ANNALES
POLONICI MATHEMATICI
92.1 (2007)

Non-uniruledness and the cancellation problem (II)

by Robert Dryło (Kraków)

Abstract. We study the following cancellation problem over an algebraically closed field $\mathbb{K}$ of characteristic zero. Let X, Y be affine varieties such that $X \times \mathbb{K}^m \cong Y \times \mathbb{K}^m$ for some m. Assume that X is non-uniruled at infinity. Does it follow that $X \cong Y$? We prove a result implying the affirmative answer in case X is either unirational or an algebraic line bundle. However, the general answer is negative and we give as a counterexample some affine surfaces.

1. Introduction. Let $\mathbb{K}$ be an algebraically closed field of characteristic zero. The *cancellation problem* asks whether two affine varieties X, Y are isomorphic if there exists an isomorphism $X \times \mathbb{K}^m \cong Y \times \mathbb{K}^m$ for some m. To study this problem the following terminology will be useful. A variety X has the *cancellation property* if every variety Y with a given isomorphism $X \times \mathbb{K}^m \cong Y \times \mathbb{K}^m$ is isomorphic to X. Furthermore, a variety X has the *strong cancellation property* if every isomorphism $f\colon X \times \mathbb{K}^m \to Y \times \mathbb{K}^m$ satisfies the condition: for each $x \in X$ there exists $y \in Y$ such that $f(\{x\} \times \mathbb{K}^m) = \{y\} \times \mathbb{K}^m$ (then f clearly induces an isomorphism between X and Y).

It is well known and easy to prove that affine curves have the cancellation property (in fact, a much more general algebraic result was proved by Abhyankar, Eakin and Heinzer in [1]). However, surfaces need not have this property, which was showed by Danielewski in [3] (see also [6] and [13]).

Zariski's cancellation problem asks whether $\mathbb{K}^n$ has the cancellation property. The affirmative answer for $\mathbb{K}^2$ is due to Fujita [7] and Miyanishi–Sugie [17]. This problem remains open for all $n \geq 3$.

Iitaka and Fujita proved in [10] that every variety of non-negative logarithmic Kodaira dimension has the strong cancellation property. Furthermore, it was shown in [4] that also every non-$\mathbb{K}$-uniruled affine variety, and every unirational affine variety non-uniruled at infinity of dimension greater than one, have this property. The aim of the present paper is to extend the last result. First we fix some terminology.

2000 *Mathematics Subject Classification*: Primary 14R10.
Key words and phrases: uniruled variety, cancellation problem, algebraic line bundle.

By a variety we will always mean an algebraic variety.

A variety X of positive dimension n is called *uniruled* (resp. $\mathbb{K}$-*uniruled*) if there exists a variety Y of dimension $n-1$ and a dominant rational map $Y \times \mathbb{P}^1 \dashrightarrow X$ (resp. a dominant morphism $Y \times \mathbb{K} \to X$). A closed subset of a variety is called *uniruled* (resp. $\mathbb{K}$-*uniruled*) if all its irreducible components are uniruled (resp. $\mathbb{K}$-uniruled).

We say that an affine variety X is *non-uniruled at infinity* if for some compactification $\overline{X}$ of X the set $\overline{X} \setminus X$ is non-uniruled. (Note that by a *compactification* of a variety X we mean any projective variety containing X as an open subset. It is well known that for any compactification $\overline{X}$ of an affine variety X the set $\overline{X} \setminus X$ is of pure codimension one in $\overline{X}$.)

Recall that a variety X is called *unirational* if there exists a dominant rational map $\mathbb{P}^n \dashrightarrow X$.

The main result of this paper is

THEOREM 1. *Let X be an affine variety which is either non-$\mathbb{K}$-uniruled or non-uniruled at infinity, unirational and of dimension > 1. Then X has the strong cancellation property, and any algebraic line bundle over X has the cancellation property.*

In this context it is natural to ask whether an affine variety non-uniruled at infinity has the cancellation property. Clearly, the above theorem gives an affirmative answer under some additional assumptions. Furthermore, it was noticed in [4] that the answer is affirmative for every affine variety having at least two components non-uniruled at infinity, since such a variety is non-$\mathbb{K}$-uniruled, which was showed by Jelonek in [12]. However, the general answer turns out to be negative. Namely, using ideas of Danielewski [3] and Fieseler [6] we construct affine surfaces non-uniruled at infinity without the cancellation property. This example may seem quite surprising if we compare it with Theorem 1 and the following result, which arose by considering the stable equivalence problem (see [5]): if H is a non-uniruled hypersurface in a smooth affine variety X and $f\colon X \times \mathbb{K}^m \to Y \times \mathbb{K}^m$ is an isomorphism satisfying $f(H \times \mathbb{K}^m) = H' \times \mathbb{K}^m$, where H' is a hypersurface in the variety Y, then for each $x \in X$ there exists $y \in Y$ such that $f(\{x\} \times \mathbb{K}^m) = \{y\} \times \mathbb{K}^m$.

2. Proof of Theorem 1. In this section π_X denotes the projection $X \times \mathbb{K}^m \ni (x,t) \mapsto x \in X$.

LEMMA 2. *Let $f\colon Y \times \mathbb{K}^m \to X$ be a dominant morphism of affine varieties and assume that* $\dim f(\{b\} \times \mathbb{K}^m) > 0$ *for some $b \in Y$. Then X is $\mathbb{K}$-uniruled. Furthermore, if Y is unirational then X is uniruled at infinity.*

Proof. Let L be a line in $\mathbb{K}^m$ such that $\dim f(\{b\} \times L) > 0$ and $g:$ $(Y \times \mathbb{K}^{m-1}) \times \mathbb{K} \to Y \times \mathbb{K}^m$ an isomorphism satisfying $g(\{b'\} \times \mathbb{K}) = \{b\} \times L$

for some $b' \in Y \times \mathbb{K}^{m-1}$. Then taking the composition $f \circ g$ we may assume that $m = 1$. Now we use induction on $r := \dim Y$. Let $n := \dim X$.

If $r = n - 1$ then X is $\mathbb{K}$-uniruled by definition. Furthermore, if Y is unirational with a dominant rational map $g \colon \mathbb{P}^r \dashrightarrow Y$ then we have a dominant morphism $f \circ (g \times \mathrm{id}_{\mathbb{K}}) \colon U \times \mathbb{K} \to X$, where U is the domain of g. So it follows from [11, Th. 4] (see also [4, Lem.1]) that X is uniruled at infinity, since $\mathbb{P}^r \times \mathbb{P}^1$ is a smooth compactification of $U \times \mathbb{K}$ such that the set $(\mathbb{P}^r \times \mathbb{P}^1) \setminus (U \times \mathbb{K})$ is uniruled.

Assume now that $r \geq n$. Observe that the set of all $y \in Y$ for which $\dim f(\{y\} \times \mathbb{K}) = 0$ is closed in Y, since if X is contained in $\mathbb{K}^N$ and $f = (f_1, \ldots, f_N)$ then the set in question equals $\bigcap_{i=1,\ldots,N} \bigcap_{s,t \in \mathbb{K}} \{y \in Y : f_i(y,s) - f_i(y,t) = 0\}$. Hence after removing some closed subset from Y we may assume that $\dim f(\{y\} \times \mathbb{K}) > 0$ for all $y \in Y$. Furthermore, if Y is unirational, we may also assume that there is an open subset U of $\mathbb{P}^r$ together with a finite morphism from U to Y. Now choose $x \in X$ such that $\dim f^{-1}(x) = r + 1 - n$ and a hypersurface H in Y satisfying $0 \leq \dim(H \cap \pi_Y(f^{-1}(x))) < \dim f^{-1}(x)$, which can be unirational in case Y is unirational. Then $\operatorname{res} f \colon H \times \mathbb{K} \to Y$ is a dominant morphism, since its fiber over x has dimension $r - n$. So the lemma follows from the induction hypothesis. ∎

LEMMA 3. *Let $p_i \colon E_i \to X$ be an algebraic line bundle over a variety X, $i = 1, 2$. Then E_1 and E_2 are isomorphic as algebraic line bundles over X provided there exists an isomorphism $f \colon E_1 \times \mathbb{K}^m \to E_2 \times \mathbb{K}^m$ for which the following diagram is commutative*:

$$\begin{array}{ccc} E_1 \times \mathbb{K}^m & \xrightarrow{\quad f \quad} & E_2 \times \mathbb{K}^m \\ \downarrow{\scriptstyle \pi_{E_1}} & & \downarrow{\scriptstyle \pi_{E_2}} \\ E_1 & & E_2 \\ & {\scriptstyle p_1} \searrow \quad \swarrow {\scriptstyle p_2} & \\ & X & \end{array}$$

Proof. Assume that E_i is given on an open cover $\{U_\alpha\}$ of X by transition functions $g^i_{\alpha,\beta} \colon U_\alpha \cap U_\beta \to \mathbb{K}^*$, $i = 1, 2$. Observe that one can identify $E_i \times \mathbb{K}^m$ with the direct sum of E_i and the trivial bundle $X \times \mathbb{K}^m$. Hence

$$G^i_{\alpha,\beta} = \begin{pmatrix} g^i_{\alpha,\beta} & 0 \\ 0 & I_m \end{pmatrix}$$

are transition functions for $E_i \times \mathbb{K}^m$ on $U_\alpha \cap U_\beta$, where I_m is the identity in $\mathrm{GL}(\mathbb{K}^m)$. Therefore f induces a family of morphisms $f_\alpha \colon U_\alpha \times \mathbb{K}^{m+1} \to \mathbb{K}^{m+1}$ such that $f_\alpha(u, \cdot)$ is an automorphism of $\mathbb{K}^{m+1}$ for each $u \in U_\alpha$ and

$$f_\alpha(u, \cdot) G^1_{\alpha,\beta}(u) = G^2_{\alpha,\beta}(u) f_\beta(u, \cdot) \quad \text{for all } u \in U_\alpha \cap U_\beta.$$

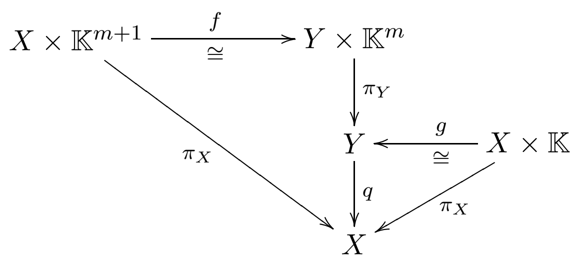
Denote by $h_\alpha(u)$ the Jacobian of $f_\alpha(u,\cdot)$ for $u \in U_\alpha$. Then

$$h_\alpha(u) g^1_{\alpha,\beta}(u) = g^2_{\alpha,\beta}(u) h_\beta(u) \quad \text{for all } u \in U_\alpha \cap U_\beta,$$

which means that the family $\{h_\alpha\}$ determines an isomorphism between E_1 and E_2. ∎

We will also need a solution of the following problem: assuming that R is a ring and A is an R-algebra together with an R-isomorphism of polynomial rings $R[T_1,\dots,T_{n+1}] \cong A[T_1,\dots,T_n]$, we ask if A is R-isomorphic to $R[T_1]$. This problem was studied in several papers. Abhyankar, Eakin and Heinzer gave in [1] an affirmative solution in case R is locally factorial. A little later Asanuma showed in [2] that the answer is affirmative if R is normal, but negative in general. In fact, he showed that the ring $k[T^n, T^{n+1}]$, where $n > 1$ and k is a field of positive characteristic, is a counterexample to this problem. On the other hand, Hamann gave in [8] an affirmative solution for any $\mathbb{Q}$-algebra R. Now we formulate the geometric version of his result and we show how it can be proved directly for smooth varieties.

Lemma 4. *Let $q\colon Y \to X$ be a morphism of affine varieties and $f\colon X \times \mathbb{K}^{m+1} \to Y \times \mathbb{K}^m$ an isomorphism satisfying $\pi_X = q \circ \pi_Y \circ f$. Then there exists an isomorphism $g\colon X \times \mathbb{K} \to Y$ such that $q \circ g = \pi_X$.*



Proof. (As mentioned above, the proof is given under the assumption that X is smooth.) Observe that all fibers of q are isomorphic to $\mathbb{K}$, since f carries $\pi_X^{-1}(x) \cong \mathbb{K}^{m+1}$ onto $q^{-1}(x) \times \mathbb{K}^m$, and affine curves have the cancellation property. Furthermore, if $s_0\colon X \ni x \mapsto (x,0) \in X \times \mathbb{K}^{m+1}$ is the null section then the map $s\colon X \ni x \mapsto \pi_Y(f(s_0(x))) \in Y$ is a section of q, i.e. $q \circ s = \mathrm{id}_X$. Now we claim that on Y one can introduce a structure of an algebraic line bundle over X with projection q and zero section s, which concludes the proof by Lemma 3.

To see this observe that the induced map $q^*\colon \operatorname{Pic}(X) \to \operatorname{Pic}(Y)$ is an isomorphism, since the maps $\pi_X^*\colon \operatorname{Pic}(X) \to \operatorname{Pic}(X \times \mathbb{K}^{m+1})$ and $\pi_Y^*\colon \operatorname{Pic}(Y) \to \operatorname{Pic}(Y \times \mathbb{K}^m)$ are isomorphisms. So for a prime divisor $\Gamma := s(X)$ on Y there exists a divisor D on X such that Γ and $q^*(D)$ are linearly equivalent (recall that on a smooth variety every divisor is locally principal). Let $\{U_i\}$ be an open affine cover of X such that $D \cap U_i$ is principal in U_i. Then

$q^*(D) \cap q^{-1}(U_i)$ is principal in $q^{-1}(U_i)$ and hence so is $\Gamma \cap q^{-1}(U_i)$. This implies that the ideal of the set $\Gamma \cap q^{-1}(U_i)$ is principal in the coordinate ring $\mathbb{K}[q^{-1}(U_i)]$; say it is generated by $F_i \in \mathbb{K}[q^{-1}(U_i)]$. Since $q^{-1}(x) \cong \mathbb{K}$ and Γ intersects $q^{-1}(x)$ transversally and at only one point, it follows that the restriction of F_i to $q^{-1}(x)$ is a coordinate for each $x \in U_i$. Now consider the map $q^{-1}(U_i) \ni y \mapsto (q(y), F_i(y)) \in U_i \times \mathbb{K}$. It is obviously bijective and hence an isomorphism by Zariski's Main Theorem. Now using these maps we introduce on Y the claimed structure of a line bundle. ∎

We will need one more elementary fact: if X and Y are affine varieties and an isomorphism $f\colon X \times \mathbb{K}^m \to Y \times \mathbb{K}^m$ is given then X dominates Y (in particular, if X is unirational then so is Y). To see this, choose a point $y \in Y$ and a morphism $p\colon X \to \mathbb{K}^m$ such that the intersection of its graph with $f^{-1}(\{y\} \times \mathbb{K}^m)$ has a component of dimension zero. Then the morphism $X \ni x \mapsto \pi_Y(f(x, p(x))) \in Y$ is dominant, since its fiber over y has a component of dimension zero.

Proof of Theorem 1. The first statement is an immediate consequence of Lemma 2. To prove the second part take an algebraic line bundle over X, $p\colon E \to X$, and an isomorphism $f\colon Y \times \mathbb{K}^m \to E \times \mathbb{K}^m$. By Lemma 2 the composition $p \circ \pi_E \circ f$ contracts subvarieties of the form $\{y\} \times \mathbb{K}^m$ to a point, for all $y \in Y$. This means that there exists a morphism $q\colon Y \to X$ making the diagram

$$\begin{array}{ccc} Y \times \mathbb{K}^m & \xrightarrow{\ f\ } & E \times \mathbb{K}^m \\ & & \downarrow{\scriptstyle \pi_E} \\ \downarrow{\scriptstyle \pi_Y} & & E \\ & & \downarrow{\scriptstyle p} \\ Y & \xrightarrow{\ q\ } & X \end{array}$$

commutative. If E is trivial over an open affine subset U of X then $\operatorname{res} q\colon q^{-1}(U) \to U$ is a trivial bundle by Lemma 4. Furthermore, as in the proof of Lemma 4 we show that q has a section $s\colon X \to Y$. These imply that on Y one can introduce a structure of an algebraic line bundle over X with projection q and zero section s. Now Lemma 3 concludes the proof. ∎

Remark 5. Theorem 1 remains true if we assume that $\operatorname{Reg} X$ is either non-$\mathbb{K}$-uniruled or unirational of dimension greater than 1 and has a non-uniruled hypersurface at infinity. (Here and in what follows, we denote by $\operatorname{Reg} X$ the set of all nonsingular points of a variety X. Furthermore, we say that a variety X has a *non-uniruled hypersurface at infinity* if for some compactification $\overline{X}$ of X the set $\overline{X} \setminus X$ has a non-uniruled irreducible component of codimension one in $\overline{X}$.) The above proof works also in this case, we only need to modify Lemma 1 slightly. Furthermore, the following obvious fact will be needed: every isomorphism $f\colon Y\times\mathbb{K}^m\to X\times\mathbb{K}^m$ induces the isomorphism $\operatorname{res} f\colon \operatorname{Reg} Y\times\mathbb{K}^m\to\operatorname{Reg} X\times\mathbb{K}^m$. The details are left to the reader.

3. Final remarks. Now applying ideas of Danielewski–Fieseler we give the announced example of affine surfaces non-uniruled at infinity without the cancellation property.

Example 6. Let X be a smooth non-rational affine curve. Assume that f and g are regular functions on X vanishing only at a point $x_0\in X$. Put $X_1=X_2=X$ and $U_1=U_2=X\setminus\{x_0\}$. Let V be the surface obtained by gluing $X_1\times\mathbb{K}$ and $X_2\times\mathbb{K}$ via the isomorphism $U_1\times\mathbb{K}\ni(x,t)\mapsto(x,t+1/f(x))\in U_2\times\mathbb{K}$. Let W be the surface obtained in the same manner as V by using g instead of f. Then V and W are affine surfaces non-uniruled at infinity, $V\times\mathbb{K}\cong W\times\mathbb{K}$, but V is not isomorphic to W in case $\operatorname{ord}_{x_0}(f)\neq\operatorname{ord}_{x_0}(g)$.

To show that V is affine consider the function

$$H(x,t):=\begin{cases} f(x)t+1, & (x,t)\in X_1\times\mathbb{K},\\ f(x)t, & (x,t)\in X_2\times\mathbb{K}.\end{cases}$$

It induces a morphism $h\colon V\to\mathbb{K}$ such that the sets $V\setminus h^{-1}(0)\cong X_1\times\mathbb{K}\setminus\{(x,t):f(x)t+1=0\}$ and $V\setminus h^{-1}(1)\cong X_2\times\mathbb{K}\setminus\{(x,t):f(x)t=1\}$ are affine. This implies that h is an affine morphism and consequently V is an affine surface.

From [11, Th. 4] it follows that V is non-uniruled at infinity.

To show that $V\times\mathbb{K}\cong W\times\mathbb{K}$ denote by $\widetilde{X}$ the curve X with a doubled x_0, i.e. $\widetilde{X}$ is obtained by gluing X_1 and X_2 along U_1 and U_2 via the identity. Clearly, V and W with the natural projections onto the prevariety $\widetilde{X}$ are principal $\mathbb{K}^+$-bundles over $\widetilde{X}$. Since the fiber product $V\times_{\widetilde{X}}W$ is a principal $\mathbb{K}^+$-bundle over both V and W, we have isomorphisms $V\times\mathbb{K}\cong V\times_{\widetilde{X}}W\cong W\times\mathbb{K}$ (this follows from the fact that isomorphism classes of principal $\mathbb{K}^+$-bundles over a variety Y are in one-to-one correspondence with elements of the group $H^1(Y,\mathcal{O}_Y)$, which is trivial in case Y is affine).

Now suppose that an isomorphism $\varphi\colon V\to W$ is given. Since X is non-rational we have the induced automorphism $\widetilde{\varphi}$ of $\widetilde{X}$ for which the diagram

$$\begin{array}{ccc} V & \xrightarrow{\ \varphi\ } & W\\ {\scriptstyle\mathrm{pr}}\big\downarrow & & \big\downarrow{\scriptstyle\mathrm{pr}}\\ \widetilde{X} & \xrightarrow{\ \widetilde{\varphi}\ } & \widetilde{X}\end{array}$$

is commutative. Let x_i denote the image of x_0 under the canonical embedding of X_i into $\widetilde{X}$, $i=1,2$. Observe that each automorphism of $\widetilde{X}$ carries the set $\{x_1, x_2\}$ onto itself, since every open subset of $\widetilde{X}$ not containing $\{x_1, x_2\}$ is separated.

In case $\widetilde{\varphi}(x_i) = x_i$ we have two induced automorphisms $\widetilde{\varphi}_i$ of X_i such that $\widetilde{\varphi}_i(x_0) = x_0$, $i = 1, 2$, and two other automorphisms φ_i of $X_i \times \mathbb{K}$ sending (x, t) to $(\widetilde{\varphi}_i(x), \alpha_i(x)t + \beta_i(x))$, where $\beta_i \in \mathbb{K}[X_i]$ and α_i is a unit in $\mathbb{K}[X_i]$, and making the diagram

$$\begin{array}{ccc} U_1 \times \mathbb{K} & \xrightarrow{(x,t)\mapsto(x,t+1/f(x))} & U_2 \times \mathbb{K} \\ {\scriptstyle\varphi_1}\downarrow & & \downarrow{\scriptstyle\varphi_2} \\ U_1 \times \mathbb{K} & \xrightarrow{(x,t)\mapsto(x,t+1/g(x))} & U_2 \times \mathbb{K} \end{array}$$

commutative. This gives the equality

$$\alpha_1(x)t + \beta_1(x) + \frac{1}{g(\widetilde{\varphi}_1(x))} = \alpha_2(x)\left(t + \frac{1}{f(x)}\right) + \beta_2(x),$$

whence

$$\frac{1}{g(\widetilde{\varphi}_1(x))} - \frac{\alpha_2(x)}{f(x)} = \beta_2(x) - \beta_1(x) \in \mathbb{K}[X].$$

Since α_2 is a unit we get

$$\operatorname{ord}_{x_0}(f) = \operatorname{ord}_{x_0}(g).$$

Similarly, in case $\widetilde{\varphi}(x_1) = x_2$ two isomorphisms $\varphi_1 \colon X_1 \times \mathbb{K} \to X_2 \times \mathbb{K}$ and $\varphi_2 \colon X_2 \times \mathbb{K} \to X_1 \times \mathbb{K}$ are induced for which the diagram

$$\begin{array}{ccc} U_1 \times \mathbb{K} & \xrightarrow{(x,t)\mapsto(x,t+1/f(x))} & U_2 \times \mathbb{K} \\ {\scriptstyle\varphi_1}\downarrow & & \downarrow{\scriptstyle\varphi_2} \\ U_2 \times \mathbb{K} & \xrightarrow{(x,t)\mapsto(x,t-1/g(x))} & U_1 \times \mathbb{K} \end{array}$$

is commutative. It again follows that $\operatorname{ord}_{x_0}(f) = \operatorname{ord}_{x_0}(g)$. So we have shown that our example is correct.

Finally, we want to ask the following question: given an affine variety X with the strong cancellation property, does it follow that $X \times \mathbb{K}$ has the cancellation property? Clearly, the answer is affirmative if X satisfies the assumptions of Theorem 1. This question was considered by Asanuma in [2], who gave a negative answer in the case of positive characteristic. His counterexample is the already mentioned rational curve with the coordinate ring $k[T^n, T^{n+1}]$, where $n > 1$. On the other hand, in characteristic zero we have the following

PROPOSITION 7. *If X and Y are affine curves then the surface $X \times Y$ has the cancellation property.*

Proof. The hardest case $X \cong Y \cong \mathbb{K}$ is done, since $\mathbb{K}^2$ has the cancellation property. If X is not isomorphic to $\mathbb{K}$ then $\operatorname{Reg} X$ is non-$\mathbb{K}$-uniruled, since every smooth affine and $\mathbb{K}$-uniruled curve is isomorphic to $\mathbb{K}$, and every non-constant morphism from $\mathbb{K}$ to an affine curve is finite and hence surjective. So $X \times \mathbb{K}$ has the cancellation property by Remark 5. Similarly, if neither $X \cong \mathbb{K}$ nor $Y \cong \mathbb{K}$ then the set $\operatorname{Reg}(X \times Y) = (\operatorname{Reg} X) \times (\operatorname{Reg} Y)$ is non-$\mathbb{K}$-uniruled and hence $X \times Y$ has the strong cancellation property, again by Remark 5. ■

References

[1] S. S. Abhyankar, P. Eakin and W. J. Heinzer, *On the uniqueness of the coefficient ring in a polynomial ring*, J. Algebra 23 (1972), 310–342.

[2] T. Asanuma, *On the strongly invariant coefficient rings*, Osaka J. Math. 11 (1974), 587–593.

[3] W. Danielewski, *On the cancellation problem and automorphism groups of affine algebraic varieties*, preprint, Warszawa, 1989.

[4] R. Dryło, *Non-uniruledness and the cancellation problem*, Ann. Polon. Math. 87 (2005), 93–98.

[5] —, *On the stable equivalence problem*, preprint IMUJ, 2005/14.

[6] K. H. Fieseler, *On complex affine surfaces with $\mathbb{C}^+$-action*, Comment. Math. Helv. 69 (1994), 5–27.

[7] T. Fujita, *On Zariski problem*, Proc. Japan Acad. Ser. A. Math. Sci. 55 (1979), 106–110.

[8] E. Hamann, *On the R-invariance of $R[X]$*, J. Algebra 35 (1975), 1–16.

[9] R. Hartshorne, *Algebraic Geometry*, Springer, Berlin, 1977.

[10] S. Iitaka and T. Fujita, *Cancellation theorem for algebraic varieties*, J. Fac. Sci. Univ. Tokyo 24 (1977), 123–127.

[11] Z. Jelonek, *Irreducible identity sets for polynomial automorphisms*, Math. Z. 212 (1993), 601–617.

[12] —, *Testing sets for properness of polynomial mappings*, Math. Ann. 315 (1999), 1–35.

[13] L. Makar-Limanov, *On the group of automorphisms of a surface $x^n y = P(z)$*, Israel J. Math. 121 (2001), 113–123.

[14] L. Makar-Limanov, P. van Rossum, V. Shpilrain and J. T. Yu, *The stable equivalence and cancellation problems*, Comment. Math. Helv. 79 (2004), 341–349.

[15] M. Miyanishi, *Some remarks on polynomial rings*, Osaka J. Math. 10 (1973), 617–624.

[16] M. Miyanishi and Y. Nakai, *Some remarks on strongly invariant rings*, ibid. 12 (1975), 1–17.

[17] M. Miyanishi and T. Sugie, *Affine surfaces containing cylinderlike open sets*, J. Math. Kyoto Univ. 20 (1980), 11–42.

Institute of Mathematics
Jagiellonian University
Reymonta 4
30-059 Kraków, Poland
E-mail: R.Drylo@impan.gov.pl

Received 10.5.2006
and in final form 29.12.2006 (1751)