}(\widetilde{T}))$$

and

$$(\otimes^q_s T_A \varphi') \circ \tau_{E'} \circ T_A(\otimes^q_s f) \circ T_A(\otimes^q_s \varphi^{-1})(\widetilde{x}, \widetilde{T}) \\ = (T_A \bar{f}(\widetilde{x}), \overline{\tau} \circ (\rho_{q,s,V})_1(T_A b(\widetilde{x})) \cdot \widetilde{T});$$

but $\overline{\tau}$ is equivariant, hence $\otimes^q_s(T_A f) \circ \tau_E = \tau_{E'} \circ T_A(\otimes^q_s f)$. Furthermore, $\tau_{V \to \mathrm{pt}} = \overline{\tau}$, where pt is a one-point manifold.

3° The map $\Psi : \overline{\tau} \mapsto \tau$ is obviously injective. The surjection can be shown as follows: given a natural vector bundle morphism $\tau : T_A \circ \otimes^q_s \to \otimes^q_s \circ T_A$ over id_{T_A}, define $\overline{\tau} : T_A(\otimes^q_s V) \to \otimes^q_s(T_A V)$ by $\overline{\tau} = \tau_{V \to \mathrm{pt}}$.

(i) For a linear automorphism φ of V, we have $\otimes^q_s(T_A\varphi) \circ \overline{\tau} = \overline{\tau} \circ T_A(\otimes^q_s \varphi)$: Indeed, φ is a $\mathcal{D}$ -morphism over $\mathrm{id}_{\mathrm{pt}}$, so $\otimes^q_s(T_A\varphi) \circ \tau_{V \to \mathrm{pt}} = \tau_{V \to \mathrm{pt}} \circ T_A(\otimes^q_s \varphi)$.

(ii) $\tau_{\mathbb{R}^m \times V \to \mathbb{R}^m} = \mathrm{id}_{T_A \mathbb{R}^m} \times \overline{\tau}$: Indeed, the projection $\mathrm{pr}_2 : \mathbb{R}^m \times V \to V$, $(x,t) \mapsto t$, is a $\mathcal{D}$-morphism (over $\mathbb{R}^m \to \mathrm{pt}$), hence

$$T_A(\mathrm{pr}_2) = \mathrm{pr}_2 : T_A \mathbb{R}^m \times T_A V \to T_A V$$

is also a $\mathcal{D}$-morphism. Moreover

$$\otimes^q_s(\mathrm{pr}_2) = \mathrm{pr}_2 : \mathbb{R}^m \times (\otimes^q_s V) \to \otimes^q_s V,$$

then

$$T_A(\otimes^q_s(\mathrm{pr}_2)) = \mathrm{pr}_2 : T_A\mathbb{R}^m \times T_A(\otimes^q_s V) \to T_A(\otimes^q_s V).$$

The relation $\otimes^q_s(T_A(\mathrm{pr}_2)) \circ \tau_{\mathbb{R}^m\times V\to\mathbb{R}^m} = \tau_{V\to\mathrm{pt}} \circ T_A(\otimes^q_s(\mathrm{pr}_2))$ can be written $\mathrm{pr}_2 \circ \tau_{\mathbb{R}^m\times V\to\mathbb{R}^m} = \overline{\tau} \circ \mathrm{pr}_2$, hence $\tau_{\mathbb{R}^m\times V\to\mathbb{R}^m} = \mathrm{id}_{T_A\mathbb{R}^m} \times \overline{\tau}$.

(iii) $\overline{\tau}$ *is equivariant*: Taking $f(x,t) = (x, a(x)\cdot t)$, where $a : \mathbb{R}^p \to GL(V)$ is C^∞, the relation

$$\otimes^q_s(T_A f) \circ \tau_{\mathbb{R}^p\times V\to\mathbb{R}^p} = \tau_{\mathbb{R}^p\times V\to\mathbb{R}^p} \circ T_A(\otimes^q_s f)$$

is equivalent to

$$\rho_{q,s,T_A V}(j_V(T_A a(\widetilde{x}))) \circ \overline{\tau} = \overline{\tau} \circ (\rho_{q,s,V})_1(T_A a(\widetilde{x})),$$

for $\widetilde{x} \in T_A\mathbb{R}^p$. But for $\widetilde{g} = j_A g \in T_A GL(V)$, one can write $\widetilde{g} = T_A a(\widetilde{x})$ with $a = g$ and $\widetilde{x} = j_A(\mathrm{id}_{\mathbb{R}^p})$.

(iv) $\Psi(\overline{\tau}) = \tau$: Indeed, each local trivialisation $\varphi : \pi^{-1}(U) \to U \times V$ of a vector bundle (E, M, π) is a $\mathcal{D}$-morphism over id_U, hence

$$\otimes^q_s(T_A\varphi) \circ \tau_{E|_U} = \tau_{U\times V\to U} \circ T_A(\otimes^q_s\varphi) = (\mathrm{id}_{T_A U} \times \overline{\tau}) \circ T_A(\otimes^q_s\varphi),$$

by (ii), i.e. $\tau_{E|_U} = \Psi(\overline{\tau})_{E|_U}$, according to (1). ∎

4. Equivariant linear maps $T_A(\otimes^q_s V) \to \otimes^q_s(T_A V)$

4.1. *The case* $q = s = 0$. An equivariant linear map $\overline{\tau} : T_A(\otimes^0_0 V) \to \otimes^0_0(T_A V)$ is simply a linear form $\overline{\tau} : T_A\mathbb{R} \to \mathbb{R}$ since $(\rho_{0,0,V})_1(\widetilde{g}) = \mathrm{id}_{T_A\mathbb{R}}$ and $\rho_{0,0,T_A V}(j_V(\widetilde{g})) = \mathrm{id}_{\mathbb{R}}$. Moreover, each linear form $i \in A^*$ defines an equivariant linear map $\overline{\tau} : T_A(\otimes^0_0 V) \to \otimes^0_0(T_A V)$ by $\overline{\tau} = i$.

4.2. *The case* $q = 1$, $s = 0$. A linear map $\overline{\tau} : T_A V \to T_A V$ is equivariant if and only if $j_V(\widetilde{g}) \circ \overline{\tau} = \overline{\tau} \circ j_V(\widetilde{g})$ for any $\widetilde{g} \in T_A GL(V)$. For a fixed element $c \in A$, one can define an equivariant linear map $\overline{\tau}_c : T_A V \to T_A V$ by

$$\overline{\tau}_c(u) = c \cdot u = T_A(\cdot)(c, u),$$

where $\cdot : \mathbb{R} \times V \to V$ is the multiplication map; moreover, $\overline{\tau}_c$ is a module endomorphism over id_A.

Proposition 4.1. *Equivariant linear maps* $T_A V \to T_A V$ *are* $\overline{\tau}_c$, $c \in A$.

Proof. Let $T_A : \mathcal{M}f \to \mathcal{FM}$ be a Weil functor and $\widetilde{T}_A : \mathcal{VB} \to \mathcal{FM}$ the product-preserving gauge bundle functor on $\mathcal{VB}$ defined as follows:

$$\begin{cases} \widetilde{T}_A(E, M, \pi) = (T_A E, M, \pi_{A,M} \circ T_A\pi), \\ \widetilde{T}_A(\bar{f}, f) = (\bar{f}, T_A f). \end{cases}$$

There is a bijective correspondence between the set of natural vector bundle morphisms $T_A \to T_A$ over id_{T_A} and the set of natural transformations $\widetilde{T}_A \to$

$\widetilde{T}_A$ over id_{T_A}, by definition. Furthermore, the pair (A', V') associated to $\widetilde{T}_A$ is (A, A). According to Theorem 2 of [5], natural transformations $\widetilde{T}_A \to \widetilde{T}_A$ over id_{T_A} correspond to module endomorphisms $A \to A$ over id_A; the induced equivariant linear maps $T_A V \to T_A V$ are exactly the maps $\overline{\tau}_c$, $c \in A$. ∎

Remark 4.1. More generally, for $q \in \mathbb{N}$ and $s = 0$, one can construct some equivariant linear maps $T_A(\otimes_0^q V) \to \otimes_0^q(T_A V)$ by using [2] for example. For $(q, s) \in \mathbb{N}^2$, one can use [2] and the result below to construct some equivariant linear maps $T_A(\otimes_s^q V) \to \otimes_s^q(T_A V)$.

Proposition 4.2. *Let* $\overline{\tau} : T_A(\otimes_0^q V) \to \otimes_0^q(T_A V)$ *be an equivariant linear map. Then there is an equivariant linear map* $\tau : T_A(\otimes_s^q V) \to \otimes_s^q(T_A V)$ $(s \in \mathbb{N})$ *defined by*

$$\tau(j_A\varphi)(j_A\eta_1, \ldots, j_A\eta_s) = \overline{\tau}(j_A(\varphi * (\eta_1, \ldots, \eta_s))),$$

where $\varphi : \mathbb{R}^p \to \otimes_s^q V$, $\eta_1, \ldots, \eta_s : \mathbb{R}^p \to V$ *are* C^∞ *and*

$$\varphi * (\eta_1, \ldots, \eta_s) : \mathbb{R}^p \to \otimes_0^q V, \qquad z \to \varphi(z)(\eta_1(z), \ldots, \eta_s(z)).$$

Proof. See [1] for $T_A = T_p^1$. ∎

5. Applications

5.1. *Prolongations of functions.* Let (E, M, π) be a vector bundle; sections of $\otimes_0^0 E = M \times \mathbb{R}$ are smooth functions on M. With such a function f, one can associate the prolongation

$$i \circ T_A f : T_A M \to \mathbb{R},$$

where $i : A \to \mathbb{R}$ is linear. In particular, assume that $A = \mathcal{E}_p/\mathcal{M}_p^{r+1} = J_0^r(\mathbb{R}^p, \mathbb{R})$; then the dual basis $\{e_\alpha^*; |\alpha| \le r\}$ of $\{e_\alpha = j_0^r(z^\alpha); |\alpha| \le r\}$ induces the prolongations of functions: $f^{(\alpha)} = e_\alpha^* \circ T_A f$, $|\alpha| \le r$ (see [6] for $T_A = T_1^r$).

5.2. *Prolongations of vector fields.* Assume that $E = TM$ is the tangent bundle of M and let $\kappa : T_A T \to T T_A$ be the canonical flow natural equivalence associated to T_A ([4]). For a natural vector bundle morphism $\tau : T_A \to T_A$ and a smooth vector field $X \in \mathfrak{X}(M)$,

$$\kappa_M \circ \tau_{TM} \circ T_A X$$

is a smooth vector field on $T_A M$. If τ comes from $c \in A$, one can write

$$\kappa_M \circ \tau_{TM} \circ T_A X = (\mathrm{af}(c) \circ \mathcal{T}_A)_M$$

where $\mathrm{af}(c) : TT_A \to TT_A$ is the natural affinor given by $[\mathrm{af}(c)]_M = \kappa_M \circ \tau_{TM} \circ \kappa_M^{-1}$ and $\mathcal{T}_A : T \rightsquigarrow TT_A$ the canonical flow operator induced by T_A. This means in particular that all linear natural operators $: T \rightsquigarrow TT_A$ can be found with natural vector bundle morphisms $T_A \circ \otimes_0^1 \to \otimes_0^1 \circ T_A$.

5.3. *Prolongations of tensor fields of type* (q,s). One can use Proposition 4.2 to find natural vector bundle morphisms $\tau : T_A \circ \otimes_s^q \to \otimes_s^q \circ T_A$. For a smooth tensor field $\varphi : M \to \otimes_s^q TM$,

$$\otimes_s^q(\kappa_M) \circ \tau_{TM} \circ T_A\varphi$$

is a tensor field of the same type (see [7] for $q = 1$). One defines in this way a natural operator

$$A : \otimes_s^q \circ T \rightsquigarrow (\otimes_s^q \circ T)T_A$$

by $A_M(\varphi) = \otimes_s^q(\kappa_M) \circ \tau_{TM} \circ T_A\varphi$.

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