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On a kth-order differential equation

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Abstract. We prove a theorem on the growth of a solution of a kth-order linear differential equation. From this we obtain some uniqueness theorems. Our results improve several known results. Some examples show that the results are best possible.

1. Introduction and main results. In this paper, by meromorphic function we shall always mean a meromorphic function in the complex plane. We adopt the standard notations in the Nevanlinna theory of meromorphic functions as explained in [5], [7], [11]. It will be convenient to let E denote any set of positive real numbers of finite linear measure, not necessarily the same at each occurrence. For any nonconstant meromorphic function $h(z)$, we denote by $S(r,h)$ any quantity satisfying

$$S(r,h)=o(T(r,h)) \quad (r\to\infty,\ r\notin E).$$

Let f and g be two nonconstant meromorphic functions and let a be a finite complex number. We say that f and g *share the value a CM* provided that $f-a$ and $g-a$ have the same zeros with the same multiplicities. Similarly, we say that f and g *share the value a IM* provided that $f-a$ and $g-a$ have the same zeros ignoring multiplicities. In addition, we say that f and g *share ∞ CM* if $1/f$ and $1/g$ share the value 0 CM, and we say that f and g *share ∞ IM* if f and g share the value 0 IM (see [13]). In this paper, we also need the following two definitions.

Definition 1.1. Let f be a nonconstant entire function. The *order* of f, denoted $\sigma(f)$, is defined by

$$\sigma(f)=\limsup_{r\to\infty}\frac{\log T(r,f)}{\log r}=\limsup_{r\to\infty}\frac{\log\log M(r,f)}{\log r},$$

where $M(r,f)=\max_{|z|=r}\{|f(z)|\}$.

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Definition 1.2. Let f be a non-constant meromorphic function. The *hyper-order* of f, denoted $\nu(f)$, is defined by

$$\nu(f)=\limsup_{r\to\infty}\frac{\log\log T(r,f)}{\log r}.$$

In 1977, L. A. Rubel and C. C. Yang proved the following theorem.

Theorem A (see [9]). *Let f be a nonconstant entire function. If f and f' share two finite distinct values CM, then $f\equiv f'$.*

In 1996, R. Brück proved the following theorems.

Theorem B (see [1]). *Let f be a nonconstant entire function satisfying $\nu(f)<\infty$, where $\nu(f)$ is not a positive integer. If f and f' share the value 0 CM, then $f\equiv cf'$ for some constant $c\neq 0$.*

Theorem C (see [1]). *Let f be a nonconstant entire function. If f and f' share the value 1 CM, and if $N(r,1/f')=S(r,f)$, then $f-1\equiv c(f'-1)$ for some constant $c\neq 0$.*

Brück made the following conjecture.

Conjecture 1.1 (see [1]). Let f be a nonconstant entire function satisfying $\nu(f)<\infty$, where $\nu(f)$ is not a positive integer. If f and f' share one finite value a CM, then $f-a\equiv c(f'-a)$ for some constant $c\neq 0$.

Regarding Conjecture 1.1, a natural question is:

Question 1.1 (see [12, Question 1]). What can be said when a nonconstant entire function f shares one finite value a with one of its derivatives $f^{(k)}$ $(k\geq 1)$?

Consider the differential equation

$$F^{(k)}-e^{Q(z)}F=1, \tag{1.1}$$

where k is a positive integer, and $Q(z)$ is an entire function.

Regarding Question 1.1, in 1999, Lian-Zhong Yang proved the following results for $a\neq 0$ under the additional assumption $\sigma(f)<\infty$.

Theorem D (see [12, Theorem 1]). *Let $Q(z)$ be a nonconstant polynomial and k be a positive integer. Then every solution F of (1.1) is an entire function of infinite order.*

Theorem E (see [12, Theorem 2]). *Let f be a nonconstant entire function of finite order, and let $a\neq 0$ be a finite constant. If f and $f^{(k)}$ share one finite value a CM, where k is a positive integer, then $f-a\equiv c(f^{(k)}-a)$ for some constant $c\neq 0$.*

In this paper, we shall prove the following results, which improve and supplement Theorems D and E.

THEOREM 1.1. *Let $Q(z)$ be a polynomial. If F is a solution of* (1.1), *then $\nu(F) = \gamma_Q$, where γ_Q is the degree of $Q(z)$.*

From Theorem 1.1 we easily obtain the following two corollaries.

COROLLARY 1.1. *Let $Q(z)$ be a polynomial. If F is a solution of* (1.1) *such that $\sigma(F) = \infty$, then $Q(z)$ is a nonconstant polynomial and $\nu(F) = \gamma_Q$, where γ_Q is the degree of $Q(z)$.*

COROLLARY 1.2. *Let $Q(z)$ be a polynomial, let $a \neq 0$ be a complex number, and let k be a positive integer. If f is a solution of the differential equation*

$$\frac{f^{(k)} - a}{f - a} = e^{Q(z)} \tag{1.2}$$

such that $\nu(f)$ is not a positive integer, then $f - a \equiv c(f^{(k)} - a)$ for some constant $c \neq 0$.

Proof of Corollary 1.2. Let

$$f/a - 1 = F. \tag{1.3}$$

From (1.2) and (1.3) we easily get (1.1). From (1.1), (1.3) and Theorem 1.1 we obtain the conclusion of Corollary 1.2.

Corollary 1.2 implies the following result.

COROLLARY 1.3. *Let f be a nonconstant entire function such that $\sigma(f) < \infty$, and let $a\,(\neq 0)$ be a complex number. If f and $f^{(k)}$ share the value a CM, where k is a positive integer, then $f - a \equiv c(f^{(k)} - a)$ for some constant $c \neq 0$.*

From Theorem 1.1 we also get the following result on the growth of a nonconstant solution of a $(k+1)$th order linear differential equation.

COROLLARY 1.4. *If F is a nonconstant solution of the differential equation*

$$F^{(k+1)} - e^{Q(z)}F' - Q'(z)e^{Q(z)}F = 0, \tag{1.4}$$

where k is a positive integer and $Q(z)$ is a polynomial, then $\nu(F) = \gamma_Q$, where γ_Q is the degree of $Q(z)$.

Proof. Since F is a nonzero solution of (1.4), we easily deduce that

$$F^{(k)} - e^{Q(z)}F = c, \tag{1.5}$$

where c is a complex constant. We discuss the following two cases.

CASE 1. Suppose that $c = 0$. Then the conclusion of Corollary 1.4 is obvious.

Case 2. Suppose that

$$c \neq 0. \tag{1.6}$$

From (1.5) and (1.6) we get

$$\left(\frac{F}{c}\right)^{(k)} - e^{Q(z)}\,\frac{F}{c} = 1. \tag{1.7}$$

From (1.7) and Theorem 1.1 we deduce the conclusion of Corollary 1.4.

Now we give the following two examples.

Example 1.1 (see [4]). Let f be a solution of the differential equation

$$\frac{f'-1}{f-1} = e^{z^n},$$

where n is a positive integer. Then f is a nonconstant entire function such that f and f' share the value 1 CM. Moreover, in the same manner as in the proof of Theorem 1.1 we can verify that $\nu(f) = \sigma(e^{z^n}) = n$. This example shows that the situation of Theorem 1.1 and that of Corollary 1.1 can occur. It also shows that the condition "$\nu(f)$ is not a positive integer" in Corollary 1.2 is best possible.

Example 1.2 (see [4]). Let

$$f(z) = \frac{2e^z + z + 1}{e^z + 1}.$$

Then f is a nonconstant meromorphic function, but not an entire function. Moreover, it is easily verified that $\sigma(f) = 1$ and $\nu(f) = 0$, and that f and f' share the value 1 CM. However,

$$\frac{f'(z)-1}{f(z)-1} = -\frac{e^z}{e^z+1}.$$

This example shows that the conclusions of Corollaries 1.2 and 1.3 are invalid if f is not an entire function.

The following three corollaries follow from Corollary 1.2; they improve several results of Lian-Zhong Yang [12].

Corollary 1.5. *Let $Q(z)$ be a polynomial, and let $a\,(\neq 0)$ be a complex number. If f is a solution of the differential equation* (1.2) *such that $\nu(f)$ is not a positive integer, and if there exists a point z_0 such that $f^{(k)}(z_0) = f(z_0) \neq a$, then $f \equiv f^{(k)}$.*

Corollary 1.6. *Let $Q(z)$ be a polynomial, and let $a\,(\neq 0)$ and $b\,(\neq 0)$ be distinct complex numbers. If f is a solution of the differential equation* (1.2) *such that $\nu(f)$ is not a positive integer, and if f and $f^{(k)}$ share the value b IM, then $f \equiv f^{(k)}$.*

Proof of Corollary 1.6. Since f and $f^{(k)}$ share the value $b\,(\neq 0)$ IM, by Hayman's inequality (see [5, Theorem 3.5]) there exists a point z_0 such that $f^{(k)}(z_0) = f(z_0) = b$. Since $a \neq b$, Corollary 1.5 yields the conclusion of Corollary 1.6.

COROLLARY 1.7. *Let $Q(z)$ be a polynomial, let $a\,(\neq 0)$ be a complex number, and let n be a positive integer. If f is a solution of the differential equation* (1.2) *such that $\nu(f)$ is not a positive integer, and if there exists a point z_0 such that $f^{(n)}(z_0) = f^{(n+k)}(z_0) \neq 0$, then $f \equiv f^{(k)}$.*

In 1995, H. X. Yi and C. C. Yang posed the following question named the question of Yi and Yang.

QUESTION 1.2 (see [13, pp. 458]). *Let f be a nonconstant meromorphic function, and let a be a nonzero complex constant. If f, $f^{(n)}$ and $f^{(m)}$ share the value a CM, where n and $m\,(n < m)$ are positive integers of different parity, can we infer that $f \equiv f^{(n)}$?*

In this paper, we shall prove the following theorem.

THEOREM 1.2. *Let $Q(z)$ be a polynomial, let $a\,(\neq 0)$ be a complex number, and let n and k be two positive integers. If f is a solution of the differential equation*

$$\frac{f^{(n+k)} - a}{f^{(n)} - a} = e^{Q(z)}, \tag{1.8}$$

where $\nu(f)$ is not a positive integer, and if the value a is shared by f and $f^{(n)}$ CM, then there exist complex constants $\lambda_j\ (\neq 0)\ (1 \leq j \leq k)$, $c\ (\neq 0)$ and b_0 satisfying

$$\lambda_j^n = \lambda_j^{n+k} = c \qquad (1 \leq j \leq k), \tag{1.9}$$

$$cb_0 + (1-c)a = 0, \tag{1.10}$$

such that

$$f(z) = \sum_{j=1}^{k} \frac{\gamma_j}{c}\, e^{\lambda_j z} + b_0, \tag{1.11}$$

where $\gamma_j\ (1 \leq j \leq k)$ are complex constants.

REMARK 1.1. If $a = 0$, then the conclusions of Corollaries 1.2, 1.3, 1.5–1.7 and Theorem 1.2 are also valid.

2. Some lemmas

LEMMA 2.1 (see [12, Theorem 2]). *Let f be a nonconstant entire function of finite order, and let $a \neq 0$ be a complex number. If f and $f^{(k)}$ share the* *value a CM, where k is a positive integer, then*

$$\frac{f^{(k)}-a}{f-a}\equiv c$$

for some nonzero constant c.

LEMMA 2.2 (see [7, Theorem 3.1] or [6, pp. 36–37]). *If f is an entire function of order $\sigma(f)$, then*

$$\sigma(f)=\limsup_{r\to\infty}\frac{\log\nu(r,f)}{\log r},$$

where $\nu(r,f)$ denotes the central index of $f(z)$.

LEMMA 2.3 (see [3, Lemma 2] or [2, Lemma 4]). *If f is a transcendental entire function of hyper-order $\nu(f)$, then*

$$\nu(f)=\limsup_{r\to\infty}\frac{\log\log\nu(r,f)}{\log r},$$

where $\nu(r,f)$ denotes the central index of $f(z)$ (for the definition see [7, p. 50]).

LEMMA 2.4 (see [8, Lemma 4]). *Let $f_1,\dots,f_n$ be nonconstant meromorphic functions satisfying*

$$\overline{N}(r,f_i)+\overline{N}(r,1/f_i)=S(r),\qquad i=1,\dots,n,$$

and

$$T(r,f_i)\neq S(r),\qquad T(r,f_i/f_j)\neq S(r),\qquad i\neq j,\ i,j=1,\dots,n.$$

Let $a_0,a_1,\dots,a_m$ $(m\le n)$ be meromorphic functions satisfying $T(r,a_i)=S(r)$, $i=0,1,\dots,m$. If

$$\sum_{i=1}^{m}a_if_i\equiv a_0,$$

then $a_0\equiv a_1\equiv\cdots\equiv a_m\equiv 0$, where $S(r)=o(T(r))$ as $r\to\infty$ and $r\notin E$, and $T(r)=\sum_{i=1}^{n}T(r,f_i)$.

3. Proofs of the theorems

Proof of Theorem 1.1. We discuss the following two cases.

CASE 1. Suppose that

$$\sigma(F)<\infty. \tag{3.1}$$

From (1.1), (3.1) and Lemma 2.1 we easily see that $\nu(F)=0$ and $Q(z)\equiv c$, where c is some complex constant. Thus the conclusion of Theorem 1.1 is valid.

CASE 2. Suppose that

$$\sigma(F) = \infty. \tag{3.2}$$

From (3.2) and Lemma 2.2 we see that

$$\sigma(F) = \limsup_{r\to\infty} \frac{\log \nu(r,F)}{\log r} = \infty. \tag{3.3}$$

Write

$$Q(z) = q_n z^n + q_{n-1} z^{n-1} + \cdots + q_1 z + q_0, \tag{3.4}$$

where q_n $(\neq 0), q_{n-1}, \ldots, q_1$ and q_0 are complex constants. From (3.4) we easily get

$$\lim_{|z|\to\infty} \frac{|Q(z)|}{|q_n z^n|} = 1. \tag{3.5}$$

Hence there exists a sufficiently large positive number r_0 such that

$$\frac{|Q(z)|}{|q_n z^n|} > \frac{1}{e} \qquad (|z| > r_0). \tag{3.6}$$

From (1.1) and (3.6) we easily deduce

$$\begin{aligned} n \log r + \log |q_n| - 1 = \log \frac{|q_n z^n|}{e} &\leq \log |Q(z)| = \log |\log e^{Q(z)}| \\ &\leq |\log \log e^{Q(z)}| \\ &= \left| \log \log \frac{F^{(k)} - 1}{F} \right| \qquad (|z| > r_0), \end{aligned} \tag{3.7}$$

On the other hand, (3.2) implies that F is a nonconstant entire function. Thus

$$M(r,F) \to \infty \qquad \text{as } r \to \infty, \tag{3.8}$$

where $M(r,F) = \max_{|z|=r} |F(z)|$. Again let

$$M(r,F) = |F(z_r)|, \tag{3.9}$$

where $z_r = re^{i\theta(r)}$ and $\theta(r) \in [0, 2\pi)$. From (3.9) and the Wiman–Valiron theory (see [7, Theorem 3.2]), we find that there exists a subset $E \subset (1,\infty)$ with finite logarithmic measure, i.e., $\int_E dt/t < \infty$, such that for some z_r as above satisfying $|z_r| = r \notin E$, we have

$$\frac{F^{(k)}(z_r)}{F(z_r)} = \left(\frac{\nu(r,F)}{z_r}\right)^k (1 + o(1)) \qquad \text{as } r \to \infty, \tag{3.10}$$

where $\nu(r,F)$ denotes the central index of $F(z)$. From (3.7)–(3.10) we easily see that

$$n \log |z_r| + \log |q_n| - 1 \leq \left| \log \log \left(\left(\frac{\nu(r,F)}{z_r} \right)^k (1 + o(1)) \right) \right| \tag{3.11}$$

and

$$\log\left(\left(\frac{\nu(r,F)}{z_r}\right)^k\left(1+o(1)\right)\right) \tag{3.12}$$
$$= k(\log\nu(r,F) - \log re^{i\theta(r)}) + o(1)$$
$$= k(\log\nu(r,F) - \log r - i\theta(r)) + o(1)$$
$$= k\left(1 - \frac{\log r}{\log\nu(r,F)} - \frac{i\theta(r)}{\log\nu(r,F)}\right)\log\nu(r,F) + o(1)$$

as $r\to\infty$. Thus, noting that $\theta(r)\in[0,2\pi)$, from (3.3), (3.12) and Lemma 2.3 we easily deduce

$$\begin{aligned}
&\limsup_{r\to\infty}\frac{|\log\log((\nu(r,F)/z_r)^k(1+o(1)))|}{\log r} \\
&\le \limsup_{r\to\infty}\frac{\log\log\nu(r,F)}{\log r} + \limsup_{r\to\infty}\frac{\left|\log\left(1-\frac{\log r}{\log\nu(r,F)}-\frac{i\theta(r)}{\log\nu(r,F)}\right)\right|}{\log r} \\
&\quad + \lim_{r\to\infty}\frac{\log 2k}{\log r} + \limsup_{r\to\infty}\frac{2k_1\pi}{\log r} \\
&= \limsup_{r\to\infty}\frac{\log\log\nu(r,F)}{\log r} = \nu(F),
\end{aligned} \tag{3.13}$$

where k_1 is some nonnegative integer. Noting that $|z_r| = r$, from (3.11), (3.13) and Lemma 2.3 we easily deduce

$$n \le \limsup_{r\to\infty}\frac{\log\log\nu(r,F)}{\log r} = \nu(F). \tag{3.14}$$

Since $Q(z)$ is the polynomial (3.4), we have

$$\sigma(e^{Q(z)}) = \gamma_Q = n. \tag{3.15}$$

From (3.14) and (3.15) we get

$$\sigma(e^{Q(z)}) \le \nu(F). \tag{3.16}$$

On the other hand, since (1.1) can be rewritten as

$$\frac{F^{(k)}}{F} - \frac{1}{F} = e^Q, \tag{3.17}$$

by substituting (3.9) into (3.17) we get

$$\left(\frac{\nu(r,F)}{z_r}\right)^k(1+o(1)) = e^{Q(z_r)} \quad \text{as } r\to\infty. \tag{3.18}$$

From (3.18) we easily deduce that

$$\limsup_{r\to\infty}\frac{\log\log\nu(r,F)}{\log r}=\limsup_{r\to\infty}\frac{\log\log\frac{(\nu(r,F))^n}{2r^n}}{\log r} \tag{3.19}$$
$$\le\limsup_{r\to\infty}\frac{\log\log\left(\frac{(\nu(r,F))^n}{|z_r|^n}\cdot|1+o(1)|\right)}{\log r}$$
$$\le\limsup_{r\to\infty}\frac{\log\log M(r,e^{Q(z)})}{\log r}.$$

From (3.19), Lemma 2.3 and the definition of the order of an entire function we get

$$\nu(F)\le\sigma(e^{Q(z)}). \tag{3.20}$$

Finally, (3.15), (3.16) and (3.20) yield the conclusion of Theorem 1.1.

Proof of Theorem 1.2. Since f and $f^{(n)}$ share the value a CM, we have

$$\frac{f^{(n)}-a}{f-a}=e^{P(z)}, \tag{3.21}$$

where $P(z)$ is a polynomial. On the other hand, from (1.8), (3.21) and Corollary 1.2 we get

$$e^{P(z)}\equiv c, \tag{3.22}$$
$$e^{Q(z)}\equiv d, \tag{3.23}$$

where c and d are nonzero complex constants. From (1.8) and (3.23) we get

$$\frac{f^{(n+k)}-a}{f^{(n)}-a}=d. \tag{3.24}$$

Let

$$f^{(n)}(z)=g(z). \tag{3.25}$$

From (3.24) and (3.25) we obtain

$$\frac{g^{(k)}-a}{g-a}=d. \tag{3.26}$$

From (3.26) we easily deduce that

$$g^{(k+1)}-dg'=0. \tag{3.27}$$

From (3.27) we obtain the characteristic equation

$$\lambda^{k+1}-d\lambda=0. \tag{3.28}$$

Since the general solution of (3.27) has the form

$$f^{(n)}(z)=g(z)=\sum_{j=1}^{k}\gamma_j e^{\lambda_j z}+b \tag{3.29}$$

with constants γ_j $(1 \le j \le k)$, where $\lambda_1, \ldots, \lambda_k$ are the nonzero solutions of (3.28), and b is a complex constant, it follows that

$$f(z) = \sum_{j=1}^{k} \frac{\gamma_j}{\lambda_j^n} e^{\lambda_j z} + \frac{bz^n}{n!} + \sum_{j=0}^{n-1} b_j z^j, \tag{3.30}$$

where $b_0, b_1, \ldots, b_{n-1}$ are complex constants. On the other hand, (3.21) and (3.22) easily imply

$$f^{(n)} - cf = (1-c)a. \tag{3.31}$$

Substituting (3.29) and (3.30) into (3.31) gives

$$\sum_{j=1}^{k} \left(1 - \frac{c}{\lambda_j^n}\right) \gamma_j e^{\lambda_j z} = \frac{cbz^n}{n!} + \sum_{j=1}^{n-1} cb_j z^j + cb_0 + (1-c)a - b. \tag{3.32}$$

Since $\lambda_1, \ldots, \lambda_k$ are k distinct nonzero complex constants satisfying (3.28) and $c \neq 0$, from (3.28), (3.32) and Lemma 2.4 we easily deduce (1.10) and

$$\lambda_j^n = c \quad (j = 1, \ldots, k), \tag{3.33}$$

$$b_j = b = 0 \quad (1 \le j \le n-1). \tag{3.34}$$

From (3.30), (3.33) and (3.34) we get (1.11). On the other hand, (3.28) and (3.29) yield

$$f^{(n+k)} = \sum_{j=1}^{k} \gamma_j d e^{\lambda_j z}. \tag{3.35}$$

Substituting (3.29) and (3.35) into (3.24) and applying (3.34) we get

$$\frac{\sum_{j=1}^{k} \gamma_j d e^{\lambda_j z} - a}{\sum_{j=1}^{k} \gamma_j e^{\lambda_j z} - a} \equiv d,$$

which implies that $d = 1$. Combining (3.28) and (3.33) gives (1.9). From (1.9)–(1.11) we get the conclusion of Theorem 1.2.

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