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An improved Chen–Ricci inequality for special slant submanifolds in Kenmotsu space forms

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Abstract. B. Y. Chen [Arch. Math. (Basel) 74 (2000), 154–160] proved a geometrical inequality for Lagrangian submanifolds in complex space forms in terms of the Ricci curvature and the squared mean curvature. Recently, this Chen–Ricci inequality was improved in [Int. Electron. J. Geom. 2 (2009), 39–45].

On the other hand, K. Arslan et al. [Int. J. Math. Math. Sci. 29 (2002), 719–726] established a Chen–Ricci inequality for submanifolds, in particular in contact slant submanifolds, in Kenmotsu space forms.

In this article, we improve the latter inequality for special slant submanifolds in Kenmotsu space forms. We also investigate the equality case.

1. Preliminaries. S. Tanno [13] has classified, into three classes, the connected almost contact Riemannian manifolds whose automorphism group has maximum dimension:

1. homogeneous normal contact Riemannian manifolds with constant ϕ-holomorphic sectional curvature;
2. global Riemannian products of a line or circle and a Kaehlerian space form;
3. warped product spaces $L \times_f F$, where L is a line and F a Kaehlerian manifold.

K. Kenmotsu [7] studied the third class and characterized it by tensor equations. Below, such a manifold is called a Kenmotsu manifold.

More precisely, a $(2m+1)$-dimensional Riemannian manifold $(\tilde{M}, g)$ is said to be a *Kenmotsu manifold* if it admits an endomorphism ϕ of its tangent bundle $T\tilde{M}$, a vector field ξ and a 1-form η, which satisfy:

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$$(1.1)\qquad \begin{cases} \phi^2 = -\mathrm{Id} + \eta \otimes \xi, \quad \eta(\xi) = 1, \quad \phi\xi = 0, \quad \eta \circ \phi = 0, \\ g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \quad \eta(X) = g(X, \xi), \\ (\tilde{\nabla}_X \phi)Y = -g(X, \phi Y)\xi - \eta(Y)\phi X, \\ \tilde{\nabla}_X \xi = X - \eta(X)\xi, \end{cases}$$

for any vector fields X, Y on $\tilde{M}$, where $\tilde{\nabla}$ denotes the Riemannian connection with respect to g (see also [12], [14]).

We denote by ω the fundamental 2-form of $\tilde{M}$, i.e. $\omega(X, Y) = g(\phi X, Y)$ for all $X, Y \in \Gamma(T\tilde{M})$. It is known that the pairing (ω, η) defines a locally conformal cosymplectic structure, i.e.

$$(1.2)\qquad d\omega = 2\omega \wedge \eta, \quad d\eta = 0.$$

A Kenmotsu manifold with constant ϕ-holomorphic sectional curvature c is called a *Kenmotsu space form.* Then its curvature tensor field $\tilde{R}$ is expressed by [7]

$$\begin{aligned}(1.3)\qquad 4\tilde{R}(X, Y)Z = {} & (c - 3)\{g(Y, Z)X - g(X, Z)Y\} \\ & + (c + 1)\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X, Z)\eta(Y)\xi \\ & - g(Y, Z)\eta(X)\xi + \omega(Y, Z)\phi X - \omega(X, Z)\phi Y - 2\omega(X, Y)\phi Z\}.\end{aligned}$$

By analogy with submanifolds in a Kaehler manifold, various classes of submanifolds in a Kenmotsu manifold have been considered (see, for example, [9], [10]).

The notion of a slant submanifold in a Hermitian manifold was introduced by B. Y. Chen [2]. The corresponding notion in an almost contact Riemannian manifold was defined by A. Lotta [8].

A submanifold M isometrically immersed in an almost contact Riemannian manifold $\tilde{M}$ is said to be a *contact slant submanifold* if the angle $\theta(X)$ between ϕX and T_pM is a constant θ, for any point $p \in M$ and any vector $X \in T_pM$ linearly independent of ξ. The angle θ of a contact slant immersion is called the *slant angle* of the immersion.

Invariant and anti-invariant submanifolds are particular cases of contact slant submanifolds (with $\theta = 0$ and $\theta = \pi/2$, respectively).

A contact slant submanifold which is neither invariant nor anti-invariant is called a *proper* contact slant submanifold.

A proper contact slant submanifold of a Kenmotsu manifold is said to be a *special* contact slant submanifold if

$$(\nabla_X T)Y = (\cos^2\theta)[-\eta(Y)TX + g(Y, TX)\xi]$$

for any vector fields X, Y tangent to M, where TX is the tangential component of ϕX.

We remark that any 3-dimensional proper contact slant submanifold of a Kenmotsu manifold is a special contact slant submanifold.

2. Chen–Ricci inequality. In [3], B. Y. Chen established a sharp relationship between the Ricci curvature Ric and the squared mean curvature $\|H\|^2$ for any n-dimensional submanifold M of a real space form $\tilde{M}(c)$ of constant sectional curvature c; namely,

$$\mathrm{Ric}(X) \leq (n-1)c + \frac{n^2}{4}\|H\|^2,$$

which is known as the *Chen–Ricci inequality*. The same inequality holds for Lagrangian submanifolds in a complex space form $\tilde{M}(4c)$ (see [4]). As a general reference for such inequalities we mention [5].

K. Arslan et al. [1] proved a similar inequality for submanifolds of Kenmotsu space forms (see also [11]).

THEOREM 2.1. *Let $\tilde{M}(c)$ be a $(2m+1)$-dimensional Kenmotsu space form and M an n-dimensional submanifold, tangent to ξ. Then:*

(i) *for any unit vector $X \in T_pM$, orthogonal to ξ,*

$$\mathrm{Ric}(X) \leq \tfrac{1}{4}\{(n-1)(c-3) + \tfrac{1}{2}(3\|TX\|^2 - 2)(c+1) + n^2\|H\|^2\},$$

where TX is the tangential component of ϕX;

(ii) *if $H(p) = 0$, then a unit vector $X \in T_pM$ orthogonal to ξ yields equality in the inequality above if and only if $X \in \mathcal{N}_p$ (the kernel of the second fundamental form)*;

(iii) *equality holds for all unit tangent vectors orthogonal to ξ at p if and only if p is a totally geodesic point.*

In particular, if M is a contact slant submanifold, one has

$$\mathrm{Ric}(X) \leq \tfrac{1}{4}\{(n-1)(c-3) + \tfrac{1}{2}(3\cos^2\theta - 2)(c+1) + n^2\|H\|^2\}.$$

The Chen–Ricci inequality was further improved by S. Deng [6] for Lagrangian submanifolds in complex space forms:

THEOREM 2.2. *Let M be a Lagrangian submanifold of dimension $n \geq 2$ in a complex space form $\widetilde{M}(4c)$ of constant holomorphic sectional curvature $4c$ and X a unit tangent vector in T_pM, $p \in M$. Then*

$$\mathrm{Ric}(X) \leq (n-1)\left(c + \frac{n}{4}\|H\|^2\right).$$

Equality holds for any unit tangent vector at p if and only if either:

(i) *p is a totally geodesic point, or*
(ii) *$n = 2$ and p is an H-umbilical point with $\lambda = 3\mu$.*

Moreover, Lagrangian submanifolds in complex space forms achieving equality were also determined in [6].

In the proof of the above inequality, S. Deng used the following lemmas.

Lemma 2.1. *Let $f_1(x_1,\dots,x_n)$ be the function on $\mathbb{R}^n$ defined by*

$$f_1(x_1,\dots,x_n)=x_1\sum_{j=2}^{n}x_j-\sum_{j=2}^{n}x_j^2.$$

If $x_1+\cdots+x_n=2na$, then

$$f_1(x_1,\dots,x_n)\le\frac{n-1}{4n}(x_1+\cdots+x_n)^2.$$

Equality holds if and only if $\frac{1}{n+1}x_1=x_2=\cdots=x_n=a$.

Lemma 2.2. *Let $f_2(x_1,\dots,x_n)$ be the function on $\mathbb{R}^n$ defined by*

$$f_2(x_1,\dots,x_n)=x_1\sum_{j=2}^{n}x_j-x_1^2.$$

If $x_1+\cdots+x_n=4a$, we have

$$f_2(x_1,\dots,x_n)\le\tfrac{1}{8}(x_1+\cdots+x_n)^2.$$

Equality holds if and only if $x_1=a$, $x_2+\cdots+x_n=3a$.

3. An improved Chen–Ricci inequality. In this section we shall improve the Chen-Ricci inequality from [1] for special slant submanifolds in a Kenmotsu space form.

Definition. A proper 3-dimensional slant submanifold M in a Kenmotsu manifold $\tilde{M}$ is called *H-umbilical* if its second fundamental form h takes the following form:

$$h(e_1,e_1)=\lambda Fe_1,\quad h(e_2,e_2)=\mu Fe_1,\quad h(e_1,e_2)=\mu Fe_2,$$

with respect to an orthonormal frame $\{e_0=\xi,e_1,e_2\}$, where FX is the normal component of ϕX.

We state the main result of this paper.

Theorem 3.1. *Let $\tilde{M}(c)$ be a $(2n+1)$-dimensional Kenmotsu space form and M an $(n+1)$-dimensional special contact θ-slant submanifold. Then, for any $p\in M$ and any unit vector $X\in T_pM$ orthogonal to ξ,*

$$\mathrm{Ric}(X)\le\frac{(n+1)^2(n-1)}{4n}\|H\|^2-1+\frac{(n-1)(c-3)}{4}+\frac{3(c+1)}{4}\cos^2\theta. \tag{3.1}$$

Moreover, equality holds in (3.1) for any $p\in M$ and any unit vector $X\in T_pM$ orthogonal to ξ if and only if either

(i) *M is a totally geodesic submanifold, or*
(ii) *$n=2$ and M is H-umbilical.*

Proof. Let $p\in M$ and $X\in T_pM$ be a unit vector orthogonal to ξ.

We choose an orthonormal basis $\{e_0=\xi,e_1,\dots,e_n,e_{n+1},\dots,e_{2n}\}$ of

$T_p\tilde{M}$ such that $e_0, e_1, \dots, e_n$ are tangent to M at p and $X = e_1$, and $e_{n+j} = \frac{1}{\sin\theta} F e_j$, $j = 1, \dots, n$.

We denote

$$h^r_{ij} = g(h(e_i, e_j), e_{n+r}), \qquad \forall i, j, r = 1, \dots, n.$$

By using the expression (1.3) of the curvature tensor of a Kenmotsu space form and the Gauss equation for $X = Z = e_1$ and $Y = W = e_j$, $j = 2, \dots, n$, we get

$$R(e_1, e_j, e_1, e_j) = \frac{c-3}{4} + \frac{3(c+1)}{4} g^2(\phi e_1, e_j) + \sum_{r=1}^{n} [h^r_{11} h^r_{jj} - (h^r_{1j})^2].$$

Summing for j from 2 to n and taking into account that $K(e_1 \wedge \xi) = -1$, we obtain

$$\begin{aligned}\mathrm{Ric}(X) = & -1 + \frac{(n-1)(c-3)}{4} + \frac{3(c+1)}{4} \sum_{j=2}^{n} g^2(\phi e_1, e_j) \\ & + \sum_{r=1}^{n} \sum_{j=2}^{n} [h^r_{11} h^r_{jj} - (h^r_{1j})^2].\end{aligned}$$

Then

$$\begin{aligned}&\mathrm{Ric}(X) + 1 - \frac{(n-1)(c-3)}{4} - \frac{3(c+1)}{4} \|Te_1\|^2 \\ &= \sum_{r=1}^{n} \sum_{j=2}^{n} [h^r_{11} h^r_{jj} - (h^r_{1j})^2] \leq \sum_{r=1}^{n} \sum_{j=2}^{n} h^r_{11} h^r_{jj} - \sum_{j=2}^{n} (h^1_{1j})^2 - \sum_{j=2}^{n} (h^j_{1j})^2.\end{aligned} \tag{3.2}$$

Since M is a special contact θ-slant submanifold, we have

$$(\nabla_X T)Y = (\cos^2\theta)[-\eta(Y)TX + g(Y, TX)\xi],$$

which implies

$$h^r_{ij} = h^i_{rj}, \qquad \forall i, j, r = \overline{1, n}.$$

The previous relation becomes

$$\begin{aligned}&\mathrm{Ric}(X) + 1 - \frac{(n-1)(c-3)}{4} - \frac{3(c+1)}{4} \cos^2\theta \\ &\qquad \leq \sum_{r=1}^{n} \sum_{j=2}^{n} h^r_{11} h^r_{jj} - \sum_{j=2}^{n} (h^1_{1j})^2 - \sum_{j=2}^{n} (h^j_{1j})^2 \\ &\qquad = \sum_{r=1}^{n} \sum_{j=2}^{n} h^r_{11} h^r_{jj} - \sum_{j=2}^{n} (h^j_{11})^2 - \sum_{j=2}^{n} (h^1_{jj})^2.\end{aligned}$$

We denote

$$f_1(h^1_{11}, h^1_{22}, \dots, h^1_{nn}) = h^1_{11}\sum_{j=2}^{n} h^1_{jj} - \sum_{j=2}^{n}(h^1_{jj})^2,$$

$$f_r(h^r_{11}, h^r_{22}, \dots, h^r_{nn}) = h^r_{11}\sum_{j=2}^{n} h^r_{jj} - (h^r_{11})^2, \qquad \forall r = \overline{2, n}.$$

Since $(n+1)H^1 = h^1_{11} + h^1_{22} + \dots + h^1_{nn}$, Lemma 2.1 yields

$$f_1(h^1_{11}, h^1_{22}, \dots, h^1_{nn}) \le \frac{n-1}{4n}((n+1)H^1)^2 = \frac{(n+1)^2(n-1)}{4n}(H^1)^2.$$

Analogously, by Lemma 2.2, we obtain, for any $2 \le r \le n$,

$$f_r(h^r_{11}, h^r_{22}, \dots, h^r_{nn}) \le \tfrac{1}{8}((n+1)H^r)^2 \le \frac{(n+1)^2(n-1)}{4n}(H^r)^2.$$

Thus,

$$\begin{aligned}\mathrm{Ric}(X) + 1 - \frac{(n-1)(c-3)}{4} - \frac{3(c+1)}{4}\cos^2\theta \\ \le \frac{(n+1)^2(n-1)}{4n}\sum_{r=1}^{n}(H^r)^2 = \frac{(n+1)^2(n-1)}{4n}\|H\|^2.\end{aligned}$$

Therefore,

$$\mathrm{Ric}(X) \le \frac{(n+1)^2(n-1)}{4n}\|H\|^2 - 1 + \frac{(n-1)(c-3)}{4} + \frac{3(c+1)}{4}\cos^2\theta,$$

i.e., (3.1).

Next we study the equality case. For $n \ge 3$, we choose Fe_1 parallel to H and we have $H^r = 0$, for $r \ge 2$; from Lemma 2.2, we get

$$h^1_{1j} = h^j_{11} = \frac{(n+1)H^j}{4} = 0, \qquad \forall j \ge 2,$$

$$h^1_{jk} = 0, \qquad \forall j, k \ge 2,\ j \ne k.$$

Lemma 2.1 yields $h^1_{11} = (n+1)a$ and $h^1_{jj} = a$ for all $j \ge 2$, with $a = \frac{(n+1)H^1}{2n}$.

In (3.2) we computed $\mathrm{Ric}(X) = \mathrm{Ric}(e_1)$. Similarly, by computing $\mathrm{Ric}(e_2)$ and using the equality case of (3.1), we get

$$h^r_{2j} = h^2_{jr} = 0, \qquad \forall r \ne 2,\ j \ne 2,\ r \ne j.$$

Then we obtain

$$\frac{h^2_{11}}{n+1} = h^2_{22} = \dots = h^2_{nn} = \frac{(n+1)H^2}{2n} = 0.$$

The argument is also valid for matrices $\left(h^r_{jk}\right)$ because equality holds for all unit tangent vectors; so, $h^2_{2j} = h^j_{22} = \frac{(n+1)H^j}{2n} = 0$ for all $j \ge 3$.

The matrix (h^2_{jk}) (respectively (h^r_{jk})) has only two possible nonzero entries $h^2_{12} = h^2_{21} = h^1_{22} = \frac{(n+1)H^1}{2n}$ (respectively $h^r_{1r} = h^r_{r1} = h^1_{rr} = \frac{(n+1)H^1}{2n}$ for $r \geq 3$). Now, by the Gauss equation we obtain

$$\widetilde{R}(e_2, e_j, e_2, e_j) = R(e_2, e_j, e_2, e_j) - \left(\frac{(n+1)H^1}{2n}\right)^2, \qquad \forall j \geq 3.$$

Similarly we get

$$\widetilde{R}(e_2, e_1, e_2, e_1) = R(e_2, e_1, e_2, e_1) - (n+1)\left(\frac{(n+1)H^1}{2n}\right)^2 + \left(\frac{(n+1)H^1}{2n}\right)^2.$$

After combining the last two relations, we find

$$\mathrm{Ric}(e_2) + 1 - (n-1)\frac{c-3}{4} - \frac{3}{4}(c+1)\cos^2\theta = 2(n-1)\left(\frac{(n+1)H^1}{2n}\right)^2.$$

On the other hand, the equality case of (3.1) implies that

$$\begin{aligned}\mathrm{Ric}(e_2) + 1 - (n-1)\frac{c-3}{4} - \frac{3}{4}(c+1)\cos^2\theta &= \frac{(n+1)^2(n-1)}{4n}\|H\|^2 \\ &= n(n-1)\left(\frac{(n+1)H^1}{2n}\right)^2.\end{aligned}$$

Since $n \neq 1, 2$, by equating the last two equations we find $H^1 = 0$. Thus, (h^r_{jk}) are all zero, i.e., M is a totally geodesic submanifold in $\tilde{M}(c)$. Now, let us assume that $n = 2$. If M is not totally geodesic, one has

$$h(e_1, e_1) = \lambda e_3, \qquad h(e_2, e_2) = \mu e_3, \qquad h(e_1, e_2) = \mu e_4,$$

with $\lambda = 3\mu = \frac{9}{4}H^1$, i.e., M is H-umbilical. ■

4. An inequality for the scalar curvature. Let M be an $(n+1)$-dimensional special contact slant submanifold of a $(2n+1)$-dimensional Kenmotsu space form $\widetilde{M}(c)$. For any vector field X tangent to M we write $\phi X = TX + FX$, where TX and FX are the tangential and normal components of ϕX, respectively. An orthonormal basis of T_pM, $p \in M$, is given by $\{e_0 = \xi, e_1, \ldots, e_n\}$ and an othonormal basis of $T^\perp_p M$ is given by $\{e^*_1, \ldots, e^*_n\}$, with $e^*_k = \frac{1}{\sin\theta} Fe_k$, $k = \overline{1, n}$.

We denote $h^k_{ij} = g(h(e_i, e_j), e^*_k)$ for $i, j = \overline{0, n}$ and $k = \overline{1, n}$.

For a special contact slant submanifold, $h^k_{ij} = h^i_{jk} = h^j_{ik}$ $(= h^k_{ji} = h^i_{kj} = h^j_{ki})$ for all $i, j, k \in \{1, \ldots, n\}$.

From the Gauss equation it follows that

$$(n+1)^2\|H\|^2 = 2\tau + \|h\|^2 - n(n+1)\frac{c-3}{4} - (3\|T\|^2 - 2n)\frac{c+1}{4}.$$

By the definition,

$$(n+1)^2\|H\|^2 = \sum_{i=1}^{n}\Big[\sum_{j=1}^{n}(h^i_{jj})^2 + 2\sum_{1\le j<k\le n} h^i_{jj}h^i_{kk}\Big].$$

We derive

$$\begin{aligned}2\tau &= n(n+1)\frac{c+3}{4} + (3\|T\|^2-2n)\frac{c+1}{4} + (n+1)^2\|H\|^2 - \|h\|^2\\ &= n(n+1)\frac{c-3}{4} + (3\|T\|^2-2n)\frac{c+1}{4}\\ &\quad + 2\sum_i\sum_{j<k} h^i_{jj}h^i_{kk} - 2\sum_{i\ne j}(h^i_{jj})^2 - 6\sum_{i<j<k}(h^k_{ij})^2 - \sum_{j,k=1}^{n}(h^k_{0j})^2.\end{aligned}$$

If we denote $m = \frac{n+2}{n-1}$, we get

$$\begin{aligned}&(n+1)^2\|H\|^2 - m\Big[2\tau - n(n+1)\frac{c-3}{4} - (3\|T\|^2-2n)\frac{c+1}{4}\Big]\\ &= \sum_i (h^i_{ii})^2 + (1+2m)\sum_{i\ne j}(h^i_{jj})^2 + 6m\sum_{i<j<k}(h^k_{ij})^2 - 2(m-1)\sum_i\sum_{j<k}h^i_{jj}h^i_{kk}\\ &= \sum_i (h^i_{ii})^2 + 6m\sum_{i<j<k}(h^k_{ij})^2 + (m-1)\sum_i\sum_{j<k}(h^i_{jj}-h^i_{kk})^2\\ &\quad + [1+2m-(n-2)(m-1)]\sum_{i\ne j}(h^i_{jj})^2 - 2(m-1)\sum_{i\ne j}h^i_{ii}h^i_{jj}\\ &= 6m\sum_{i<j<k}(h^k_{ij})^2 + (m-1)\sum_{i\ne j,k}\sum_{j<k}(h^i_{jj}-h^i_{kk})^2\\ &\quad + \frac{1}{n-1}\sum_{i\ne j}[h^i_{ii} - (n-1)(m-1)h^i_{jj}]^2 \ge 0.\end{aligned}$$

For a contact θ-slant submanifold we have $\|T\|^2 = n\cos^2\theta$.

Summing up, we have derived the following.

THEOREM 4.1. *Let M be an $(n+1)$-dimensional special contact slant submanifold of a $(2n+1)$-dimensional Kenmotsu space form $\widetilde{M}(c)$. Then*

$$\begin{aligned}\|H\|^2 \ge{}& \frac{2(n+2)}{(n-1)(n+1)}\tau - \frac{n(n+2)}{(n-1)(n+1)}\cdot\frac{c-3}{4}\\ &- \frac{n(n+2)}{(n-1)(n+1)^2}(3\cos^2\theta-2)\frac{c+1}{4}.\end{aligned}$$

Equality holds at all $p\in M$ if and only if there exists a real function μ on M

such that the second fundamental form satisfies the relations

$$h(e_1, e_1) = 3\mu e_1^*, \quad h(e_2, e_2) = \cdots = h(e_n, e_n) = \mu e_1^*,$$
$$h(e_i, e_j) = \mu e_i^*, \quad h(e_j, e_k) = 0 \quad (2 \leq j \neq k \neq n),$$

with respect to a suitable orthonormal frame $\{e_0 = \xi, e_1, \ldots, e_n\}$ *on* M, *where* $e_k^* = \frac{1}{\sin\theta} F e_k$, $k \in \{1, \ldots, n\}$.

References

[1] K. Arslan, R. Ezentas, I. Mihai, C. Murathan and C. Ozgur, *Ricci curvature of submanifolds in Kenmotsu space forms*, Int. J. Math. Math. Sci. 29 (2002), 719–726.
[2] B. Y. Chen, *Geometry of Slant Submanifolds*, Katholieke Univ. Leuven, 1990.
[3] B. Y. Chen, *Relations between Ricci curvature and shape operator for submanifolds with arbitrary codimensions*, Glasgow Math. J. 41 (1999), 33–44.
[4] B. Y. Chen, *On Ricci curvature of isotropic and Lagrangian submanifolds in complex space forms*, Arch. Math. (Basel) 74 (2000), 154–160.
[5] B. Y. Chen, *Pseudo-Riemannian Geometry, δ-invariants and Applications*, World Sci., Hackensack, NJ, 2011.
[6] S. Deng, *An improved Chen–Ricci inequality*, Int. Electron. J. Geom. 2 (2009), 39–45.
[7] K. Kenmotsu, *A class of almost contact Riemannian manifolds*, Tôhoku Math. J. 24 (1972), 93–103.
[8] A. Lotta, *Slant submanifolds in contact geometry*, Bull. Math. Soc. Sci. Math. Roumanie 39 (1996), 183–198.
[9] K. Matsumoto, I. Mihai and R. Rosca, *A certain locally conformal almost cosymplectic manifold and its submanifolds*, Tensor (N.S.) 51 (1992), 91–102.
[10] K. Matsumoto, I. Mihai and M. H. Shahid, *Certain submanifolds of a Kenmotsu manifold*, in: The Third Pacific Rim Geometry Conference (Seoul, 1996), Monogr. Geom. Topology 25, Int. Press, Cambridge, MA, 1998, 183–193.
[11] I. Mihai, *Ricci curvature of submanifolds in Sasakian space forms*, J. Austral. Math. Soc. 72 (2002), 247–256.
[12] Gh. Pitiş, *Geometry of Kenmotsu Manifolds*, Publishing House of Transilvania Univ. Braşov, 2007.
[13] S. Tanno, *The automorphism groups of almost contact Riemannian manifolds*, Tôhoku Math. J. 21 (1969), 21–38.
[14] K. Yano and M. Kon, *Structures on Manifolds*, World Sci., Singapore, 1984.

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