BULLETIN OF THE POLISH
ACADEMY OF SCIENCES
MATHEMATICS
Vol. 56, No. 1, 2008

*PROBABILITY THEORY AND STOCHASTIC PROCESSES*

# On Talagrand's Admissible Net Approach to Majorizing Measures and Boundedness of Stochastic Processes

by

Witold BEDNORZ

*Presented by Stanisław KWAPIEŃ*

**Summary.** We show that the main result of [1] on sufficiency of existence of a majorizing measure for boundedness of a stochastic process can be naturally split in two theorems, each of independent interest. The first is that the existence of a majorizing measure is sufficient for the existence of a sequence of admissible nets (as recently introduced by Talagrand [5]), and the second that the existence of a sequence of admissible nets is sufficient for sample boundedness of a stochastic process with bounded increments.

**1. Introduction.** Let $(T,d)$ be a compact metric space, and let $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ be a *Young function*, i.e. convex, increasing, continuous and such that $\varphi(0)=0$. We say that a stochastic process $X(t)$, $t\in T$, *has bounded increments* if

$$\mathbf{E}\varphi\left(\frac{|X(s)-X(t)|}{d(s,t)}\right) \le 1 \quad \text{for } s,t\in T, \tag{1}$$

Without losing generality one can assume that $\varphi$ is normalized, i.e. $\varphi(1)=1$. Note that under (1) there exists a separable modification of $X(t)$, $t\in T$, which we always refer to when considering a process with bounded increments.

We say that a Borel probability measure $m$ on $(T,d)$ is *majorizing* if

$$\mathcal{M}(m,\varphi) := \sup_{t\in T} \int_0^{D(t,T)} \varphi^{-1}\left(\frac{1}{m(B(t,\varepsilon))}\right) d\varepsilon < \infty, \tag{2}$$

2000 *Mathematics Subject Classification*: 60G17, 28A99.
*Key words and phrases*: majorizing measure, sample boundedness.
Partially supported by Grant MENiN 1 P03A 01229.

and *weakly majorizing* if

$$\overline{\mathcal{M}}(m,\varphi) := \int\limits_T \int\limits_0^{D(t,T)} \varphi^{-1}\Big(\frac{1}{m(B(t,\varepsilon))}\Big)\, d\varepsilon\, m(dt) < \infty,$$

where $B(t,\varepsilon) := \{s \in T : d(s,t) \le \varepsilon\}$ and $D(t,T) := \sup\{d(s,t) : s \in T\}$. The concept of majorizing measure was introduced by Fernique [2] for the purpose of proving boundedness of stochastic processes. For the historical background on the sample boundedness of stochastic processes under the bounded increment assumption we refer to [2], [3] and [5]. The following theorem proved in [1] is a generalization of Fernique's result as well as Talagrand's:

THEOREM 1. *If $\varphi$ is a Young function and $m$ a majorizing measure on $T$ then, for each separable stochastic process $X(t)$, $t \in T$, which satisfies* (1),

$$\mathbf{E} \sup_{s,t\in T} |X(s) - X(t)| \le 32\mathcal{M}(m,\varphi).$$

In this paper we pursue a new approach to Theorem 2 using the language of admissible nets (cf. Definition 1.2.3 in [3]). Below we give a definition of admissible nets suitable for our purposes. Let $(N_k)_{k\ge0}$ be a sequence of positive reals such that $N_0 = 1$ and

$$c\varphi^{-1}(N_k) \le \varphi^{-1}(N_{k+1}) \le C\varphi^{-1}(N_k) \quad \text{for } k \ge 1, \tag{3}$$

where $2 < c \le C$ (the usual choice is $N_k := \varphi(R^k)$, where $R > 2$). We will say that $\mathcal{T} := (T_k)_{k\ge0}$ is an *admissible sequence of nets* if $|T_k| \le N_k$ and

$$\mathcal{A}(\mathcal{T},\varphi) := \sup_{u\in T} \sum_{k=0}^{\infty} d(u,T_k)\varphi^{-1}(N_k) < \infty,$$

$$\overline{\mathcal{A}}(\mathcal{T},\varphi) := \sum_{k=0}^{\infty} \sum_{u\in T_{k+1}} \frac{d(u,T_k)\varphi^{-1}(N_k)}{N_{k+1}} < \infty.$$

Theorem 1 can be obtained as a corollary of the following two theorems, which are of independent interest:

THEOREM 2. *For each sequence of admissible nets $\mathcal{T} = (T_k)_{k\ge0}$ and any stochastic process $X(t)$, $t \in T$, satisfying* (1),

$$\mathbf{E} \sup_{s,t\in T} |X(s) - X(t)| \le \frac{4cC}{c-2}\,\mathcal{A}(\mathcal{T},\varphi) + 2C\overline{\mathcal{A}}(\mathcal{T},\varphi). \tag{4}$$

THEOREM 3. *If $(T,d)$ admits a majorizing measure $m$ then there exists a sequence of nets $\mathcal{T} = (T_k)_{k\ge0}$ such that $|T_k| \le N_k$ for $k \ge 0$ and*

$$\mathcal{A}(\mathcal{T},\varphi) \le \frac{4c}{c-1}\,\mathcal{M}(\mathcal{T},\varphi), \qquad \overline{\mathcal{A}}(\mathcal{T},\varphi) \le \frac{4c}{c-1}\,\overline{\mathcal{M}}(m,\varphi).$$

Indeed, since clearly $\overline{\mathcal{M}}(m,\varphi) \le \mathcal{M}(m,\varphi)$, Theorems 2 and 3 show that the existence of a majorizing measure implies the sample boundedness of any stochastic process with bounded increments, so in this way we reprove Theorem 1.

**2. Sample boundedness via admissible nets.** Let $\pi_k(t)$ be any point in $T_k$ which satisfies $d(t,T_k) = d(t,\pi_k(t))$, i.e. a point in $T_k$ closest to $t$.

*Proof of Theorem 2.* Fix $l \ge 0$ and $t \in T$. Clearly one may assume that $\lim_{k\to\infty} d(t,T_k) = 0$ since otherwise the right hand side in (4) is infinite and there is nothing to prove. We define $t_l = \pi_l(t)$ and by reverse induction, $t_k = \pi_k(t_{k+1})$. By the chain argument we obtain

$$|f(t_l) - f(t_0)| \le \sum_{j=0}^{l-1} |f(t_j) - f(t_{j+1})|. \tag{5}$$

For all Young functions $\varphi$ we clearly have

$$\frac{x}{y} \le 1 + \frac{\varphi(x)}{\varphi(y)}, \qquad x,y > 0. \tag{6}$$

Setting $x = |f(t_j) - f(t_{j+1})|/d(t_j,t_{j+1})$ and $y = \varphi^{-1}(N_{j+1})$ in (6), we derive that

$$\frac{|f(t_j) - f(t_{j+1})|}{d(t_j,t_{j+1})\varphi^{-1}(N_{j+1})} \le 1 + \frac{1}{N_{j+1}}\varphi\biggl(\frac{|f(t_j) - f(t_{j+1})|}{d(t_j,t_{j+1})}\biggr).$$

Since by (3) we have $\varphi^{-1}(N_{j+1}) \le C\varphi^{-1}(N_j)$, we can see that

$$|f(t_j) - f(t_{j+1})| \le Cd(t_j,t_{j+1})\varphi^{-1}(N_j)\biggl(1 + \frac{1}{N_{j+1}}\varphi\biggl(\frac{|f(t_j) - f(t_{j+1})|}{d(t_j,t_{j+1})}\biggr)\biggr).$$

This implies that

$$\begin{aligned} |f(t_l) - f(t_0)| &\le C\sum_{j=0}^{l-1} d(t_j,t_{j+1})\varphi^{-1}(N_j) \\ &\quad + C\sum_{k=0}^{\infty}\sum_{u\in T_{k+1}} \frac{d(u,T_k)\varphi^{-1}(N_k)}{N_{k+1}}\varphi\biggl(\frac{|f(u) - f(\pi_k(u))|}{d(u,\pi_k(u))}\biggr). \end{aligned} \tag{7}$$

LEMMA 1. *The following inequality holds:*

$$\sum_{j=0}^{l-1} d(t_j,t_{j+1})\varphi^{-1}(N_j) \le \frac{2c}{c-2}\sum_{k=0}^{l} d(t,\pi_k(t))\varphi^{-1}(N_k).$$

*Proof.* We first show that for each $0 \le j \le l$ we have

$$d(t,t_j)\varphi^{-1}(N_j) \le \sum_{k=j}^{l} \left(\frac{2}{c}\right)^{j-k} d(t,T_k)\varphi^{-1}(N_k), \tag{8}$$

where $c$ is the constant in (3). The proof goes by reverse induction. The case $j = l$ is trivial, so we may assume that

$$d(t,t_{j+1})\varphi^{-1}(N_{j+1}) \le \sum_{k=j+1}^{l} \left(\frac{2}{c}\right)^{j+1-k} d(t,T_k)\varphi^{-1}(N_k). \tag{9}$$

Note that the definition of $\pi_j$ implies that

$$d(t_j,t_{j+1}) = d(\pi_j(t_{j+1}),t_{j+1}) \le d(\pi_j(t),t_{j+1}) \le d(t,\pi_j(t)) + d(t,t_{j+1}),$$

which combined with $d(t,t_j) \le d(t,t_{j+1}) + d(t_j,t_{j+1})$ results in

$$d(t,t_j) \le d(t,\pi_j(t)) + 2d(t,t_{j+1}).$$

From (3) we obtain

$$\begin{aligned} d(t,t_j)\varphi^{-1}(N_j) &\le (d(t,\pi_j(t)) + 2d(t,\pi_{j+1}(t)))\varphi^{-1}(N_j) \\ &\le d(t,\pi_j(t))\varphi^{-1}(N_j) + \frac{2}{c}\, d(t,\pi_{j+1}(t))\varphi^{-1}(N_{j+1}). \end{aligned}$$

The induction assumption (9) now yields (8). We finish the proof of the lemma by first checking that

$$\sum_{j=0}^{l-1} d(t_j,t_{j+1})\varphi^{-1}(N_j) \le 2\sum_{j=0}^{l} d(t,t_j)\varphi^{-1}(N_j) \tag{10}$$

and then applying (8) so that

$$\begin{aligned} \sum_{j=0}^{l} d(t,t_j)\varphi^{-1}(N_j) &\le \sum_{j=0}^{l}\Bigg(\sum_{k=j}^{l}\left(\frac{c}{2}\right)^{j-k} d(t,\pi_k(t))\varphi^{-1}(N_k)\Bigg) \\ &\le \sum_{k=0}^{l}\Bigg(\sum_{j=k}^{l}\left(\frac{c}{2}\right)^{j-k}\Bigg) d(t,\pi_k(t))\varphi^{-1}(N_k) \\ &\le \frac{c}{c-2}\sum_{k=0}^{l} d(t,\pi_k(t))\varphi^{-1}(N_k). \ \blacksquare \end{aligned}$$

We use (7) and Lemma 1 to show that

$$\begin{aligned} |f(t_l) - f(t_0)| \le \frac{2cC}{c-2}&\sum_{k=0}^{l} d(t,T_k)\varphi^{-1}(N_k) \\ &+ C\sum_{k=0}^{\infty}\sum_{u\in T_{k+1}} \frac{d(u,T_k)\varphi^{-1}(N_k)}{N_{k+1}}\varphi\bigg(\frac{|f(u)-f(\pi_k(u))|}{d(u,\pi_k(u))}\bigg). \end{aligned}$$

From the property $\lim_{k\to\infty} d(t,T_k)=0$ we deduce that

$$|f(t)-f(t_0)| \le \frac{4cC}{c-2}\sup_{u\in T}\sum_{k=0}^{\infty} d(u,T_k)\varphi^{-1}(N_k) + 2C\sum_{k=0}^{\infty}\sum_{u\in T_{k+1}} \frac{d(u,T_k)\varphi^{-1}(N_k)}{N_{k+1}}\varphi\Big(\frac{|f(u)-f(\pi_k(u))|}{d(u,\pi_k(u))}\Big).$$

Since $t_0=\pi_0(T)$ is the only point in $T_0$ which does not depend on $t$, it is clear that for any $s,t\in T$ we have

$$(11)\qquad |f(s)-f(t)| \le \frac{4cC}{c-2}\sup_{u\in T}\sum_{k=0}^{\infty} d(u,T_k)\varphi^{-1}(N_k) + 2C\sum_{k=0}^{\infty}\sum_{u\in T_{k+1}} \frac{d(u,T_k)\varphi^{-1}(N_k)}{N_{k+1}}\varphi\Big(\frac{|f(u)-f(\pi_k(u))|}{d(u,\pi_k(u))}\Big).$$

Having thus established the result for any continuous functions $f$ on $(T,d)$ we turn to its stochastic version. By a standard argument (see Theorem 2.3 of [3] or Theorem 3.1 of [1]) it suffices to prove Theorem 2 for processes with a.s. Lipschitz samples (with respect to $d$). By the Fubini theorem and (1) we obtain

$$\begin{aligned}\mathbf{E}\sup_{s,t\in T}|X(s)-X(t)| &\le \frac{4cC}{c-2}\sup_{u\in T}\sum_{k=0}^{\infty} d(u,T_k)\varphi^{-1}(N_k) \\ &\quad + 2C\sum_{k=0}^{\infty}\sum_{u\in T_{k+1}} \frac{d(u,T_k)\varphi^{-1}(N_k)}{N_{k+1}}\,\mathbf{E}\varphi\Big(\frac{|f(u)-f(\pi_k(u))|}{d(u,\pi_k(u))}\Big) \\ &\le \frac{4cC}{c-2}\sup_{u\in T}\sum_{k=0}^{\infty} d(u,T_k)\varphi^{-1}(N_k) + 2C\sum_{k=0}^{\infty}\sum_{u\in T_{k+1}} \frac{d(u,T_k)\varphi^{-1}(N_k)}{N_{k+1}}. \quad \blacksquare\end{aligned}$$

**3. Construction of a sequence of admissible nets.** We describe how to construct a sequence of admissible nets when we have a majorizing measure $m$ on $(T,d)$ (thus in particular $\mathrm{supp}(m)=T$). Let

$$r_k(t) := \inf\{\varepsilon>0 : m(B(t,\varepsilon))\ge 1/N_k\},$$

where $(N_k)_{k\ge0}$ satisfies (3). Clearly $m(B(t,r_k(t)))\ge 1/N_k$ and $r_0(t)\le D(t,T)$. In [1] two simple properties of $r_k$ are given; we repeat their proofs for completeness.

LEMMA 2. *The functions $r_k$, $k\ge0$, are 1-Lipschitz for all $t\in T$.*

*Proof.* A geometrical argument shows that

$$B(s, r_k(t) + d(s,t)) \supset B(t, r_k(t)),$$

and consequently $m(B(s, r_k(t) + d(s,t))) \geq 1/N_k$. Hence $r_k(s) \leq r_k(t) + d(s,t)$ and similarly $r_k(t) \leq r_k(s) + d(s,t)$, which implies that $r_k$ is 1-Lipschitz. ∎

LEMMA 3. *For each* $0 \leq \delta \leq D(t,T)$ *we have*

$$\sum_{k=0}^{\infty} \min\{r_k(t), \delta\}\varphi^{-1}(N_k) \leq \frac{c}{c-1}\int_0^{\delta} \varphi^{-1}\left(\frac{1}{m(B(t,\varepsilon))}\right) d\varepsilon.$$

*Proof.* Observe that there exists $k_0 \geq 0$ such that $r_{k_0+1}(t) < \delta \leq r_{k_0}(t)$. Clearly

$$\int_{r_{k+1}(t)}^{r_k(t)} \varphi^{-1}\left(\frac{1}{m(B(t,\varepsilon))}\right) d\varepsilon \geq (r_k(t) - r_{k+1}(t))\varphi^{-1}(N_k),$$

and in the same way we show that

$$\int_{r_{k_0+1}(t)}^{\delta} \varphi^{-1}\left(\frac{1}{m(B(t,\varepsilon))}\right) d\varepsilon \geq (\delta - r_{k_0+1}(t))\varphi^{-1}(N_{k_0}).$$

Thus using (3) we deduce that

$$\begin{aligned}
&\int_0^{\delta} \varphi^{-1}\left(\frac{1}{m(B(t,\varepsilon))}\right) d\varepsilon \\
&\qquad \geq \sum_{k=k_0+1}^{\infty} (r_k(t) - r_{k+1}(t))\varphi^{-1}(N_k) + (\delta - r_{k_0+1}(t))\varphi^{-1}(N_{k_0}) \\
&\qquad \geq \sum_{k=k_0+1}^{\infty} r_k(t)(\varphi^{-1}(N_k) - \varphi^{-1}(N_{k-1})) + \delta\varphi^{-1}(N_{k_0}) \\
&\qquad \geq \frac{c-1}{c} \sum_{k=k_0+1}^{\infty} r_k(t)\varphi^{-1}(N_k) + \delta\varphi^{-1}(N_{k_0}).
\end{aligned}$$

Since

$$\sum_{k=0}^{k_0} \varphi^{-1}(N_k) \leq \sum_{k=0}^{k_0} c^{-k}\varphi^{-1}(N_{k_0}) \leq \frac{c}{c-1}\,\varphi^{-1}(N_{k_0})$$

we finally obtain

$$\int_0^{\delta} \varphi^{-1}\left(\frac{1}{m(B(t,\varepsilon))}\right) d\varepsilon \geq \frac{c-1}{c} \sum_{k=0}^{\infty} \min\{r_k(t), \delta\}\varphi^{-1}(N_k). \quad ∎$$

The construction of a sequence of admissible nets $\mathcal{T} = (T_k)_{k\geq 0}$, assuming the existence of a majorizing measure, is based on the following intermediate result:

THEOREM 4. *There exists a sequence of nets $\mathcal{T} = (T_k)_{k\geq 0}$, $T_k \subset T$, that satisfies the following conditions*:

1. $|T_0| = 1$, $|T_k| \leq N_k$;
2. $B(t, r_k(t))$ *are disjoint for* $t \in T_k$;
3. *for each* $t \in T$ *we have* $d(t, T_k) \leq 4r_k(t)$;
4. $r_k(t) \leq 2r_k(x)$ *for each* $t \in T_k$ *and* $x \in B(t, r_k(t))$.

*Proof.* Fix $k \geq 0$. We define $t_1$ as a minimum point of $r_k$, that is, $r_k(t_1) = \inf_{t\in T} r_k(t)$ (we use the fact that $(T, d)$ is compact). Then we define an open subset $A_1$ in $T$ by

$$A_1 := \{s \in T : 2(r_k(s) + r_k(t_1)) > d(s, t_1)\}.$$

Suppose we have constructed points $t_1, \ldots, t_l$ and open sets $A_1, \ldots, A_l$. If $T \setminus \bigcup_{j=1}^{l} A_j$ is non-empty, then we define $t_{l+1}$ as a minimum point of $r_k$ on this set (which is again compact), and set

$$A_{l+1} := \{s \in T : 2(r_k(s) + r_k(t_{l+1})) > d(s, t_{l+1})\}.$$

Note that by the definition $d(t_j, t_l) \geq 2(r_k(t_j) + r_k(t_l))$ if $j \neq l$, and hence $B(t_j, r_k(t_j))$ and $B(t_l, r_k(t_l))$ are disjoint. It follows that

$$1 = m(T) \geq \sum_{j=1}^{|T_k|} m(B(t_j, r_k(t_j))) \geq \frac{|T_k|}{N_k}.$$

Thus $|T_k| \leq N_k$, which implies that our construction stops after a finite number of steps. Clearly $N_0 = 1$ implies that $|T_0| = 1$. For each $t \in T$ there exists the smallest $l = l_0$ such that $t \in A_l$. By the construction we have $2(r_k(t) + r_k(t_{l_0})) > d(t, t_{l_0})$ and $r_k(t_{l_0}) \leq r_k(t)$, hence $d(t, T_k) < 4r_k(t)$.

To prove the last assertion we consider $x \in B(t_{l_0}, r_k(t_{l_0}))$ with $t_{l_0} \in T_k$. There exists the smallest $l = l_1$ such that $x \in A_l$; if $l_0 \leq l_1$ then $r_k(t_{l_0}) \leq r_k(t_{l_1}) \leq r_k(x)$, which ends the proof in this case. If $l_1 < l_0$, then $t_{l_0} \notin \bigcup_{j=1}^{l_1} A_j$ and so $d(t_{l_0}, t_{l_1}) \geq 2(r_k(t_{l_0}) + r_k(t_{l_1}))$. Consequently, by the triangle inequality,

$$2(r_k(t_{l_0}) + r_k(t_{l_1})) \leq d(t_{l_0}, t_{l_1}) \leq d(x, t_{l_1}) + d(x, t_{l_0}) \leq d(x, t_{l_1}) + r_k(t_{l_0}),$$

where the last inequality follows because $x \in B(t_{l_0}, r_k(t_{l_0}))$. On the other hand, $x \in A_{l_1}$, so

$$d(x, t_{l_1}) < 2(r_k(x) + r_k(t_{l_1})).$$

It follows that

$$2(r_k(t_{l_0}) + r_k(t_{l_1})) \le d(x, t_{l_1}) + r_k(t_{l_0}) \le 2(r_k(x) + r_k(t_{l_1})) + r_k(t_{l_0}),$$

and hence $r_k(t_{l_0}) \le 2r_k(x)$ as desired. ■

*Proof of Theorem 3.* By Theorem 4 there exists an admissible net $\mathcal{T} = (T_k)_{k\ge 0}$ such that $d(t, T_k) \le 4r_k(t)$. Consequently, Lemma 3 shows that for each $t \in T$ we have

$$\begin{aligned}\sum_{k=0}^{\infty} d(t, T_k)\varphi^{-1}(N_k) &\le 4\sum_{k=0}^{\infty} r_k(t)\varphi^{-1}(N_k)\\ &\le \frac{4c}{c-1}\int_0^{D(t,T)} \varphi^{-1}\Big(\frac{1}{m(B(t,\varepsilon))}\Big)\, d\varepsilon,\end{aligned}$$

which implies that

$$\sup_{t\in T}\sum_{k=0}^{\infty} d(t, T_k)\varphi^{-1}(N_k) \le \frac{4c}{c-1}\,\mathcal{M}(m, \varphi).$$

To show the second claim we first check that since $1/N_{k+1} \le m(B(t, r_{k+1}(t)))$ and $d(t, T_k) \le 4r_k(t)$ one can see that

$$\sum_{t\in T_{k+1}} \frac{d(t, T_k)\varphi^{-1}(N_k)}{N_{k+1}} \le 4\sum_{t\in T_k}\int_{B(t,r_{k+1}(t))} r_k(t)\varphi^{-1}(N_k)\, m(dx). \tag{12}$$

By the Lipschitz property of $r_k$ (Lemma 2) we derive that $r_k(t) \le r_k(x) + r_{k+1}(t)$ for $x \in B(t, r_{k+1}(t))$. Therefore

$$\int_{B(t,r_{k+1}(t))} r_k(t)\varphi^{-1}(N_k)\, m(dx) \le \int_{B(t,r_{k+1}(t))} (r_k(x) + r_{k+1}(t))\varphi^{-1}(N_k)\, m(dx).$$

The last assertion in Theorem 4 implies that $r_{k+1}(t) \le 2r_{k+1}(x)$ for any $t \in T_{k+1}$ and $x \in B(t, r_{k+1}(t))$. Hence

$$\int_{B(t,r_{k+1}(t))} r_k(t)\varphi^{-1}(N_k)\, m(dx) \le \int_{B(t,r_{k+1}(t))} (r_k(x) + 2r_{k+1}(x))\varphi^{-1}(N_k)\, m(dx).$$

Since $B(t, r_{k+1}(t))$ are disjoint for $t \in T_k$ (the second claim in Theorem 4), we derive

$$\sum_{t\in T_k}\int_{B(t,r_{k+1}(t))} r_k(t)\varphi^{-1}(N_k)\, m(dx) \le 2\int_T (r_k(x) + 2r_{k+1}(x))\varphi^{-1}(N_k)\, m(dx).$$

Combining the above inequality with (12) and (3) (with $c > 2$) we deduce that

$$\sum_{k=0}^{\infty} \frac{d(t, T_k)\varphi^{-1}(N_k)}{N_{k+1}} \le 4\int_T \sum_{k=0}^{\infty} r_k(x)\varphi^{-1}(N_k)\, m(dx).$$

It remains to use Lemma 3, which yields

$$\sum_{k=0}^{\infty} \frac{d(t,T_k)\varphi^{-1}(N_k)}{N_{k+1}} \le \frac{4c}{c-1}\,\overline{\mathcal{M}}(m,\varphi). \ \blacksquare$$

**References**

[1] W. Bednorz, *A theorem on majorizing measures*, Ann. Probab. 34 (2006), 1771–1781.
[2] X. Fernique, *Régularité des trajectoires des fonctions aléatoires gaussiennes*, in: École d'Été de Probabilités de Saint-Flour, IV-1974, Lecture Notes in Math. 480, Springer, Berlin, 1975, 1–96.
[3] M. Talagrand, *Sample boundedness of stochastic processes under increment conditions*, Ann. Probab. 18 (1990), 1–49.
[4] —, *Majorizing measures without measures*, ibid. 29 (2001), 411–417.
[5] —, *The Generic Chaining. Upper and Lower Bounds of Stochastic Processes*, Springer, Berlin, 2005.

Witold Bednorz
Department of Mathematics
University of Warsaw
Banacha 2
02-097 Warszawa, Poland
E-mail: wbednorz@mimuw.edu.pl

*Received October 3, 2007;*
*received in final form May 16, 2008* (7620)