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Definable stratification satisfying the Whitney property with exponent 1

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Abstract. We prove that for a finite collection of sets $A_1, \dots, A_s \subset \mathbb{R}^{k+n}$ definable in an o-minimal structure there exists a compatible definable stratification such that for any stratum the fibers of its projection onto $\mathbb{R}^k$ satisfy the Whitney property with exponent 1.

Introduction. K. Kurdyka proved (in [4]) that for any locally finite family of subanalytic sets in $\mathbb{R}^n$ there exists a subanalytic stratification of $\mathbb{R}^n$ compatible with every element of the family and such that all strata satisfy the Whitney property with exponent 1. The aim of our note is to prove a version with parameter of the above theorem for an o-minimal structure on $(\mathbb{R}, +, \cdot)$.

Theorem 1. *Let $\mathcal{S}$ be an o-minimal structure on $(\mathbb{R}, +, \cdot)$ and let $A_1, \dots, A_s \subset \mathbb{R}^{k+n}$ be definable sets in $\mathcal{S}$. Then there exists a finite definable stratification of $\mathbb{R}^{k+n}$ compatible with the sets $A_1, \dots, A_s$ and such that for any stratum Q of this stratification and any point $y \in \pi(Q)$ the fiber Q_y is* (*in some coordinate system in $\mathbb{R}^n$*) *a definable cell satisfying the Whitney property with exponent* 1 (*and coefficient depending only on n*).

In the proofs we shall use properties of the closure of a definable cell and extensions of definable functions to the boundary.

1. Basic properties of o-minimal structures. In this section we collect some basic properties of o-minimal structures on $(\mathbb{R}, +, \cdot)$, crucial for further considerations. Let us start with some definitions.

Definition 1 ([2]). A *structure $\mathcal{S}$ on $\mathbb{R}$* consists of a collection $\mathcal{S}_n$ of subsets of $\mathbb{R}^n$, for each $n \in \mathbb{N}$, such that

(1) $\mathcal{S}_n$ is a boolean algebra of subsets of $\mathbb{R}^n$,

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(2) $\mathcal{S}_n$ contains the diagonals $\{(x_1, \ldots, x_n) \in \mathbb{R}^n : x_i = x_j\}$ for $1 \le i < j \le n$,
(3) if $A \in \mathcal{S}_n$, then $A \times \mathbb{R}$ and $\mathbb{R} \times A$ belong to $\mathcal{S}_{n+1}$,
(4) if $A \in \mathcal{S}_{n+1}$, then $\pi(A) \in \mathcal{S}_n$, where $\pi : \mathbb{R}^{n+1} \to \mathbb{R}^n$ is the projection on the first n coordinates.

We say that a set $A \subset \mathbb{R}^n$ is *definable* iff $A \in \mathcal{S}_n$. A function $f : A \to \mathbb{R}^m$ with $A \subset \mathbb{R}^n$ is called *definable* iff its graph is definable.

DEFINITION 2 ([2]). A structure $\mathcal{S}$ on $\mathbb{R}$ is *o-minimal* iff

(1) $\{(x, y) : x < y\} \in \mathcal{S}_2$ and $\{a\} \in \mathcal{S}_1$ for each $a \in \mathbb{R}$,
(2) each set in $\mathcal{S}_1$ is a finite union of intervals (a, b), $-\infty \le a < b \le +\infty$, and points $\{a\}$.

A *structure on* $(\mathbb{R}, +, \cdot)$ is a structure on $\mathbb{R}$ containing the graphs of both addition and multiplication.

2. Cell decomposition and stratification

DEFINITION 3 ([1]). *Cells* in $\mathbb{R}^n$ are definable sets defined in the following inductive way:

(1) The cells in $\mathbb{R}^1$ are exactly points and open intervals,
(2) Let $C \subset \mathbb{R}^n$ be a cell and let f, $g : C \to \mathbb{R}$ be continuous definable functions such that $f < g$ on C. Then

$$(f, g) := \{(x, r) \in C \times \mathbb{R} : \ f(x) < r < g(x)\}$$

is a cell in $\mathbb{R}^{n+1}$. Also, given a continuous definable function $f : C \to \mathbb{R}$ on a cell C in $\mathbb{R}^n$, the graph

$$\mathit{\Gamma}(f) = \{(x, r) \in C \times \mathbb{R} : \ r = f(x)\}$$

and the sets

$$\{(x, r) \in C \times \mathbb{R} : f(x) < r\},\ \{(x, r) \in C \times \mathbb{R} : r < f(x)\},\ C \times \mathbb{R}$$

are cells in $\mathbb{R}^{n+1}$.

DEFINITION 4 ([1]). *A cell decomposition* of $\mathbb{R}^n$ is a partition of $\mathbb{R}^n$ into finitely many cells defined in the following inductive way:

(1) A decomposition of $\mathbb{R}^1$ is a collection of open intervals and points of the following form:

$$\{(-\infty, a_1),\ (a_1, a_2), \ldots, (a_k, +\infty), \{a_1\}, \ldots, \{a_k\}\}.$$

(2) A decomposition of $\mathbb{R}^{n+1}$ is a finite partition of $\mathbb{R}^{n+1}$ into cells A such that the set of projections $\pi(A)$ is a decomposition of $\mathbb{R}^n$, where $\pi : \mathbb{R}^{n+1} \to \mathbb{R}^n$ is the projection on the first n coordinates.

In a similar way we define a $\mathcal{C}^k$ cell and $\mathcal{C}^k$ cell decomposition, by requiring that the functions in part (2) of Definition 3 are $\mathcal{C}^k$ functions.

PROPOSITION 2 ([2]). *Any o-minimal structure $\mathcal{S}$ on $(\mathbb{R}, +, .)$ admits $\mathcal{C}^1$ cell decompositions, i.e.:*

(1) *If $A_1, \ldots, A_k \subset \mathbb{R}^n$ are definable sets then there exists a $\mathcal{C}^1$ cell decomposition of $\mathbb{R}^n$ compatible with $A_1, \ldots, A_k$.*
(2) *For each definable function $f : A \to \mathbb{R}$ with $A \subset \mathbb{R}^n$ there exists a cell decomposition of $\mathbb{R}^n$ partitioning A and such that for every $C \subset A$ in the decomposition the restriction $f|C : C \to \mathbb{R}$ is a $\mathcal{C}^1$ function.*

REMARK 3. Every o-minimal structure on $(\mathbb{R}, +, \cdot)$ admits $\mathcal{C}^k$ cell decompositions (for any positive integer k), i.e. the above proposition holds with $\mathcal{C}^1$ replaced by $\mathcal{C}^k$.

DEFINITION 5. We call a definable subset of $\mathbb{R}^n$ which is a $\mathcal{C}^k$ submanifold of $\mathbb{R}^n$ a *definable $\mathcal{C}^k$ stratum* in $\mathbb{R}^n$.

A *definable $\mathcal{C}^k$ stratification* of $\mathbb{R}^n$ is a finite partition of $\mathbb{R}^n$ into definable $\mathcal{C}^k$ strata satisfying the following boundary condition: for any two strata S, T of the partition, if $S \cap \partial T \neq \emptyset$ then $S \subset \partial T$.

DEFINITION 6. A set T definable in an o-minimal structure *satisfies the Whitney property with exponent α* (cf. [5]) if there exists a positive constant C such that any points p and q in T can be joined by a definable curve γ with $\text{length}(\gamma) \leq C|p - q|^\alpha$.

3. Angle between linear subspaces

DEFINITION 7. The *angle between a linear subspace X and a line P* in $\mathbb{R}^n$ is the number

$$\delta(P, X) = \inf\{\sin(P, S) : S \text{ a line in } X\}$$

where $\sin(P, S)$ denotes the sine of the angle between the lines P and S.

The *angle between linear subspaces X and Y* in $\mathbb{R}^n$ is the number

$$\delta(Y, X) := \sup\{\delta(P, X) : P \text{ a line in } Y\}.$$

If $Y = 0$ we put $\delta(0, X) = 0$.

REMARK 4 ([4]).

(1) If $\dim X = \dim Y$ then $\delta(X, Y) = \delta(Y, X)$.
(2) If $\dim X \leq \dim Y \leq \dim Z$ then $\delta(Z, X) \leq \delta(Z, Y) + \delta(Y, X)$.
(3) Let $\mathbb{G}(k, m)$ be the Grassmannian of k-dimensional subspaces in $\mathbb{R}^m$. The mapping $\mathbb{G}(k, m) \times \mathbb{G}(k, m) \ni (X, Y) \mapsto \delta(X, Y) \in \mathbb{R}$ is continuous and semialgebraic.

(4) For any $\alpha > 0$ there exists $M > 0$ such that if $\delta(P, X) > \alpha$ for some linear hyperplane X and a line P then X is the graph of a linear map $\phi : P^{\perp} \to P$ satisfying $\|\phi\| \leq M$.

LEMMA 5 ([4, Lem. 3]). *For any nonnegative integers r, n there exist $\varepsilon, m > 0$ such that for any hyperplanes $X_1, \ldots, X_r$ in $\mathbb{R}^n$ there exists a line P such that for any hyperplanes $Y_1, \ldots, Y_r$ satisfying $\delta(X_i, Y_i) < \varepsilon$ we have $\delta(P, Y_i) > m$.*

4. Closure of a cell. The closure of a definable cell is also definable. In this section we shall give a description of the closure of a cell. We shall consider separately cells of graph and band types.

EXAMPLE 6. Consider the following cell of graph type in $\mathbb{R}^3$:

$$Q = \{(x, y, z) : 0 < x < 1, 0 < y < 1, z = x/y\}.$$

The closure $\overline{Q}$ of Q is **not** a graph, its fiber over any point from the closure of the projection of $\overline{\pi(Q)} = [0, 1]^2$ different from $(0, 0)$ consists of one point, whereas the fiber over $(0, 0)$ is the half-line $[0, \infty)$.

We shall show that for any cell of graph type the set of points over which the fiber of the closure is infinite has small dimension.

LEMMA 7. *Let $f : Q \to \mathbb{R}$ be a continuous definable function defined on a cell of dimension d in $\mathbb{R}^n$. There is a definable set $Z \subset \partial Q$ of dimension $\leq d - 2$ such that f has a continuous extension to $\overline{Q} \setminus Z$.*

Proof. Let $Z := \{x \in \partial Q : \lim_{y \to x,\, y \in Q} f(y)$ does not exist$\}$. To prove that $\dim Z \leq d - 2$, assume to the contrary that Z contains a cell W of dimension $d - 1$. The boundary of the graph of f has dimension smaller than d, so the set of points in the closure of Q for which the fiber of the closure of the graph is infinite (i.e. the function has infinitely many accumulation points) has dimension smaller than $d - 1$.

Using the cell decomposition we may assume that at any $x \in Z$ the function f has finitely many accumulation points and that the definable functions $\limsup_{y \to x,\, y \in Q} f(y)$ and $\liminf_{y \to x,\, y \in Q} f(y)$ are continuous on Z. Fix $x_0 \in Z$ and set $a = \limsup_{y \to x_0,\, y \in Q} f(y)$, $b = \liminf_{y \to x_0,\, y \in Q} f(y)$. There exist numbers $c \in (a, b)$ and $e > 0$ such that $|f(y) - c| > e$ in a neighborhood of x_0 in $\overline{Q}$. Let $Q_1 \subset Q$ be a cell such that $\overline{Q}_1$ is a neighborhood of x_0 in $\overline{Q}$ and $|f(x) - c| > e$ on Q_1. This contradicts the connectedness of Q_1. ■

If

$$Q = \{(x, x_n) \in \mathbb{R}^{n-1} \times \mathbb{R} : x \in Q_1,\ f(x) < x_n < g(x)\}$$

is a *cell of band type* then

$$\overline{Q} = Q \cup \operatorname{graph} f \cup \operatorname{graph} g \cup (\overline{Q} \cap (\partial Q_1 \times \mathbb{R})).$$

The cell Q is "bounded from below and above" by cells of graph type, graph g and graph f, which we shall call the *top* and *bottom decks* of Q. The closure $\overline{Q}$ of Q is bounded by the closures of the top and bottom decks. In the case of cells of band type with only one deck or without a deck the closure is described similarly.

LEMMA 8. *Under the assumptions of Theorem* 1, *for any* $\varepsilon > 0$ *there exists a cell decomposition* $\mathcal{T}$ *of* $\mathbb{R}^k \times \mathbb{R}^n$ *compatible with* $A_1, \ldots, A_s$ *and satisfying the following conditions*:

(1) *for any cell* $Q \in \mathcal{T}$ *such that* $\dim Q_y = n - 1$ *and any points* $(x', y'), (x'', y'') \in Q$ *we have*
$$\delta(T_{x'}Q_{y'}, T_{x''}Q_{y''}) < \varepsilon,$$

(2) *for any cell* $Q \in \mathcal{T}$ *such that* $\dim Q_y = n$ *for some* $y \in \pi Q$ *there exist cells* $B_1, \ldots, B_p \in \mathcal{T}$ $(p \leq 2n)$ *such that* $\dim (B_i)_y = n - 1$, $(B_i)_y \subset \overline{Q}_y \setminus Q_y$ *and the set* $\partial Q_y \setminus \bigcup (B_i)_y$ *is a finite union of cells of dimension* $\leq n - 2$.

Proof. We use induction on n.

There exists a cell decomposition of $\mathbb{R}^k \times \mathbb{R}^n$ compatible with $A_1, \ldots, A_s$ and such that the corresponding decomposition of $\mathbb{R}^k \times \mathbb{R}^{n-1}$ satisfies the assertion of the lemma. Consequently, condition (1) holds for any cell of band type. For any cell $Q \subset \mathbb{R}^k \times \mathbb{R}^n$ such that $\dim Q_y = n-1$ and Q_y is of graph type consider the map
$$Q \ni (x, y) \mapsto T_x Q_y \in \mathbb{G}(n-1, n).$$
Since this map is definable we can assume, after refining the decomposition in $\mathbb{R}^k \times \mathbb{R}^{n-1}$, that condition (1) holds for any cell Q such that $\dim Q_y = n-1$ and Q_y is of graph type.

Fix a cell Q such that Q_y is an open cell. Clearly Q is of band type, so
$$Q = \{(y, x) \in Q_1 \times \mathbb{R} : f(y, x_1, \ldots, x_{n-1}) < x_n < g(y, x_1, \ldots, x_{n-1})\}$$
where Q_1 is the projection of Q onto $\mathbb{R}^k \times \mathbb{R}^{n-1}$.

By Lemma 7 there exists a definable subset $Z \subset \partial Q_1$ such that for any y we have $\dim Z_y < n-2$ and the functions $f(y, \cdot)$ and $g(y, \cdot)$ extend continuously to $(\partial \overline{Q}_1 \setminus Z)_y$. After refining we may assume that the decomposition of $\mathbb{R}^k \times \mathbb{R}^{n-1}$ is compatible with Q_1, $\overline{Q}_1$ and Z and satisfies the assertions of the lemma.

We have constructed a cell decomposition of $\mathbb{R}^k \times \mathbb{R}^n$ such that

- for any cell Q of this decomposition such that Q_y is of graph type and $\dim Q_y = n - 1$ condition (1) holds,
- for any cell Q such that Q_y is open,
$$Q = \{(y, x) \in Q_1 \times \mathbb{R} : f(y, x_1, \ldots, x_{n-1}) < x_n < g(y, x_1, \ldots, x_{n-1})\},$$

there exist definable cells $\widetilde{B}_1, \ldots, \widetilde{B}_p$ such that $(\widetilde{B}_i)_y \subset \partial(Q_1)_y$ and $(\partial Q_1)_y \setminus \bigcup(\widetilde{B}_i)_y$ is a union of cells of dimension $< n-2$ and the functions $f(y,\cdot)$, $g(y,\cdot)$ have continuous extensions $\widetilde{f}(y,\cdot)$, $\widetilde{g}(y,\cdot)$ onto $(Q_1)_y \cup \bigcup(\widetilde{B}_i)_y$.

Put

$$B_i = \{(y,x) \in \mathbb{R}^k \times \mathbb{R}^n : (y, x_1, \ldots, x_{n-1}) \in \widetilde{B}_i, \\ \widetilde{f}(y, x_1, \ldots, x_{n-1}) < x_n < \widetilde{g}(y, x_1, \ldots, x_{n-1})\}$$

for $i = 1, \ldots, p$, and

$$B_{p+1} = \operatorname{graph} f, \quad B_{p+2} = \operatorname{graph} g.$$

Clearly $(B_1)_y, \ldots, (B_{p+2})_y$ are cells of dimension $n-1$, and $p+2 \le 2n$. We now show that $\dim(\partial Q_y \setminus \bigcup(\widetilde{B}_i)_y) < n-1$. Assume that $\partial Q_y \setminus \bigcup(\widetilde{B}_i)_y$ contains a cell C of dimension $n-1$; we can assume (after refining the decomposition) that C is a cell of the decomposition. If $C \subset \overline{(B_{p+1})_y} \cup \overline{(B_{p+2})_y}$ then by Lemma 7 we would get $C \subset (B_{p+1})_y$ or $C \subset (B_{p+2})_y$, contrary to our assumptions. Consequently, $C \cap \big(\overline{(B_{p+1})_y} \cup \overline{(B_{p+2})_y}\big) = \emptyset$. This means that the projection of C onto $\mathbb{R}^{n-1}$ is contained in one of the sets $(\widetilde{B}_i)_y$, $i = 1, \ldots, p$. But then $C \subset (B_i)_y \cup \operatorname{graph} \widetilde{g}|(\widetilde{B}_i)_y \cup \operatorname{graph} \widetilde{f}|(\widetilde{B}_i)_y$, which contradicts the choice of C. ∎

LEMMA 9. *Let $A \subset \mathbb{R}^k \times \mathbb{R}^n$ be a definable set and let $d := \max \dim A_y$. For any $\varepsilon > 0$ there exists a cell decomposition $\mathcal{T}$ of $\mathbb{R}^k \times \mathbb{R}^n$ compatible with A such that for any cell Q of $\mathcal{T}$ satisfying $\dim Q_y = d$ and any points $(x', y'), (x'', y'') \in Q$ we have*

$$\delta(T_{x'} Q_{y'}, T_{x''} Q_{y''}) < \varepsilon.$$

Proof. The proof is similar to the proof of (1) in Lemma 8. ∎

5. Proof of Theorem 1. We shall prove the theorem using induction on n. Since in $\mathbb{R}$ every cell is a point, segment, half-line or line, the theorem is obvious for $n = 1$.

We shall construct a sequence $\mathcal{T}_i$ of definable stratifications compatible with sets $A_1, \ldots, A_s$ and such that for each stratum $Q \in \mathcal{T}_i$ with $\dim Q > n+k-i$ and any point $y \in \pi(Q)$ the fiber Q_y is a cell satisfying the Whitney property with exponent 1. We can take as $\mathcal{T}_0$ any definable stratification compatible with $A_1, \ldots, A_s$. Then $\mathcal{T}_{i+1}$ is constructed by refinement of strata from $\mathcal{T}_i$ of dimension at most $n+k-i$.

Using Lemmata 8 and 9 it is enough to prove that for any cell Q satisfying the assertions of the lemma there are subsets $Q_1, \ldots, Q_r \subset Q$ such that $\dim(Q \setminus \bigcup_i Q_i) < \dim Q$ and for any point $y \in \pi(Q_i)$ the fiber $(Q_i)_y$ is a cell satisfying the Whitney property with exponent 1.

Case I. If $\dim Q_y = n$ (i.e. Q_y is an open cell in $\mathbb{R}^n$) then there exist cells $B_1, \ldots, B_p$ ($p \leq 2n$) such that

- for any points (x', y'), $(x'', y'') \in B_i$ we have
$$\delta(T_{x'}(B_i)_{y'}, T_{x''}(B_i)_{y''}) < \varepsilon,$$
- $\dim (B_i)_y = n - 1$,
- $\partial Q_y \setminus \bigcup (B_i)_y$ is a finite sum of cells of dimension $\leq n - 2$.

By Lemma 5 there exists a line L in $\mathbb{R}^n$ such that for any point $(x, y) \in Q$ we have $\delta(L, T_x(B_i)_y) > \alpha$, where $i = 1, \ldots, p$, and α is a constant depending only on n. Changing coordinates in $\mathbb{R}^n$ we can assume that L is the x_n-axis. Every cell $(B_i)_y$ is locally the graph of a definable function with derivative bounded by a constant M_n depending only on n.

Using cell decomposition and the inductive hypothesis we get a cell decomposition $\mathcal{C}$ of $\mathbb{R}^k \times \mathbb{R}^n$ compatible with Q and B_i such that the induced decomposition $\mathcal{C}_1$ of $\mathbb{R}^k \times \mathbb{R}^{n-1}$ satisfies the assertion of the theorem. Let $\widetilde{\mathcal{C}}$ be a cell decomposition of $\mathbb{R}^k \times \mathbb{R}^n$ given by the cell decomposition $\mathcal{C}_1$ of $\mathbb{R}^k \times \mathbb{R}^{n-1}$ and the sets B_i (this means that for any cell of $\widetilde{\mathcal{C}}$ its projection is an element of $\mathcal{C}_1$, and each cell of graph type is a subset of some B_i).

For any cell $K \in \widetilde{\mathcal{C}}$ of graph type such that $\dim K_y = n - 1$ we have $K_y \subset (B_i)_y$ for some i, and so K_y is the graph of a function with derivative bounded by the constant M_n and defined on some cell in $\mathbb{R}^{n-1}$ satisfying the Whitney property with exponent 1 and coefficient $L_n := L_{n-1}\sqrt{1 + M_{n-1}^2}$ depending only on n.

Consequently, each cell $K \in \widetilde{\mathcal{C}}$ satisfies the Whitney property with exponent 1 and coefficient depending only on n because its projection and decks satisfy the Whitney property.

Let $Q_1, \ldots, Q_r$ be cells of the decomposition $\widetilde{\mathcal{C}}$ such that $\dim (Q_i)_y = n$ and $Q_i \cap Q \neq \emptyset$. Clearly $\dim(Q_y \setminus \bigcup_i (Q_i)_y) \leq n - 1$ and $(Q_i)_y$ satisfies the Whitney property with exponent 1 and coefficient depending only on n. Since $\partial Q_y \setminus \bigcup (B_j)_y$ is a finite sum of cells of dimension $\leq n - 2$ and $Q_i \cap B_j = \emptyset$ we get $Q_i \subset Q$.

Case II. If $d = \dim Q_y < n$ then there exists a line L in $\mathbb{R}^n$ such that $\delta(L, T_x Q_y) \geq 1 - \varepsilon$ for any $(x, y) \in Q$. After a change of variables in $\mathbb{R}^n$ we can assume that $L = (x_1 = \cdots = x_{n-1} = 0)$. Then every Q_y is the graph of a $\mathcal{C}^1$ function with derivative bounded by an arbitrarily small positive constant (depending on ε) defined on the set $\widetilde{Q}_y$, where $\widetilde{Q}$ is the projection of Q onto $\mathbb{R}^k \times \mathbb{R}^{n-1}$.

Applying the inductive hypothesis we can find $\widetilde{Q}_1, \ldots, \widetilde{Q}_r \subset \widetilde{Q}$ which are definable cells satisfying the Whitney property with exponent 1 and coefficient depending only on d, and such that $\dim(\widetilde{Q} \setminus \bigcup_i \widetilde{Q}_i) < \dim \widetilde{Q}$. Now, put $Q_i = Q \cap (\widetilde{Q}_i \times \mathbb{R})$. ∎

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