A correction to and a remark on our paper
“Remarks on the proof of a generalized Hartogs Lemma”

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1. Correction. In our paper [1] on page 45 the definition of the space \( E \) should have been the following (after restricting \( \psi \) to be in \( \mathcal{F}_0 \)): \( f \in \mathcal{E} \) if \( f \) is a continuous function defined on \( \mathbb{C} \) that tends to 0 at \( \infty \), and \( \partial f / \partial \bar{z} \) (in the sense of distributions) is a continuous function with support in the closed unit disc.

Then the remark “Due to the support condition on \( \theta \)”, line 4 of page 47, makes sense. We wrote instead that \( \partial f / \partial \bar{z} \) should simply tend to 0 at \( \infty \). The definition given in [1] allows a proof by a continuity argument. But we prefer the finitely many steps proof (based on (d), line –2 of page 45) that uses the Lemma for which we should have taken the above definition.

We thank Jernej Tonejc for pointing out the inconsistency in our paper.

2. A remark. At several points in our paper we mention that uniqueness, which obviously breaks down in higher dimensions (see 2.3 in [1]), is crucial in our proof of existence of solutions to the differential equation \( \partial f / \partial \bar{z} = \psi(z, f(z)) \). However, L. Lempert made a very disturbing remark: In higher dimensions (i.e. for \( f \) vector-valued) one can still prove the existence of solution to the differential equation by a simple application of the Schauder fixed point theorem, after transforming the equation into an integral equation as in [1]. This is contrary to what we thought. But what one does not get is good dependence on parameters, and thus the solution cannot be used in Kontinuitätssatz arguments. In fact, the generalized Hartogs Lemma is known to fail in dimension \( > 2 \) [3].

2000 Mathematics Subject Classification: Primary 32F20.
Key words and phrases: Hartogs Lemma, Schauder fixed point theorem.
A somewhat similar situation where the Schauder fixed point theorem again gives existence of solutions but does not allow good control of the solutions is discussed in [2].

References

