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REAL FUNCTIONS

On Lipschitz Selections of Multifunctions with Decomposable Values

by

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Summary. Some conditions for existence of Lipschitz selections of multifunctions with decomposable values are given.

1. Introduction. The definition of a decomposable set was formulated in the 1960's. Decomposability is an essential property in many problems associated with differential inclusions. This notion appears in papers of Cz. Olech, F. Hiai–H. Umegaki and A. Fryszkowski. The characteristic feature of decomposable sets is their similarity to convex sets. It appears that we can replace convexity by decomposability in many classical theorems. In this paper we consider the problem of existence of a Lipschitz selection of a multifunction with decomposable values.

2. Preliminaries. In this section we give basic definitions and quote some known facts needed in this paper. Let $(X, \|\cdot\|)$ be a separable Banach space, (Ω, Σ, μ) be a measure space with a complete measure and $\mathcal{L}_p(\Omega, X)$ be the Banach space of equivalence classes of Σ-measurable functions $f : \Omega \to X$ with the norm $\|f\|_{\mathcal{L}_p} = (\int_\Omega \|f\|^p \, d\mu)^{1/p} < \infty$ for $1 \le p < \infty$. We denote the characteristic function of a set $A \in \Sigma$ by χ_A.

Definition 1. A set $\mathcal{K} \subset \mathcal{L}_p$ is called *decomposable* if for any $u, v \in \mathcal{K}$ and each $A \in \Sigma$, $\chi_A u + \chi_{\Omega \setminus A} v \in \mathcal{K}$.

F. Hiai and H. Umegaki showed that a decomposable set can be represented as a set of selections of some multifunction [4].

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Theorem 1 . *Let* (Ω, Σ, μ) *be a* σ*-finite measure space and* $1 \leq p < \infty$*. If* $\mathcal{K} \subset \mathcal{L}_p(\Omega, X)$ *is a nonempty closed and decomposable set, then there exists a weakly measurable multifunction* $F : \Omega \to 2^X$ *with nonempty and closed values such that* $\mathcal{K} = \{f \in \mathcal{L}_p(\Omega, X) : f(\omega) \in F(\omega)$ *a.e. in* $\Omega\}$.

Sets of selections of multifunctions have several obvious properties:

Theorem 2. *Let* (Ω, Σ, μ) *be a* σ*-finite measure space and let* $F : \Omega \to 2^X$ *be a weakly measurable multifunction with closed nonempty values. Then the set* $\{f \in \mathcal{L}_p(\Omega, X) : f(\omega) \in F(\omega)$ *a.e. in* $\Omega\}$ *is bounded if and only if there exists* $\rho \in \mathcal{L}_1(\Omega, \mathbb{R})$ *such that* $\sup_{x \in F(\omega)} \|x\|^p \leq \rho(\omega)$ *for every* $\omega \in \Omega$.

Theorem 3. *Let* (Ω, Σ, μ) *be a* σ*-finite measure space and* $F : \Omega \to 2^X$ *be a weakly measurable multifunction with closed nonempty values. Then the set* $\{f \in \mathcal{L}_p(\Omega, X) : f(\omega) \in F(\omega)$ *a.e. in* $\Omega\}$ *is convex if and only if* $F(\omega)$ *is convex for almost all* $\omega \in \Omega$.

Now let (T, d), (Z, d_1) be metric spaces and $F : T \to 2^Z$ be a multifunction with nonempty values.

Definition 2. A selection η of the multifunction F is called a *Lipschitz selection* if there exists $L > 0$ such that $d_1(\eta(t_1), \eta(t_2)) \leq Ld(t_1, t_2)$ for all $t_1, t_2 \in T$.

Definition 3. A multifunction with closed, nonempty and bounded values is called *h-Lipschitz* if there exists $L > 0$ such that

$$h_Z(F(t_1), F(t_2)) \leq Ld(t_1, t_2) \quad \text{for all } t_1, t_2 \in T,$$

where h_Z denotes the Hausdorff distance of subsets of Z.

We also have the following theorem due to A. A. and D. A. Tolstonogov [7].

Theorem 4. *Let* (Ω, Σ, μ) *be a* σ*-finite measure space. Let* $\mathcal{K}_1, \mathcal{K}_2 \in \mathcal{L}_p(\Omega, X)$ *be nonempty, closed and bounded decomposable sets. If* F_1, F_2 *are weakly measurable multifunctions with closed values such that* $\mathcal{K}_1 = \{f \in \mathcal{L}_p(\Omega, X) : f(\omega) \in F_1(\omega)$ *a.e. in* $\Omega\}$ *and* $\mathcal{K}_2 = \{f \in \mathcal{L}_p(\Omega, X) : f(\omega) \in F_2(\omega)$ *a.e. in* $\Omega\}$, *then*

$$h_{\mathcal{L}_p}(\mathcal{K}_1, \mathcal{K}_2) \leq \Big(\int_\Omega (h_X(F_1(\omega), F_2(\omega)))^p \, d\mu\Big)^{1/p} \leq 2^{1/p} h_{\mathcal{L}_p}(\mathcal{K}_1, \mathcal{K}_2).$$

One of the main problems in the theory of multifunctions is the existence of Lipschitz selections. It is known that every h-Lipschitz multifunction with nonempty, closed, bounded and convex values in $\mathbb{R}^n$ has a Lipschitz selection. This is no longer true in infinite-dimensional spaces. However, we have the following theorem [5, Th. 4.33].

THEOREM 5. *Let (T, d) be a compact metric space and Z be a normed space. If $F : T \to 2^Z \setminus \{\emptyset\}$ is a multifunction with convex values such that $F^{-1}(\{z\})$ is an open set in T for each $z \in Z$ then F admits a Lipschitz selection.*

3. Main results. It turns out that in the space $\mathcal{L}_1(\Omega, X)$, decomposability is a good substitute of convexity. We shall prove the following theorem:

THEOREM 6. *Let (T, d) be a compact metric space. Let (Ω, Σ, μ) be a maeasure space with a finite nonatomic measure. If $\mathcal{F} : T \to 2^{\mathcal{L}_1(\Omega,X)} \setminus \{\emptyset\}$ is a multifunction with decomposable values such that the set $\mathcal{F}^{-1}(\{v\})$ is open in T for every $v \in \mathcal{L}_1(\Omega, X)$, then $\mathcal{F}$ has a Lipschitz selection.*

Before proving Theorem 6 we state a lemma.

LEMMA 1. *Suppose that (Ω, Σ, μ) is a measure space with a finite measure and $n \in \mathbb{N}$. Let $A_1, \ldots, A_n \in \Sigma$ be pairwise disjoint with $\bigcup_{i=1}^n A_i = \Omega$. Similarly, let $B_1, \ldots, B_n \in \Sigma$ be pairwise disjoint with $\bigcup_{i=1}^n B_i = \Omega$. Define $C_s = \bigcup_{i=1}^s A_i$ and $D_s = \bigcup_{i=1}^s B_i$ for $s = 1, \ldots, n$. If $C_s \subset D_s$ or $D_s \subset C_s$ for each $s = 1, \ldots, n$, then*

$$\sum_{i,k=1,\, i\neq k}^{n} \mu(A_k \cap B_i) \leq \sum_{i=1}^{n} |\mu(D_i) - \mu(C_i)|.$$

Proof. It is easy to see that

$$\sum_{i,k=1,\, i\neq k}^{n} \mu(A_k \cap B_i) = \mu(\Omega) - \sum_{i=1}^{n} \mu(A_i \cap B_i).$$

Moreover,

$$\Omega \subset \Big(\bigcup_{i=1}^{n} A_i \cap B_i\Big) \cup \Big(\bigcup_{i=1}^{n} D_i \triangle C_i\Big)$$

where $A \triangle B = (A \setminus B) \cup (B \setminus A)$, and the lemma follows. ■

To make the proof of Theorem 6 clearer we introduce some notation and state some simple properties. Fix $n \in \mathbb{N}$. Define a norm in $\mathbb{R}^n$ by

$$\|\vec{x}\|_1 = |x_1| + |x_1 + x_2| + \cdots + |x_1 + \cdots + x_n|$$

for $\vec{x} = (x_1, \ldots, x_n) \in \mathbb{R}^n$. Let μ_1 be a nonatomic finite measure on Ω, to be defined later. Set

$$D = \Big\{(x_1, \ldots, x_n) \in \mathbb{R}^n : x_i \geq 0 \text{ for } i = 1, \ldots, n;\ \sum_{i=1}^{n} x_i = \mu_1(\Omega)\Big\},$$

$$D_{\{i_1,\ldots,i_k\}} = \Big\{(x_1, \ldots, x_n) \in \mathbb{R}^n : x_{i_1} = \ldots = x_{i_k} = 0;$$
$$x_j \geq 0 \text{ for } j \notin \{i_1, \ldots, i_k\};\ \sum_{i=1}^{n} x_i = \mu_1(\Omega)\Big\},$$

where $\{i_1,\dots,i_k\} \subsetneq \{1,\dots,n\}$. We allow $\{i_1,\dots,i_k\}$ to be an empty set. It is easy to show directly that $D_{\{i_1,\dots,i_k\}}$ is nonempty and convex for all $\{i_1,\dots,i_k\} \subsetneq \{1,\dots,n\}$.

It is known that there exists a function $\xi : [0,\mu_1(\Omega)] \to \Sigma$ with the following properties (see [3]):

(i) $\xi(0) = \emptyset$,
(ii) $\xi(\mu_1(\Omega)) = \Omega$,
(iii) if $\alpha,\beta \in [0,\mu_1(\Omega)]$ and $\alpha < \beta$, then $\xi(\alpha) \subset \xi(\beta)$,
(iv) $\mu_1(\xi(\alpha)) = \alpha$ for $\alpha \in [0,\mu_1(\Omega)]$.

For any $\vec{x} = (x_1,\dots,x_n) \in D$ we define a partition of Ω into subsets

$$A_{1,\vec{x}} = \xi(x_1), \qquad A_{2,\vec{x}} = \xi(x_1+x_2) \setminus \xi(x_1),$$
$$A_{i,\vec{x}} = \xi(x_1+\dots+x_i) \setminus \xi(x_1+\dots+x_{i-1}), \qquad i = 1,\dots,n,$$

and define a function $H : D \to \Sigma^n$ by

$$H(\vec{x}) = (A_{1,\vec{x}},\dots,A_{n,\vec{x}}) \quad \text{for } \vec{x} \in D.$$

Notice that for $\vec{x} = (x_1,\dots,x_n) \in D$ we get:

(w.1) If $x_i = 0$, then $A_{i,\vec{x}} = \emptyset$, $i = 1,\dots,n$.
(w.2) $\bigcup_{i=1}^{s} A_{i,\vec{x}} = \xi(x_1+\dots+x_s)$ for $s = 1,\dots,n$.
(w.3) $\mu_1(A_{i,\vec{x}}) = x_i$ for $i = 1,\dots,n$ and $\mu_1(\bigcup_{i=1}^{s} A_{i,\vec{x}}) = \sum_{i=1}^{s} x_i$ for $s = 1,\dots,n$.

Proof of Theorem 6. Set $\mathcal{C} = \{\mathcal{F}^{-1}(\{g\}) : g \in \mathcal{F}(T)\}$, where $\mathcal{F}(T) = \bigcup_{t\in T} \mathcal{F}(t)$. Notice that $\mathcal{C}$ is an open cover of T. By compactness there exists an $n \in \mathbb{N}$ and $g_1,\dots,g_n \in \mathcal{F}(T)$ such that $T = \bigcup_{i=1}^{n} \mathcal{F}^{-1}(\{g_i\})$. Define $h : \Omega \to \mathbb{R}_+ \cup \{0\}$ by $h(\omega) = \max\{\|g_i(\omega) - g_j(\omega)\| : 1 \le i,j \le n,\}$, $\omega \in \Omega$. The function h is obviously measurable and

$$\int_\Omega h(\omega)\,d\mu \le \int_\Omega \sum_{i=1}^{n}\sum_{j=1}^{n} \|g_i(\omega) - g_j(\omega)\|\,d\mu \le \sum_{i=1}^{n}\sum_{j=1}^{n} (\|g_i\|_{\mathcal{L}_1} + \|g_j\|_{\mathcal{L}_1}) < \infty.$$

Hence $h \in \mathcal{L}_1(\Omega,\mathbb{R})$. Consider a nonatomic finite measure $\mu_1 : \Sigma \to \mathbb{R}$ given by $\mu_1(A) = \int_A h(\omega)\,d\mu$, $A \in \Sigma$. Let $G : T \to 2^{\mathbb{R}^n}$ be a multifunction such that for $t \in T$,

$$G(t) = D_{\{i_1,\dots,i_k\}}$$
$$\text{if } g_{i_1},\dots,g_{i_k} \notin \mathcal{F}(t) \text{ and } g_j \in \mathcal{F}(t) \text{ for } j \in \{1,\dots,n\} \setminus \{i_1,\dots,i_k\}.$$

Because $\bigcup_{i=1}^{n} \mathcal{F}^{-1}(\{g_i\}) = T$ the multifunction G is correctly defined and furthermore it has nonempty and convex values.

We will show that G has a Lipschitz selection. If $\vec{x} = (x_1,\dots,x_n) \in D$, where $x_{i_1} = \dots = x_{i_k} = 0$ and $x_j \ne 0$ for $j \in \{1,\dots,n\} \setminus \{i_1,\dots,i_k\} = \{j_1,\dots,j_s\}$, then the conditions $\vec{x} \in G(t)$ and $g_{j_1},\dots,g_{j_s} \in \mathcal{F}(t)$ are equivalent for each $t \in T$. Hence $G^{-1}(\vec{x}) = \bigcap_{l=1}^{s} \mathcal{F}^{-1}(\{g_{j_l}\})$ is an open set in T. If

$\vec{x} \notin D$, then $G^{-1}(\vec{x}) = \emptyset$ is also an open set. Thus G satisfies the assumptions of Theorem 5 and hence there exists a Lipschitz selection γ with respect to the norm $\|\cdot\|_1$.

Now let $J : \Sigma^n \to \mathcal{L}_1(\Omega, X)$ be defined by

$$J(A_1, \ldots, A_n) = \sum_{i=1}^{n} \chi_{A_i} g_i \quad \text{for } (A_1, \ldots, A_n) \in \Sigma^n.$$

We will show that $\eta = J \circ H \circ \gamma$ is a Lipschitz selection of $\mathcal{F}$.

First we show that η is a selection of $\mathcal{F}$. Let $t \in T$ and $\gamma(t) = \vec{x} = (x_1, \ldots, x_n) \in D$, where $x_{i_1} = \cdots = x_{i_k} = 0, x_{j_1}, \ldots, x_{j_s} \neq 0$, $\{i_1, \ldots, i_k\} \cup \{j_1, \ldots, j_s\} = \{1, \ldots, n\}$. Then $g_{j_1}, \ldots, g_{j_s} \in \mathcal{F}(t)$. Notice that $H(\gamma(t)) = (A_{1,\vec{x}}, \ldots, A_{n,\vec{x}})$ is a measurable partition of Ω, and only the sets $A_{j_1,\vec{x}}, \ldots, A_{j_s,\vec{x}}$ are nonempty (property (w.1)). Thus $J(H(\gamma(t))) = \sum_{l=1}^{s} \chi_{A_{j_l,\vec{x}}} g_{j_l} \in \mathcal{F}(t)$ and so η is a selection of $\mathcal{F}$.

It remains to show that η is a Lipschitz function. Let $t_1, t_2 \in T, \gamma(t_1) = \vec{x} \in D, \gamma(t_2) = \vec{y} \in D$. Then $\eta(t_1) = \sum_{i=1}^{n} \chi_{A_i} g_i$ and $\eta(t_2) = \sum_{i=1}^{n} \chi_{B_i} g_i$, where $(A_1, \ldots, A_n) = H(\vec{x})$ and $(B_1, \ldots, B_n) = H(\vec{y})$ are measurable partitions of Ω (we allow some of these sets to be empty). We get

$$\begin{aligned}
\|\eta(t_1) - \eta(t_2)\|_{\mathcal{L}_p} &= \int_{\Omega} \Big\| \sum_{i=1}^{n} \chi_{A_i} g_i - \sum_{i=1}^{n} \chi_{B_i} g_i \Big\| \, d\mu \\
&= \sum_{i=1}^{n} \sum_{j=1}^{n} \int_{A_i \cap B_j} \|g_i - g_j\| \, d\mu = \sum_{i,j=1,\, i\neq j}^{n} \int_{A_i \cap B_j} \|g_i - g_j\| \, d\mu \\
&\le \sum_{i,j=1,\, i\neq j}^{n} \int_{A_i \cap B_j} h(\omega) \, d\mu = \sum_{i,j=1,\, i\neq j}^{n} \mu_1(A_i \cap B_j).
\end{aligned}$$

Denoting $\vec{x} = (x_1, \ldots, x_n) \in D$, $\vec{y} = (y_1, \ldots, y_n) \in D$, $C_s = \bigcup_{i=1}^{s} A_i$, $D_s = \bigcup_{i=1}^{s} B_i, s = 1, \ldots, n$, we conclude that

$$C_s = \xi(x_1 + \cdots + x_s), \qquad D_s = \xi(y_1 + \cdots + y_s)$$

(property (w.2)), hence $C_s \subset D_s$ or $D_s \subset C_s$ (condition (iii)). Thus by Lemma 1,

$$\|\eta(t_1) - \eta(t_2)\|_{\mathcal{L}_1} \le \sum_{i=1}^{n} |\mu_1(D_i) - \mu_1(C_i)|.$$

Simultaneously from the definition of $\|\cdot\|_1$ and property (w.3) it follows that

$$\begin{aligned}
&\|\gamma(t_1) - \gamma(t_2)\|_1 \\
&= |x_1 - y_1| + |(x_1 + x_2) - (y_1 + y_2)| + \cdots + |(x_1 + \cdots + x_n) - (y_1 + \cdots + y_n)| \\
&= \sum_{i=1}^{n} |\mu_1(C_i) - \mu_1(D_i)|.
\end{aligned}$$

Hence $\|\eta(t_1)-\eta(t_2)\|_{\mathcal{L}_1} \leq \|\gamma(t_1)-\gamma(t_2)\|_1$. Because γ is a Lipschitz function, so is η, which completes the proof. ∎

From Theorem 6 we obtain the following corollary.

COROLLARY 1. *Under the assumptions of Theorem* 6 *for each point* (t, v) *belonging to the graph of the multifunction* $\mathcal{F}$, *there exists a Lipschitz selection* η *of* $\mathcal{F}$ *such that* $\eta(t) = v$.

Proof. Fix $t_0 \in T$ and $v \in \mathcal{F}(t_0)$. It is enough to notice that a multifunction $\mathcal{F}_1 : T \to 2^{\mathcal{L}_1(\Omega,X)}$ such that $\mathcal{F}_1(t_0) = v$, $\mathcal{F}_1(t) = \mathcal{F}(t)$ for $t \in T \setminus \{t_0\}$ satisfies the assumptions of Theorem 6. Thus $\mathcal{F}_1$ has a Lipschitz selection η. Obviously $\eta(t_0) = v$. ∎

As stated earlier, closedness, boundness and convexity of values of an h-Lipschitz multifunction in an infinite-dimensional space do not guarantee that the multifunction has a Lipschitz selection. It turns out that it is enough to assume additionally that the values of $\mathcal{F}$ are decomposable in $\mathcal{L}_p(\Omega, \mathbb{R}^n)$ to deduce the existence of such a selection.

THEOREM 7. *Let* (Ω, Σ, μ) *be a* σ*-finite measure space and* (T, d) *be a metric space. If* $\mathcal{F} : T \to 2^{\mathcal{L}_p(\Omega,\mathbb{R}^n)}, n \in \mathbb{N}$, *is an h-Lipschitz multifunction with nonempty*, *closed*, *convex*, *bounded and decomposable values*, *then* $\mathcal{F}$ *has a Lipschitz selection.*

Let P be the space of all compact nonempty subsets of $(\mathbb{R}^n, \|\cdot\|)$ with the Hausdorff distance, and P_c be the space of compact nonempty and convex subsets of $\mathbb{R}^n$ with the same distance.

Let $s(A)$ denote the Steiner point of a set $A \in P_c$. To prove Theorem 7 we are going to use the following properties of Steiner points [6]:

(x) $s(A) \in A$ for $A \in P_c$.

(xx) There exists $L \in \mathbb{R}_+$ such that $\|s(A) - s(B)\| \leq L h_{\mathbb{R}^n}(A, B)$ for all $A, B \in P_c$.

Proof of Theorem 7. Let $F : T \times \Omega \to 2^{\mathbb{R}^n}$ be a multifunction with nonempty and closed values such that for each $t \in T$, $\mathcal{F}(t, \cdot)$ is a weakly measurable multifunction and $\mathcal{F}(t) = \{f \in \mathcal{L}_p(\Omega, X) : f(\omega) \in F(t, \omega)\ \mu\text{-a.e. in } \Omega\}$. Such an F exists (Th. 1), and without loss of generality, we can assume that its values are compact and convex (Ths. 2, 3). Fix $t \in T$ and define $g_t : \Omega \to \mathbb{R}^n$ by

$$g_t(\omega) = s(F(t, \omega)), \qquad \omega \in \Omega.$$

We have $g_t(\omega) \in F(t, \omega)$ for each $\omega \in \Omega$ (property (x)).

We will show that g_t is a measurable function. Let $\mathbb{U}$ be a fixed open subset of $\mathbb{R}^n$. Define $\Delta : P_c \to \mathbb{R}^n$ by $\Delta(A) = s(A)$ for $A \in P_c$, and $\Gamma : \Omega \to P_c$ by $\Gamma(\omega) = F(t, \omega)$ for $\omega \in \Omega$. Thus $g_t = \Delta \circ \Gamma$. By the property (xx) it follows that the function Δ is continuous and so $\Delta^{-1}(\mathbb{U})$ is open in P_c. Let $B(A,\varepsilon)$ denote the open ball in P with center $A \in P$ and radius $\varepsilon > 0$, and $B_c(A,\varepsilon)$ the open ball in P_c. Let $V \in P$ be the union of all balls $B(A,\varepsilon)$ such that $B_c(A,\varepsilon) \subset \Delta^{-1}(\mathbb{U})$. Notice that V is open in P and $V \cap P_c = \Delta^{-1}(\mathbb{U})$. Thus $g_t^{-1}(\mathbb{U}) = \Gamma^{-1}(\Delta^{-1}(\mathbb{U})) = \Gamma^{-1}(V)$. Because the multifunction $F(t,\cdot)$ is weakly measurable and has compact values, the function Γ is measurable [1, Th. III.2] and $\Gamma^{-1}(V) \in \Sigma$. These facts imply that g_t is a measurable function.

Hence, by Theorem 2, $g_t \in \mathcal{L}_p(\Omega,\mathbb{R}^n)$ and consequently $g_t \in \mathcal{F}(t)$. Now define $\eta : T \to \mathcal{L}_p(\Omega,\mathbb{R}^n)$ by $\eta(t) = g_t$. Fix $t_1, t_2 \in T$. Using the definition of η, property (xx) and Theorem 4 we obtain

$$\begin{aligned}\|\eta(t_1) - \eta(t_2)\|_{\mathcal{L}_p} &= \Big(\int_\Omega \|g_{t_1}(\omega) - g_{t_2}(\omega)\|^p \, d\mu\Big)^{1/p} \\ &\le \Big(\int_\Omega (Lh_{\mathbb{R}^n}(F(t_1,\omega), F(t_2,\omega)))^p \, d\mu\Big)^{1/p} \le 2^{1/p} Lh_{\mathcal{L}_p}(\mathcal{F}(t_1), \mathcal{F}(t_2)).\end{aligned}$$

Because the multifunction $\mathcal{F}$ is h-Lipschitz the above inequalities mean that the function η is a Lipschitz selection. ∎

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