On natural vector bundle morphisms
\[ T_A \circ \otimes^q_s \to \otimes^q_s \circ T_A \] over \( \text{id}_{T_A} \)

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Abstract. Some properties and applications of natural vector bundle morphisms
\[ T_A \circ \otimes^q_s \to \otimes^q_s \circ T_A \] over \( \text{id}_{T_A} \) are presented.

1. Introduction. Let \( T_A : \mathcal{M}f \to \mathcal{F}M \) be a Weil functor. For any vector bundle \((E, M, \pi)\) with the standard fibre \( V \), the bundle \((T_A E, T_A M, T_A \pi)\) is a well-defined vector bundle since \( T_A \) is product-preserving. One can consider the following vector bundles:
\[ \otimes^q_s \pi : \otimes^q_s E \to M, \]
\[ T_A(\otimes^q_s \pi) : T_A(\otimes^q_s E) \to T_A M, \]
\[ \otimes^q_s (T_A \pi) : \otimes^q_s T_A E \to T_A M. \]

Let \( S : M \to \otimes^q_s E \) be a tensor field of type \((q, s)\) and \( \Phi : T_A(\otimes^q_s E) \to \otimes^q_s T_A E \) a vector bundle morphism over \( \text{id}_{T_A M} \); then \( \tilde{S} := \Phi \circ T_A S \) is a tensor field of the same type on \((T_A E, T_A M, T_A \pi)\). When \( \Phi \) comes from a natural vector bundle morphism \( T_A \circ \otimes^q_s \to \otimes^q_s \circ T_A \) over \( \text{id}_{T_A M} \) and \( E = TM \) the tangent bundle of a manifold \( M \), one can define some interesting natural operators \( \otimes^q_s \circ T \sim (\otimes^q_s \circ T)T_A \) (see [4]) by using the canonical flow natural equivalence \( \kappa : T_A \circ T \to T \circ T_A \) (see [7] for the case \( q = 1 \)).

The main result of this paper is Proposition 3.1 that reduces the research of natural vector bundle morphisms \( T_A \circ \otimes^q_s \to \otimes^q_s \circ T_A \) over \( \text{id}_{T_A} \) to that of equivariant linear maps \( T_A(\otimes^q_s V) \to \otimes^q_s (T_A V) \).

2. Weil functor

2.1. Weil algebra

Definition 2.1. A Weil algebra is a finite-dimensional quotient of the algebra of germs \( \mathcal{E}_p = C_0^\infty(\mathbb{R}^p, \mathbb{R}) \) \((p \in \mathbb{N}_*)\).

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We denote by $\mathcal{M}_p$ the ideal of germs vanishing at 0; $\mathcal{M}_p$ is the maximal ideal of the local algebra $\mathcal{E}_p$.

**Example 2.1.** (1) $\mathbb{R}$ is a Weil algebra since it is canonically isomorphic to the quotient $\mathcal{E}_p/\mathcal{M}_p$.

(2) $J_0^r(\mathbb{R}^p, \mathbb{R}) = \mathcal{E}_p/\mathcal{M}_p^{r+1}$ is a Weil algebra.

**2.2. Covariant description of a Weil functor $T_A : \mathcal{M}f \to \mathcal{F}M$.** We write $\mathcal{M}f$ for the category of finite-dimensional differential manifolds and mappings of class $C^\infty$; furthermore, $\mathcal{F}M$ is the category of fibred manifolds and fibred manifold morphisms.

Let $A = \mathcal{E}_p/I$ be a Weil algebra and consider a manifold $M$. In the set of $\varphi \in C^\infty(\mathbb{R}^p, M)$ such that $\varphi(0) = x$, one defines an equivalence relation $\mathcal{R}_x$ by: $\varphi \mathcal{R}_x \psi$ if and only if $[h]_x \circ [\psi]_0 - [h]_x \circ [\varphi]_0 \in I$ for any $[h]_x \in C^\infty_x(M, \mathbb{R})$.

The equivalence class of $\varphi$ is denoted by $j_A \varphi$ and is called the $A$-velocity of $\varphi$ at 0; the class $j_A \varphi$ depends only on the germ of $\varphi$ at 0. The quotient set is denoted by $(T_A M)_x$ and the disjoint union of $(T_A M)_x$, $x \in M$, by $T_AM$.

The mapping $\pi_{A,M} : T_A M \to M$, $j_A \varphi \mapsto \varphi(0)$, defines a bundle structure on $T_A M$ and for any differentiable mapping $f : M \to N$, one defines a bundle morphism $T_A f : T_A M \to T_A N$ (over $f$) by $T_A f(j_A \varphi) = j_A(f \circ \varphi)$.

The correspondence $T_A : \mathcal{M}f \to \mathcal{F}M$ is a product-preserving bundle functor ([4]).

**Example 2.2.** If $A = J_0^r(\mathbb{R}^p, \mathbb{R})$, then $T_A$ is equivalent to the functor $T_p^r$ of $(p, r)$-velocities, and if $A = \mathcal{E}_1/M^2_1$, then $T_A = T$, the tangent functor.

**2.3. The canonical flow natural equivalence $\kappa : T_A \circ T \to T \circ T_A$.** Let $T_A$, $T_B$ be two Weil functors with $A = \mathcal{E}_p/I$, $B = \mathcal{E}_q/J$; let $M$ be a manifold. For any $\zeta = j_A \varphi \in T_A T_B M$, there is a differentiable mapping $\Phi : \mathbb{R}^p \times \mathbb{R}^q \to M$ such that $\varphi(z) = j_B \Phi_z$ in a neighbourhood of $0 \in \mathbb{R}^p$ (see [4]). By this result, one can define a natural equivalence

$$\kappa : T_A \circ T_B \to T_B \circ T_A$$

as follows:

$$\kappa_M(\zeta) = j_B \eta,$$

where $\eta : \mathbb{R}^q \to T_A M$, $t \mapsto j_A \Phi^t$. In particular, for $T_B = T$, we obtain the canonical flow natural equivalence $\kappa : T_A \circ T \to T \circ T_A$ associated to the bundle functor $T_A$.

**3. Natural vector bundle morphisms $T_A \circ \otimes_s^q \to \otimes_s^q \circ T_A$ over $\text{id}_{T_A}$.**

In this section, $A = \mathcal{E}_p/I$ is a Weil algebra with $\mathcal{M}_p \supset I \supset \mathcal{M}_p^{r+1}$, $r$ minimal; $(q, s) \in \mathbb{N}^2$; $V$ is a finite-dimensional real vector space.
3.1. Preliminaries. We write $\mathcal{VB}$ for the subcategory of $\mathcal{FM}$ of vector bundles and vector bundle morphisms; $\mathcal{D}$ is the subcategory of $\mathcal{VB}$ of vector bundles with the standard fibre $V$ and morphisms of vector bundles which are isomorphisms on fibres.

Let us consider the following vector spaces:

\[
T_A(\otimes^q_s V) := T_A((\otimes^s V^*) \otimes (\otimes^q V)),
\]

\[
\otimes^q_s(T_A(V)) := (\otimes^s(T_A V)^*) \otimes (\otimes^q T_A V).
\]

If $\varphi$ is a linear automorphism of $V$, one can consider the following linear automorphisms:

\[
T_A(\otimes^q_s \varphi) := T_A(\otimes^s(\lambda^{-1}) \otimes (\otimes^q \varphi)),
\]

\[
\otimes^q_s(T_A \varphi) := \otimes^s(\lambda^{-1}(T_A \varphi)) \otimes (\otimes^q T_A \varphi)
\]

respectively on $T_A(\otimes^q_s V)$ and $\otimes^q_s(T_A(V))$.

Finally, consider the functors $T_A \circ \otimes^q_s : \mathcal{D} \rightarrow \mathcal{VB}$ and $\otimes^q_s \circ T_A : \mathcal{D} \rightarrow \mathcal{VB}$ defined as follows:

\[
\begin{align*}
T_A \circ \otimes^q_s((E, M, \pi)) &= (T_A(\otimes^q_s E), T_A M, T_A(\otimes^q_s \pi)), \\
T_A \circ \otimes^q_s((f, j)) &= (T_A f, T_A(\otimes^q_s f)),
\end{align*}
\]

and

\[
\begin{align*}
\otimes^q_s \circ T_A((E, M, \pi)) &= (\otimes^q_s(T_A E), T_A M, \otimes^q_s(T_A \pi)), \\
\otimes^q_s \circ T_A((f, j)) &= (T_A f, \otimes^q_s(T_A f)).
\end{align*}
\]

3.2. Natural vector bundle morphisms $T_A \circ \otimes^q_s \rightarrow \otimes^q_s \circ T_A$ over id$_{T_A}$.

Let us consider the representation $\rho_{q,s,V} : GL(V) \rightarrow GL(\otimes^q_s V)$ given by $\rho_{q,s,V}(u) = \otimes^q_s(u)$. Let $\lambda_V : GL(V) \times V \rightarrow V$, $(u, x) \mapsto u(x)$, denote the canonical linear action; the map $T_A \lambda_V : T_A GL(V) \times T_A V \rightarrow T_A V$ is also a linear action, so there is a unique representation $j_V : T_A GL(V) \rightarrow GL(T_A V)$ defined by $j_V(j_A \varphi)(j_V \eta) = T_A \lambda_V(j_A \varphi, j_A \eta)$. The representation $j_V \circ T_A \rho_{q,s,V} : T_A GL(V) \rightarrow GL(T_A(\otimes^q_s V))$ will be denoted $(\rho_{q,s,V})_1$; $(\rho_{q,s,V})_1$ induces a left action of $T_A GL(V)$ on $T_A(\otimes^q_s V)$ defined by $\tilde{g} \cdot T = (\rho_{q,s,T_A V})_1(\tilde{g})(T)$. The representations $\rho_{q,s,T_A V}$ and $j_V$ also induce a left action of $T_A GL(V)$ on $\otimes^q s T_A V$ defined by $\tilde{g} \cdot \tilde{T} = \rho_{q,s,T_A V}(j_V(\tilde{g}))(\tilde{T})$. The particular case $T_A = J^1_p$ (the bundle functor of $(p, 1)$-velocities) is treated in [1].

Definition 3.1. A linear map $\overline{\tau} : T_A(\otimes^q_s V) \rightarrow \otimes^q_s(T_A V)$ is said to be equivariant if it is $T_A GL(V)$-equivariant with respect to the previous actions, i.e.

\[
\rho_{q,s,T_A V}(j_V(\tilde{g})) \circ \overline{\tau} = \overline{\tau} \circ (\rho_{q,s,V})_1(\tilde{g})
\]

for all $\tilde{g} \in T_A GL(V)$.

Definition 3.2. A natural vector bundle morphism $\overline{\tau} : T_A \circ \otimes^q_s \rightarrow \otimes^q_s \circ T_A$ over id$_{T_A}$ is a system of base-preserving vector bundle morphisms, $\tau_E :$
Let us put the set of all equivariant linear maps as follows: given a natural vector bundle morphism \( \tau : T_A \circ \otimes_s^q \rightarrow \otimes_s^q \circ T_A \) over \( \text{id}_{T_A} \) and the set of all equivariant linear maps \( T_A(\otimes_s^q V) \rightarrow \otimes_s^q (T_A V) \).

**Proposition 3.1.** There is a bijective correspondence between the set of all natural vector bundle morphisms \( \tau : T_A \circ \otimes_s^q \rightarrow \otimes_s^q \circ T_A \) over \( \text{id}_{T_A} \) and the set of all equivariant linear maps \( T_A(\otimes_s^q V) \rightarrow \otimes_s^q (T_A V) \).

**Proof.** Let \( \varphi : \pi^{-1}(U) \rightarrow U \times V \) be a local trivialisation of a vector bundle \((E, M, \pi)\) and put

\[
\tau_E \mid_{[T_A(\otimes_s^q \pi)]^{-1}(T_A U)} = (\otimes_s^q T_A \varphi^{-1}) \circ (\text{id}_{T_A U} \times \pi) \circ T_A(\otimes_s^q \varphi).
\]

1° The right hand side of (1) does not depend on \( \varphi \): Indeed, let \( \varphi_1 : \pi^{-1}(U) \rightarrow U \times V \) be another local trivialisation such that \( (\varphi_1 \circ \varphi^{-1})(x, t) = (x, a(x) \cdot t) \); one has

\[
\begin{align*}
(\otimes_s^q \varphi_1) \circ (\otimes_s^q \varphi^{-1})(x, T) &= (x, \rho_{q,s,V}(a(x)) \cdot T), \\
(T_A \varphi_1 \circ T_A \varphi^{-1})(\vec{x}, \vec{t}) &= (\vec{x}, j_V(T_A a(\vec{x})) \cdot \vec{t}), \\
T_A(\otimes_s^q \varphi_1) \circ T_A(\otimes_s^q \varphi^{-1})(\vec{x}, \vec{T}) &= (\vec{x}, (\rho_{q,s,V})_1(T_A a(\vec{x})) \cdot \vec{T}), \\
(\otimes_s^q (T_A \varphi_1) \circ (\otimes_s^q (T_A \varphi)^{-1})(\vec{x}, T_1) &= (\vec{x}, \rho_{q,s,T_A V}(j_V(T_A a(\vec{x}))) \cdot T_1).
\end{align*}
\]

2° \( \tau \) is a natural vector bundle morphism: Indeed, let \( f : E \rightarrow E' \) be a \( \mathcal{D} \)-morphism over \( \bar{f}, \varphi : \pi^{-1}(U) \rightarrow U \times V \) a local trivialisation of \( E \), and \( \varphi' : (\pi')^{-1}(U') \rightarrow U' \times V \) a local trivialisation of \( E' \) such that \( \bar{f}(U) \subset U' \). Let us put \( \frac{(\varphi' \circ f \circ \varphi^{-1})(x, t) = (\bar{f}(x), b(x) \cdot t)}{\times} \). For any \( (\vec{x}, \vec{T}) \in T_A(\otimes_s^q \varphi) \circ (T_A(\otimes_s^q \pi))^{-1}(T_A U) \),

\[
(\otimes_s^q (T_A \varphi') \circ (\otimes_s^q (T_A f) \circ \tau_E \circ T_A(\otimes_s^q \varphi^{-1})(\vec{x}, \vec{T}) = (T_A \bar{f}(\vec{x}), \rho_{q,s,T_A V}(j_V(T_A a(\vec{x}))) \cdot \pi(\vec{T})),
\]

and

\[
(\otimes_s^q (T_A \varphi') \circ \tau_{E'} \circ T_A(\otimes_s^q f) \circ T_A(\otimes_s^q \varphi^{-1})(\vec{x}, \vec{T}) = (T_A \bar{f}(\vec{x}), \tau \circ (\rho_{q,s,V})_1(T_A b(\vec{x})) \cdot \vec{T});
\]

but \( \pi \) is equivariant, hence \( \otimes_s^q (T_A f) \circ \tau_E = \tau_{E'} \circ \rho_{q,s,V} \circ T_A(\otimes_s^q f) \). Furthermore, \( \tau_{V \rightarrow \text{pt}} = \pi \), where \( \text{pt} \) is a one-point manifold.

3° The map \( \Psi : \pi \mapsto \tau \) is obviously injective. The surjection can be shown as follows: given a natural vector bundle morphism \( \tau : T_A \circ \otimes_s^q \rightarrow \otimes_s^q \circ T_A \) over \( \text{id}_{T_A} \), define \( \pi : T_A(\otimes_s^q V) \rightarrow \otimes_s^q (T_A V) \) by \( \pi = \tau_{V \rightarrow \text{pt}} \).

(i) For a linear automorphism \( \varphi \) of \( V \), we have \( \otimes_s^q (T_A \varphi) \circ \pi = \pi \circ T_A(\otimes_s^q \varphi) \): Indeed, \( \varphi \) is a \( \mathcal{D} \)-morphism over \( \text{id}_{\text{pt}} \), so \( \otimes_s^q (T_A \varphi) \circ \tau_{V \rightarrow \text{pt}} = \tau_{V \rightarrow \text{pt}} \circ T_A(\otimes_s^q \varphi) \).

(ii) \( \tau_{R^n \times V \rightarrow R^m} = \text{id}_{T_A R^n} \times \pi \): Indeed, the projection \( \text{pr}_2 : R^m \times V \rightarrow V \), \((x, t) \mapsto t\), is a \( \mathcal{D} \)-morphism (over \( R^m \rightarrow \text{pt} \)), hence

\[
T_A(\text{pr}_2) = \text{pr}_2 : T_A R^m \times T_A V \rightarrow T_A V
\]
is also a \( D \)-morphism. Moreover
\[
\otimes^q_s(\text{pr}_2) = \text{pr}_2 : \mathbb{R}^m \times (\otimes^q_s V) \to \otimes^q_s V,
\]
then
\[
T_A(\otimes^q_s(\text{pr}_2)) = \text{pr}_2 : T_A\mathbb{R}^m \times T_A(\otimes^q_s V) \to T_A(\otimes^q_s V).
\]
The relation \( \otimes^q_s(T_A(\text{pr}_2)) \circ \tau_{\mathbb{R}^m \times V, \mathbb{R}^m} = \tau_{V, \mathbb{R}^m} \circ \text{pr}_2 \) can be written as \( \text{pr}_2 \circ \tau_{\mathbb{R}^m \times V, \mathbb{R}^m} = \tau \circ \text{pr}_2 \), hence \( \tau_{\mathbb{R}^m \times V, \mathbb{R}^m} = \text{id}_{T_A \mathbb{R}^m \times \tau} \).

(iii) \( \tau \) is equivariant: Taking \( f(x, t) = (x, a(x) \cdot t) \), where \( a : \mathbb{R}^p \to GL(V) \) is \( C^\infty \), the relation
\[
\otimes^q_s(T_A f) \circ \tau_{V, \mathbb{R}^p \times V, \mathbb{R}^p} = \tau_{V, \mathbb{R}^p \times V, \mathbb{R}^p} \circ T_A(\otimes^q_s f)
\]
is equivalent to
\[
\rho_{q,s, T_A V}(j_V(T_A a(\tilde{x}))) \circ \tau = \tau \circ (\rho_{q,s,V} 1)(T_A a(\tilde{x})),
\]
for \( \tilde{x} \in T_A \mathbb{R}^p \). But for \( \tilde{g} = j_A g \in T_A GL(V) \), one can write \( \tilde{g} = T_A a(\tilde{x}) \) with \( a = g \) and \( \tilde{x} = j_A(\text{id}_{\mathbb{R}^p}) \).

(iv) \( \Psi(\tau) = \tau \): Indeed, each local trivialisation \( \varphi : \pi^{-1}(U) \to U \times V \) of a vector bundle \( (E, M, \pi) \) is a \( D \)-morphism over \( \text{id}_U \), hence
\[
\otimes^q_s(T_A \varphi) \circ \tau_{E|U} = \tau_{U \times V, -U} \circ T_A(\otimes^q_s \varphi) = (\text{id}_{T_A U \times \tau}) \circ T_A(\otimes^q_s \varphi),
\]
by (ii), i.e. \( \tau_{E|U} = \Psi(\tau)|_{E|U} \), according to (1).

4. Equivariant linear maps \( T_A(\otimes^q_s V) \to \otimes^q_s(T_A V) \)

4.1. The case \( q = s = 0 \). An equivariant linear map \( \tau : T_A(\otimes^0_0 V) \to \otimes^0_0(T_A V) \) is simply a linear form \( \tau : T_A \mathbb{R} \to \mathbb{R} \) since \( (\rho_{0,0,V})_{1}(g) = \text{id}_{T_A \mathbb{R}} \) and \( \rho_{0,0,T_A V}(j_V(g)) = \text{id}_{\mathbb{R}} \). Moreover, each linear form \( i \in A^* \) defines an equivariant linear map \( \tau : T_A(\otimes^0_0 V) \to \otimes^0_0(T_A V) \) by \( \tau = i \).

4.2. The case \( q = 1, s = 0 \). A linear map \( \tau : T_A V \to T_A V \) is equivariant if and only if \( j_V(\tilde{g}) \circ \tau = \tau \circ j_V(\tilde{g}) \) for any \( \tilde{g} \in T_A GL(V) \). For a fixed element \( c \in A \), one can define an equivariant linear map \( \tau_c : T_A V \to T_A V \) by
\[
\tau_c(u) = c \cdot u = T_A(\cdot)(c, u),
\]
where \( \cdot : \mathbb{R} \times V \to V \) is the multiplication map; moreover, \( \tau_c \) is a module endomorphism over \( \text{id}_A \).

**Proposition 4.1.** Equivariant linear maps \( T_A V \to T_A V \) are \( \tau_c, c \in A \).

**Proof.** Let \( T_A : \mathcal{M}f \to \mathcal{F}M \) be a Weil functor and \( \tilde{T}_A : \mathcal{V}B \to \mathcal{F}M \) the product-preserving gauge bundle functor on \( \mathcal{V}B \) defined as follows:
\[
\begin{align*}
\tilde{T}_A(E, M, \pi) &= (T_A E, M, \pi_{A,M} \circ T_A \pi), \\
\tilde{T}_A(\tilde{f}, f) &= (\tilde{f}, T_A f).
\end{align*}
\]
There is a bijective correspondence between the set of natural vector bundle morphisms \( T_A \to T_A \) over \( \text{id}_{T_A} \) and the set of natural transformations \( \tilde{T}_A \to \).
\( \tilde{T}_A \) over \( \text{id}_{T_A} \), by definition. Furthermore, the pair \((A', V')\) associated to \( \tilde{T}_A \) is \((A, A)\). According to Theorem 2 of [5], natural transformations \( \tilde{T}_A \to \tilde{T}_A \) over \( \text{id}_{T_A} \) correspond to module endomorphisms \( A \to A \) over \( \text{id}_{A} \); the induced equivariant linear maps \( T_A V \to T_A V \) are exactly the maps \( \tau_c, c \in A \). □

**Remark 4.1.** More generally, for \( q \in \mathbb{N} \) and \( s = 0 \), one can construct some equivariant linear maps \( T_A(\otimes^q_0 V) \to \otimes^q_0(T_A V) \) by using [2] for example. For \( (q, s) \in \mathbb{N}^2 \), one can use [2] and the result below to construct some equivariant linear maps \( T_A(\otimes^q_0 V) \to \otimes^q_0(T_A V) \).

**Proposition 4.2.** Let \( \tau : T_A(\otimes^q_0 V) \to \otimes^q_0(T_A V) \) be an equivariant linear map. Then there is an equivariant linear map \( \tau : T_A(\otimes^q_0 V) \to \otimes^q_0(T_A V) \) \((s \in \mathbb{N})\) defined by

\[
\tau(j_A \varphi)(j_A \eta_1, \ldots, j_A \eta_s) = \eta(j_A(\varphi \ast (\eta_1, \ldots, \eta_s))),
\]

where \( \varphi : \mathbb{R}^p \to \otimes^q_0 V, \eta_1, \ldots, \eta_s : \mathbb{R}^p \to V \) are \( C^\infty \) and

\[
\varphi \ast (\eta_1, \ldots, \eta_s) : \mathbb{R}^p \to \otimes^q_0 V, \quad z \mapsto \varphi(z)(\eta_1(z), \ldots, \eta_s(z)).
\]

**Proof.** See [1] for \( T_A = T^1_p \). □

5. Applications

**5.1. Prolongations of functions.** Let \((E, M, \pi)\) be a vector bundle; sections of \( \otimes^q_0 E = M \times \mathbb{R} \) are smooth functions on \( M \). With such a function \( f \), one can associate the prolongation

\[
i \circ T_A f : T_A M \to \mathbb{R},
\]

where \( i : A \to \mathbb{R} \) is linear. In particular, assume that \( A = \mathcal{E}_p/\mathcal{M}_p^{r+1} = J^r_0(\mathbb{R}^p, \mathbb{R}) \); then the dual basis \( \{ e^*_\alpha; |\alpha| \leq r \} \) of \( \{ e_\alpha = j_0^r(z^\alpha); |\alpha| \leq r \} \) induces the prolongations of functions: \( f^r(\alpha) = e^*_\alpha \circ T_A f, |\alpha| \leq r \) (see [6] for \( T_A = T^1_p \)).

**5.2. Prolongations of vector fields.** Assume that \( E = TM \) is the tangent bundle of \( M \) and let \( \kappa : T_A T \to TT_A \) be the canonical flow natural equivalence associated to \( T_A \) ([4]). For a natural vector bundle morphism \( \tau : T_A \to T_A \) and a smooth vector field \( X \in \mathfrak{X}(M) \),

\[
\kappa_M \circ \tau_{TM} \circ T_A X
\]

is a smooth vector field on \( T_A M \). If \( \tau \) comes from \( c \in A \), one can write

\[
\kappa_M \circ \tau_{TM} \circ T_A X = (af(c) \circ T_A)_M
\]

where \( af(c) : TT_A \to TT_A \) is the natural affinor given by \( [af(c)]_M = \kappa_M \circ \tau_{TM} \circ \kappa^{-1}_M \) and \( T_A : T \leadsto TT_A \) the canonical flow operator induced by \( T_A \). This means in particular that all linear natural operators : \( T \leadsto TT_A \) can be found with natural vector bundle morphisms \( T_A \circ \otimes^1_0 \to \otimes^1_0 \circ T_A \).
5.3. Prolongations of tensor fields of type \((q, s)\). One can use Proposition 4.2 to find natural vector bundle morphisms \(\tau : T_A \circ \otimes_s^q \rightarrow \otimes_s^q \circ T_A\). For a smooth tensor field \(\varphi : M \rightarrow \otimes_s^q TM\),

\[
\otimes_s^q (\kappa_M) \circ \tau_{TM} \circ T_A \varphi
\]

is a tensor field of the same type (see [7] for \(q = 1\)). One defines in this way a natural operator

\[
A : \otimes_s^q \circ T \rightsquigarrow (\otimes_s^q \circ T)T_A
\]

by

\[
A_M(\varphi) = \otimes_s^q (\kappa_M) \circ \tau_{TM} \circ T_A \varphi.
\]

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